Notes on Intuitionistic Fuzzy Sets
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Trigonometric-based operators over intuitionistic fuzzy sets

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Abstract: In the present paper we introduce two new operators over intuitionistic fuzzy sets and study their properties.
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1 Introduction

Intuitionistic fuzzy sets (IFS) were introduced in 1983 by K. Atanassov [1] as an extension of the fuzzy sets. Fuzzy sets, introduced by L. Zadeh in 1965 [6], generalized the classical characteristic function $\chi_A(x)$ by proposing a membership degree for a given element of the set $\mu_A(x)$. IFS further introduced a non-membership degree $\nu_A(x)$ which reflects the extent to which an element does not belong to the set. The complement of the sum of the membership and non-membership degrees to 1 ($\pi_A(x)$) is called *hesitancy degree* or *index of indeterminacy* [2]. A formal definition is the following:

Definition 1 (cf. [1, 3, 5]). Let X be a universe set, $A \subset X$. Then an intuitionistic fuzzy set generated by the set A is an object of the form:

$$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{1.1}$$

where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ are mappings, such that for any $x \in X$,

$$\mu_A(x) + \nu_A(x) \leq 1. \tag{1.2}$$

Let us further denote the class of all IFSs over the same universe X, by IFS(X). If S is a mapping S : IFS$(X) \to$ IFS(X), we call S an operator defined over IFS(X).

Some examples of operators previously defined include but are not limited to (see [5]):

$$D_\alpha(A^*) = \{\langle x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + (1-\alpha)\pi_A(x)\rangle | x \in X\},$$

where $\alpha \in [0, 1]$;

$$F_{\alpha,\beta}(A^*) = \{\langle x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x)\rangle | x \in X\},$$

where $\alpha, \beta, \alpha + \beta \in [0, 1]$;

$$G_{\alpha,\beta}(A^*) = \{\langle x, \alpha\mu_A(x), \beta\nu_A(x)\rangle | x \in X\},$$

where $\alpha, \beta \in [0, 1]$.

In what follows we propose two new operators based on the trigonometric functions $\sin(t)$ and $\cos(t)$.

2 The proposed operators

First we start with the following simple observations:

- In the interval $[0, \frac{\pi}{2}]$, the function $\cos(t)$ is non-negative and strictly decreasing.
- In the interval $[0, \frac{\pi}{2}]$, the function $1 - \sin(t)$ is non-negative and strictly decreasing.
- Both function are bounded from below by 0 and from above by 1 on the interval $[0, \frac{\pi}{2}]$.

Another key point to note is that due to (1.2), we have $\mu_A(x) \leq 1 - \nu_A(x)$ and $\nu_A(x) \leq 1 - \mu_A(x)$, which means that:

$$\cos\left(\mu_A(x)\frac{\pi}{2}\right) \geq \cos\left((1 - \nu_A(x))\frac{\pi}{2}\right) \tag{2.1}$$

$$1 - \sin\left(\mu_A(x)\frac{\pi}{2}\right) \geq 1 - \sin\left((1 - \nu_A(x))\frac{\pi}{2}\right). \tag{2.2}$$

Now we are ready to define our two operators. Let A^* be the IFS defined by (1.1).

Definition 2. We define the operator $Z_{\cos}$: IFS$(X) \to$ IFS(X) as follows:

$$Z_{\cos}(A^*) = \{\langle x, \frac{\sqrt{2}}{2}\cos\left((1 - \nu_A(x))\frac{\pi}{2}\right), \frac{\sqrt{2}}{2}\cos\left((1 - \mu_A(x))\frac{\pi}{2}\right)\rangle | x \in X\} \tag{2.3}$$

Definition 3. We define the operator $Z_{\sin} : \mathrm{IFS}(X) \to \mathrm{IFS}(X)$ as follows:

$$Z_{\sin}(A^*) = \{\langle x, 1 - \sin\left((1-\nu_A(x))\frac{\pi}{2}\right), 1 - \sin\left((1-\mu_A(x))\frac{\pi}{2}\right)\rangle | x \in X\} \tag{2.4}$$

