Notes on Intuitionistic Fuzzy Sets
Print ISSN 1310–4926, Online ISSN 2367–8283
2025, Volume 31, Number 4, 522–531
DOI: 10.7546/nifs.2025.31.4.522-531

Intuitionistic fuzzy approach to bias correction of users' QoE estimation in information service networks

Peter Georgiev [1], **Stoyan Poryazov** [2], **Krassimir Atanassov** [3] and **Velin Andonov** [4]

[1] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 8, Sofia-1113, Bulgaria
e-mail: p.georgiev@math.bas.bg

[2] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 8, Sofia-1113, Bulgaria
e-mail: stoyan@math.bas.bg

[3] Department of Bioinformatics and Mathematical Modelling, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 105, Sofia-1113, Bulgaria
e-mail: krat@bas.bg

[4] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 8, Sofia-1113, Bulgaria
e-mail: velin_andonov@math.bas.bg

Received: 11 October 2025 **Revised:** 12 November 2025
Accepted: 21 November 2025 **Online First:** 27 November 2025

Abstract: In the paper, an approach to bias correction of users' Quality of Experience (QoE) estimation in information service networks is described. Numerical opinion scales for degree of confidence of the users and for the user's level of competence (self-estimation) are used. One approach to the correction of the incorrect expert's evaluation is presented. Geometrical

interpretations of the corrections of the user's estimations are also presented.

Keywords: Intuitionistic fuzzy set, Quality of service, Quality of experience, Estimation.
2020 Mathematics Subject Classification: 03E72, 68M10.

1 Introduction

Dealing with experts' bias is an important topic in expert systems. Many real-world problems can be represented as decision-making problems which require multiple expert judgements. Experts' opinions often deviate. Finding the best alternative among a set of alternatives can be regarded as a collective decision-making problem the solution of which requires the aggregation of various independent experts judgments. Various types of biases and approaches to dealing with them are described in [1, 2, 25].

The Intuitionistic Fuzzy Sets (IFSs, see [3]) represent a mathematical tool for modelling of uncertainty and vagueness, and they can be used to mitigate expert bias by providing a mathematical framework to aggregate and weigh the opinions of multiple experts. While classical fuzzy sets only consider the degree of membership, IFSs include both membership and non-membership degrees, allowing for a more nuanced representation of expert's opinion, which is especially useful when experts may have different experiences and knowledge. An application of IFSs in the area of human reliability analysis is described in [16], where reference task-based human error analysis model is introduced aimed at correction of experts' bias.

In the present paper, an approach to bias correction of users' QoE estimation in information service networks is proposed. In most published papers on the QoE evaluation in telecommunication systems, the QoE is presented via mean opinion score (MOS) for a group of users [9]. Here, we consider evaluations of information services in the form of Intuitionistic Fuzzy Pairs (IFPs). A correction of the bias in the evaluations of the services is proposed and a geometrical interpretation is presented in the intuitionistic fuzzy triangle.

2 Quality of service concept

We consider an information service system consisting of a service network, for instance, a telecommunication network and users.

Quality of Service (QoS) according to the documents of the International Telecommunication Union (ITU) is (see [21]): Totality of characteristics of a telecommunications service that bear on its ability to satisfy stated and implied needs of the user of the service.

There are at least four viewpoints of QoS [21]:

1. QoS requirements of user/customer (QoSR).
2. QoS offered/planned by service provider (QoSO).
3. QoS delivered/achieved by service provider (QoSD).
4. QoS experienced/perceived by customer/user (QoSE).

In this paper, we consider QoSE only. QoSE is a statement expressing the level of quality that customers/users believe they have experienced. It reflects subjective (qualitative) parameters.

3 Intuitionistic fuzzy quality evaluation

In recent years, in a series of papers (see [18–20]) an approach to quantification of the uncertainty of the service of requests in service systems has been developed. It is based on an intuitionistic fuzzy characterization of the service of requests by virtual service devices and the theory of the Intuitionistic Fuzzy Sets (IFSs, [3]) and in particular the notion of an Intuitionistic Fuzzy Pair (IFP, see [5]). Intuitionistic fuzzy evaluations of the uncertainty of the service of requests of various compositions of services have been obtained.

In this paper, using the definition of an IFS we shall consider a set A which characterizes the degrees to which the quality experienced by every user belongs to "excellent quality", the degree to which the quality does not belong to "excellent quality" and the degree of non-determinacy. Formally, let U be the set of all users of a certain information service. Then

$$A(u) = \{\langle u, \mu_A(u), \nu_A(u)\rangle | u \in U\}\,, \tag{1}$$

where the functions $\mu_A(u)$ and $\nu_A(u)$ are defined as follows:

$$\mu_A(u) : U \to [0, 1]\,, \tag{2}$$

$$\nu_A(u) : U \to [0, 1]\,, \tag{3}$$

and satisfy the following condition:

$$\mu_A(u) + \nu_A(u) \leq 1\,. \tag{4}$$

$A(u)$ is an IFS and the degree of uncertainty is given by

$$\pi_A(u) = 1 - \mu_A(u) - \nu_A(u)\,. \tag{5}$$

In our context, $\pi_A(u)$ is the degree of indeterminacy concerning whether the service experienced by user u belongs to the excellent quality service.

4 Numerical representation of the QoSE

The level of QoSE is presented numerically by Mean Opinion Score (MOS) values on a predefined opinion scale that users assign to their opinion of the performance of the service system. The MOS reflects the estimation of many aspects of the service systems.

We consider the evaluation of QoSE obtained through a questionnaire to the users. To every question in the questionnaire, the user assigns a number (evaluation) chosen from a predefined scale with evaluations. There are several opinion scales recommended in the documents of ITU. Some scales in [22] are of five or two grades. In [23], there are 9- and 11-grade numerical quality scales. Almost all scales are discrete.

We will use intervals to represent the Intuitionistic Fuzzy Ratings (IFRs) regarding QoSE. In our approach, we employ a scale corresponding to the 5-level MOS scale, excluding the "no service" rating for answering the question

"To what degree the service belongs to the excellent quality services?"

Depending on their assessment (Table 1, column 2 — "Experienced quality"), the respondent selects a value from the intervals in the first column of Table 1. The customers initial ("raw") rating of the product is shown in the Table 1.

Table 1. Numerical opinion scale for the QoSE representation

Degree of membership to excellent quality, μ_A^{Raw}	**Experienced quality**
$(0.80, 1.00]$	Excellent
$(0.60, 0.80]$	Good
$(0.40, 0.60]$	Fair
$(0.20, 0.40]$	Poor
$(0.01, 0.20]$	Very Poor
0.00	No Service

The respondent answers the question:

"How sure are you about your answer to the previous question?"

with a value of $\alpha \in [0, 1]$ as shown in Table 2.

Table 2. Numerical opinion scale for the degree of confidence

Degree of confidence in the answer, α	**Confidence level**
$(0.80, 1.00]$	Complete confidence
$(0.60, 0.80]$	High confidence
$(0.40, 0.60]$	Moderate confidence
$(0.20, 0.40]$	Low confidence
$[0.00, 0.20]$	No confidence

Now, let us consider a questionnaire in which questions related to the self-evaluation of the competence of the users are included. Each user gives a self-evaluation of his/her competence and experience regarding the studied indicator. For the answer of such questions, the scale presented in Table 3 can be used.

Table 3. Numerical opinion scale for the User Level of Competence (self-) evaluation

Competence, β	**Competence level**
$(0.80, 1.00]$	Completely competent
$(0.60, 0.80]$	Very competent
$(0.40, 0.60]$	Moderately competent
$(0.20, 0.40]$	Slightly competent
$[0.00, 0.20]$	No competent at all

The user's opinion evaluated using the scale from Table 3 can be considered as a base for determination of the parameter "indeterminacy" in the IF evaluation of the user. When the user is incompetent the indeterminacy of his/her evaluation is highest. When the user is completely competent the indeterminacy of his/her evaluation is lowest (0.00).