We will show that $Z_{\cos}(A^*) \in \mathrm{IFS}(X)$ and $Z_{\sin}(A^*) \in \mathrm{IFS}(X)$, i.e. the operators are correctly defined. It is clear that

$$\frac{\sqrt{2}}{2}\cos\left((1-\nu_A(x))\frac{\pi}{2}\right) + \frac{\sqrt{2}}{2}\cos\left((1-\mu_A(x))\frac{\pi}{2}\right) \geq 0$$

since $0 \leq (1-\nu_A(x))\frac{\pi}{2} \leq \frac{\pi}{2}$ and $0 \leq (1-\mu_A(x))\frac{\pi}{2} \leq \frac{\pi}{2}$. So it remains only to show that

$$\frac{\sqrt{2}}{2}\cos\left((1-\nu_A(x))\frac{\pi}{2}\right) + \frac{\sqrt{2}}{2}\cos\left((1-\mu_A(x))\frac{\pi}{2}\right) \leq 1.$$

We have (due to (2.1))

$$\cos\left((1-\nu_A(x))\frac{\pi}{2}\right) + \cos\left((1-\mu_A(x))\frac{\pi}{2}\right) \leq \cos\left((1-\nu_A(x))\frac{\pi}{2}\right) + \cos\left(\nu_A(x)\frac{\pi}{2}\right).$$

It is easy to see that

$$\cos\left((1-\nu_A(x))\frac{\pi}{2}\right) + \cos\left(\nu_A(x)\frac{\pi}{2}\right) \leq \sqrt{2}.$$

Thus, we obtain that

$$\frac{\sqrt{2}}{2}\left[\cos\left((1-\nu_A(x))\frac{\pi}{2}\right) + \cos\left((1-\mu_A(x))\frac{\pi}{2}\right)\right] \leq \frac{\sqrt{2}}{2}\sqrt{2} \leq 1$$

and hence (2.3) is an IFS.

Similarly, (due to (2.2))

$$2 - \sin\left((1-\nu_A(x))\frac{\pi}{2}\right) - \sin\left((1-\mu_A(x))\frac{\pi}{2}\right) \leq 2 - \sin\left((\mu_A(x))\frac{\pi}{2}\right) - \sin\left((1-\mu_A(x))\frac{\pi}{2}\right)$$

We can easily see that:

$$2 - \sin\left((\mu_A(x))\frac{\pi}{2}\right) - \sin\left((1-\mu_A(x))\frac{\pi}{2}\right) = 2 - \left[\sin\left(\mu_A(x)\frac{\pi}{2}\right) + \cos\left(\mu_A(x)\frac{\pi}{2}\right)\right] \leq 1.$$

and hence (2.4) is an IFS.

In order to see how these operators behave we consider the following intuitionistic fuzzy set

$$B = \{\langle x_1, 0.67, 0.3\rangle, \langle x_2, 0.6, 0.37\rangle, \langle x_3, 0.4, 0.45\rangle, \langle x_4, 0.2, 0.73\rangle, \langle x_5, 0.9, 0.05\rangle, \langle x_6, 0.5, 0.5\rangle\}$$

We have

$$Z_{\cos}(B) = \{\langle x_1, 0.321, 0.614\rangle, \langle x_2, 0.388, 0.572\rangle, \langle x_3, 0.459, 0.415\rangle, \langle x_4, 0.644, 0.218\rangle, \langle x_5, 0.055, 0.698\rangle, \langle x_6, 0.5, 0.5\rangle\}$$

$$Z_{\sin}(B) = \{\langle x_1, 0.108, 0.504\rangle, \langle x_2, 0.164, 0.412\rangle, \langle x_3, 0.239, 0.190\rangle, \langle x_4, 0.588, 0.489\rangle, \langle x_5, 0.003, 0.843\rangle, \langle x_6, 0.292, 0.292\rangle\}$$

We can see that the new operators act similarly to the classical negation in the sense that if $\mu_B(x) > \nu_B(x)$, we have $\mu_Z(x) < \nu_Z(x)$, and vice versa.