In this approach, it is necessary that the evaluations given by the users according to Table 1, Table 2 and Table 3 be obtained by taking into account the main dependency between the parameters of an IF evaluation (see (5)): the sum of the membership, non-membership and uncertainty is equal to 1.

Let $\mu_A^{Raw}(x) \in [0,1]$, $\alpha(x) \in [0,1]$ and $\beta(x) \in [0,1]$ for user $x \in U$. If U is the set of all of the interviewed users of the service, then we have the following IFS:

$$P = \{\langle x, \mu_P(x), \nu_P(x)\rangle | x \in U\}\,, \tag{6}$$

We can explore different ways to determine the initial estimate $P(x)$. Below, we present two possible approaches for this purpose.

Approach 1:

$$\mu_P(x) = \mu_A^{Raw}(x).\alpha(x).\beta(x)\,, \tag{7}$$

$$\nu_P(x) = (1 - \mu_A^{Raw}(x)).(1 - \alpha(x)).(1 - \beta(x))\,. \tag{8}$$

Obviously, set P is an IFS and represents an IF evaluation. This can be verified in the following way:

$$\mu_P(x), \nu_P(x) \in [0,1]$$

and

$$\mu_P(x) + \nu_P(x) = \mu_A^{Raw}(x).\alpha(x).\beta(x) + (1 - \mu_A^{Raw}(x)).(1 - \alpha(x)).(1 - \beta(x)) \leq 1$$

because $\mu_A^{Raw}(x).\alpha(x).\beta(x) \leq \mu_A^{Raw}(x)$ and $(1-\mu_A^{Raw}(x)).(1-\alpha(x)).(1-\beta(x)) \leq 1-\mu_A^{Raw}(x)$.

Approach 2:

$$\mu_P(x) = \mu_A^{Raw}(x).\alpha(x).\beta(x)\,, \tag{9}$$

$$\nu_P(x) = 1 - \max(\alpha(x), \beta(x))\,, \tag{10}$$

$$\pi_P(x) = 1 - \mu_P(x) - \nu_P(x)\,. \tag{11}$$

Obviously, set P is an IFS and represents an IF evaluation. This can be verified in the following way:

$$\mu_P(x), \nu_P(x) \in [0,1]$$

and

$$\begin{aligned}\mu_P(x) + \nu_P(x) &= \mu_A^{Raw}(x).\alpha(x).\beta(x) + (1 - \max(\alpha(x), \beta(x))) \\ &\leq \max(\alpha(x), \beta(x)) + 1 - \max(\alpha(x), \beta(x)) \\ &\leq 1\,,\end{aligned}$$

because $\mu_A^{Raw}(x).\alpha(x).\beta(x) \leq \max(\alpha(x), \beta(x))$. The two approaches described above can be used for intuitionistic fuzzy evaluation of the QoE of users of telecommunication services.

5 Correction of user's evaluation

Many of the good questionnaires for evaluation of the quality of the services by the users contain additional, direct or indirect, control questions which give the possibility for an outside expert evaluator–mediator (umpire) to correct the user's evaluation by making it more reliable before giving it to the service provider. For this purpose, the evaluator–mediator evaluates the reliability of the opinion of the user mainly on the basis of his/her answers to the basic and additional questions in the questionnaire. Additional information related to the education, profession, social or work status, uncontroversial answers, etc. The evaluator determines the reliability of the user's opinion. Here, an average evaluation π_{av} will be used which is based on the answers of a set of "reliable" users.

Let us have n users who have evaluated some object with IFPs $\langle \mu_i, \nu_i \rangle$, where $1 \leq i \leq n$. Let k of these users are correct and let π_{av} be the average evaluation of the degrees of uncertainty of their evaluations, where these degrees are $\pi_j = 1 - \mu_j - \nu_j, (1 \leq j \leq k)$.

Each IFP $\langle \mu, \nu \rangle$ can be represented as a point in the IF interpretation triangle (see Figure 1).

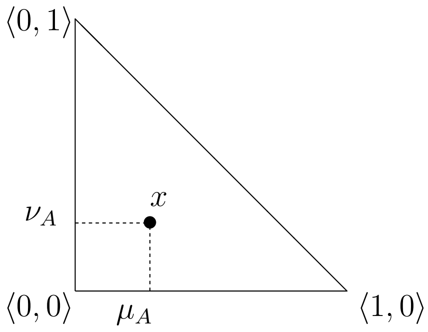

Figure 1. Geometrical interpretation of an element $x \in U$

Let us put the segment shown on Figure 2 for which point Z is at a distance π_{av} from point $\langle 1, 0 \rangle$, where π_{av} is the average value of the uncertainty values $\pi_P(x)$ of each reliable user $x \in U$, i.e.:

$$\pi_{av} = \sum_{i=1}^{k} \frac{\pi_P(x_i)}{k}. \tag{12}$$

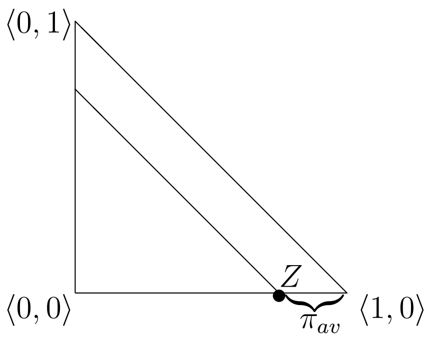

Figure 2. Geometrical interpretation of the segment from point Z

For the place of point E there are two cases. When it is to the left of the line segment (see Figure 3) and when it is to the right of the line segment (see Figure 4). In both cases, we change the incorrect expert's evaluation with the IFP $\langle \frac{\mu_P(1-\pi_{av})}{\mu_P+\nu_P}, \frac{\nu_P(1-\pi_{av})}{\mu_P+\nu_P} \rangle$ – see point F in Figures 3 and 4, respectively.

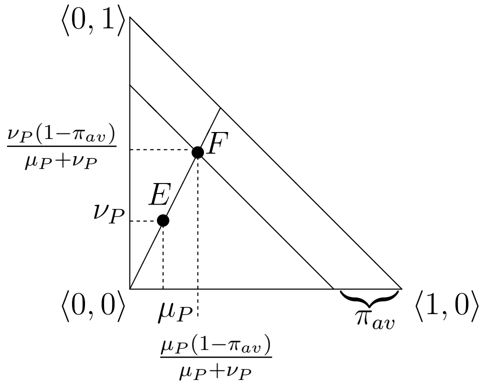

Figure 3. Geometrical interpretation of the incorrect expert's evaluation — first case

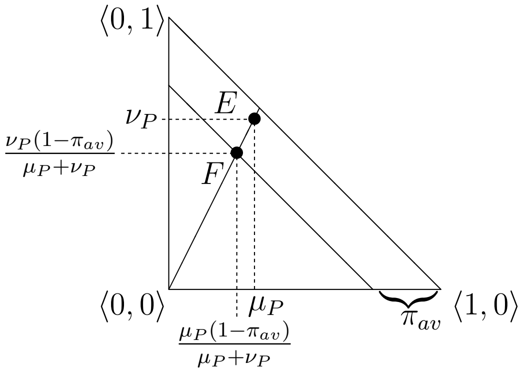

Figure 4. Geometrical interpretation of the incorrect expert's evaluation — second case