Proposition 1. *The following relationships hold:*

$$Z_{\cos}(\neg(A^*)) = \neg(Z_{\cos}(A^*));$$
$$Z_{\sin}(\neg(A^*)) = \neg(Z_{\sin}(A^*)),$$

where

$$\neg(A^*) = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}.$$

Proof. This follows directly from (2.3) and (2.4). □

Remark 1. If we denote $\overline{Z_{\cos}}(A^*) = \neg(Z_{\cos}(A^*))$, and $\overline{Z_{\sin}}(A^*) = \neg(Z_{\sin}(A^*))$, we obtain another two operators $\overline{Z_{\cos}}$ and $\overline{Z_{\sin}}$ over the intuitionistic fuzzy sets.

Proposition 2. *The following relationships are fulfilled:*

$$G_{1,0}(F_{0,1}(Z_{\sin}(A^*))) = G_{1,0}(Z_{\sin}(F_{1,0}(A^*)))$$
$$G_{1,0}(F_{0,1}(Z_{\cos}(A^*))) = G_{1,0}(Z_{\cos}(F_{1,0}(A^*)))$$
$$G_{0,1}(F_{1,0}(Z_{\sin}(A^*))) = G_{0,1}(Z_{\sin}(F_{0,1}(A^*)))$$
$$G_{0,1}(F_{1,0}(Z_{\cos}(A^*))) = G_{0,1}(Z_{\cos}(F_{0,1}(A^*))).$$

Proof. We will only go through the first equality since the others are checked in the same manner. For the left-hand side we obtain:

$$\begin{aligned}
&G_{1,0}\left(F_{0,1}\left(\left\{\left\langle x, 1-\sin\left((1-\nu_A(x))\frac{\pi}{2}\right), 1-\sin\left((1-\mu_A(x))\frac{\pi}{2}\right)\right\rangle | x \in X\right\}\right)\right) \\
&\quad = G_{1,0}\left(\left\{\left\langle x, 1-\sin\left((1-\nu_A(x))\frac{\pi}{2}\right), \sin\left((1-\nu_A(x))\frac{\pi}{2}\right)\right\rangle | x \in X\right\}\right) \\
&\quad = \left\{\left\langle x, 1-\sin\left((1-\nu_A(x))\frac{\pi}{2}\right), 0\right\rangle | x \in X\right\}
\end{aligned}$$

For the right-hand side:

$$\begin{aligned}
&G_{1,0}\left(Z_{\sin}\left(\{\langle x, 1-\nu, \nu\rangle | x \in X\}\right)\right) \\
&\quad = G_{1,0}\left(\left\{\left\langle x, 1-\sin\left((1-\nu_A(x))\frac{\pi}{2}\right), 1-\sin\left(\nu_A(x)\frac{\pi}{2}\right)\right\rangle | x \in X\right\}\right) \\
&\quad = \left\{\left\langle x, 1-\sin\left((1-\nu_A(x))\frac{\pi}{2}\right), 0\right\rangle | x \in X\right\}
\end{aligned}$$

Thus both sides yield the same IFS. □

3 Conclusion and an open problem

In the present work we introduced two new operators over intuitionistic fuzzy sets based on the trigonometric functions $\sin$ and $\cos$. An interesting **Open problem** is the following:

Can the operator $X_{a,b,c,d,e,f}$, defined by (see [5])

$$\begin{aligned}
X_{a,b,c,d,e,f}(A^*) = \{\langle x, &a\mu_A(x) + b(1-\mu_A(x) - c\nu_A(x)), \\
&d\nu_A(x) + e(1 - f\mu_A(x) - \nu_A(x))\rangle | x \in X\}
\end{aligned}$$

reproduce the action of the operators $Z_{\sin}$ and $Z_{\cos}$ for a suitable choice of the parameters a, b, c, d, e and f?

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