The final evaluation Q for a given indicator has the form:

$$Q = \{\langle x, \mu_Q(x), \nu_Q(x)\rangle | x \in U\}, \tag{13}$$

where

$$\mu_Q(x) = \frac{\mu_P(x).(1-\pi_{av})}{\mu_P(x)+\nu_P(x)},$$
$$\nu_Q(x) = \frac{\nu_P(x).(1-\pi_{av})}{\mu_P(x)+\nu_P(x)},$$

and the degree of uncertainty is given by:

$$\begin{aligned}
\pi_Q(x) &= 1 - \mu_Q(x) - \nu_Q(x) \\
&= 1 - \frac{\mu_P(x).(1-\pi_{av})}{\mu_P(x)+\nu_P(x)} - \frac{\nu_P(x).(1-\pi_{av})}{\mu_P(x)+\nu_P(x)} \\
&= 1 - \frac{(\mu_P(x)+\nu_P(x)).(1-\pi_{av})}{\mu_P(x)+\nu_P(x)} \\
&= 1 - (1-\pi_{av}) \\
&= \pi_{av}.
\end{aligned}$$

There are many studies treating the topic of unconscientous experts' evaluations and how to correct them using IFSs (see Vassilev [26] and Dworniczak *et al.* [6, 11–14]).

6 Generalized evaluation of a service

When we have questionnaires of many users in order to obtain an aggregated evaluation it is necessary to generalize the corrected individual evaluations. Various approaches can be used. Here, we suggest using an average value of the evaluations. The opinions of all n interviewed users are taken with the same weight after the correction.

By applying the weight-operator W defined in [3], the overall score of a service is an IFP $\langle \mu_W, \nu_W \rangle$, where:

$$\mu_W = \frac{1}{n}\sum_{i=1}^{n} \mu_Q(x_i), \tag{14}$$

$$\nu_W = \frac{1}{n}\sum_{i=1}^{n} \nu_Q(x_i). \tag{15}$$

If the users of the service prefer linguistic values, we could de-i-fuzzify the evaluations in a similar way to the ones described in [7, 8, 17].

7 Conclusion

Most of the research on QoE estimation in the field of telecommunications is based on the definitions of ITU. As a result, the majority of the work in this field deals with the problems of subjective measurements of users' experience. The most typical approaches result in a MOS evaluation of the perceived quality [10]. While being widely accepted and used as a reference quality indicator, MOS is often applied without taking into account its limitations and drawbacks such as the users' bias [24]. The IFSs are a suitable tool for correcting the users' bias in QoE evaluation using questionnaires. In future, we intend to study the problem of users' bias in the estimation of a set of services. One approach to this problem is based on the user-service interaction matrix [15]. We expect that the theory of the Index Matrices (IMs, see [4]) will provide suitable tools for estimation of the QoE of telecommunication services.

Acknowledgements

The work of S. Poryazov, K. Atanassov and V. Andonov is supported by the research project "Perspective Methods for Quality Prediction in the Next Generation Smart Informational Service Networks" (KP-06-N52/2) financed by the Bulgarian National Science Fund.

References

[1] Abels, A., Lenaerts, T., Trianni, V. & Nowé, A. (2023). Dealing with expert bias in collective decision-making. *Artificial Intelligence*, 320, Article ID 103921.

[2] Anon, S. Y., Amin, S. H., & Baki, F. (2024). Third-party reverse logistics selection: A literature review. *Logistics*, 8(2), Article ID 35.

[3] Atanassov, K. (2012). *On Intuitionistic Fuzzy Sets Theory*. Springer, Berlin.

[4] Atanassov, K. (2014). *Index Matrices: Towards an Augmented Matrix Calculus*. Springer, Cham.

[5] Atanassov, K., Szmidt, E., & Kacprzyk, J. (2013). On intuitionistic fuzzy pairs. *Notes on Intuitionistic Fuzzy Sets*, 19(3), 1–13.

[6] Atanassova, L., & Dworniczak, P. (2021). Operation Δ as the tool for the de-i-fuzzification procedure and for the correction of the unconscientious experts' evaluations. *Notes on Intuitionistic Fuzzy Sets*, 27(4), 9–19.

[7] Atanassova, V., & Sotirov, S. (2012). A new formula for de-i-fuzzification of intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets*, 18(3), 49–51.

[8] Ban, A., Kacprzyk, J.,& Atanassov, K. (2008). On de-I-fuzzification of intuitionistic fuzzy sets. *Comptes Rendus de l'Academie bulgare des Sciences*, 61(12), 1535–1540.

[9] Banjanin, M. K., Stojčić, M., Danilović, D., Ćurguz, Z., Vasiljević, M., & Puzić, G. (2022). Classification and prediction of sustainable quality of experience of telecommunication service users using machine learning models. *Sustainability*, 14, Article ID 17053.

[10] Díaz-Zayas, A., Panizo, L., Baños, J., Cárdenas, C., & Dieudonne, M. (2018). QoE Evaluation: The TRIANGLE testbed approach. *Wireless Communications and Mobile Computing*, 2018, Article ID 6202854.

[11] Dworniczak, P. (2012). A note on the unconscientious experts evaluations in the intuitionistic fuzzy environment. *Notes on Intuitionistic Fuzzy Sets*, 18(3), 23–29.

[12] Dworniczak, P. (2013). Further remarks about the unconscientious experts' evaluations in the intuitionistic fuzzy environment. *Notes on Intuitionistic Fuzzy Sets*, 19(1), 27–31.

[13] Dworniczak, P. (2015). A distance based correction of the unconscientious experts' evaluations. *Notes on Intuitionistic Fuzzy Sets*, 21(3), 5–17.

[14] Dworniczak, P. (2022). The algorithm of correction of the unconscientious experts' evaluations in the interval-valued intuitionistic fuzzy sets case. *Notes on Intuitionistic Fuzzy Sets*, 28(3), 293–305.

[15] Fu, S., Li, J., & Wang L. (2025). RA-QoS: A robust autoencoder-based QoS predictor for highly accurate web service QoS prediction. *PeerJ Computer Science*, 11, Article ID e2928. DOI: 10.7717/peerj-cs.2928.

[16] Pan, X., Ding, S., Zhao, X., Zhang, W., Zuo, D., & Sun, L. (2024). Human error probability evaluation based on reference task using intuitionistic fuzzy theory. *Computers & Industrial Engineering*, 191, Article ID 110105.

[17] Patrascu, V. (2024). Two de-I-fuzzification procedures for intuitionistic fuzzy information. *Notes on Intuitionistic Fuzzy Sets*, 30(1), 18–25.

[18] Poryazov, S., Andonov, V., & Saranova, E. (2022). Intuitionistic fuzzy representation of uncertainty in biomedical operations. In: *Sotirov, S. S., Pencheva, T., Kacprzyk, J., Atanassov, K. T., Sotirova, E., & Staneva, G. (Eds.). Contemporary Methods in Bioinformatics and Biomedicine and Their Applications*. Lecture Notes in Networks and Systems, Volume 374, Springer, Cham, Switzerland, pp. 269–278.

[19] Poryazov, S., Andonov, V., & Saranova, E. (2022). Three intuitionistic fuzzy estimations of uncertainty in service compositions. In: *Uncertainty and Imprecision in Decision Making and Decision Support: New Advances, Challenges, and Perspectives*; IWIFSGN BOS/SOR 2020. Lecture Notes in Networks and Systems, Volume 338. Springer, Cham, Switzerland, pp. 72–84.

[20] Poryazov, S., Andonov, V., Saranova, E., & Atanassov, K. (2022). Two Approaches to the traffic quality intuitionistic fuzzy estimation of service compositions. *Mathematics*, 10(23), Article ID 4439.

[21] Recommendation ITU-T E.800. (09/2008). Definitions of terms related to quality of service. Available online at: `https://handle.itu.int/11.1002/1000/9524`

[22] Recommendation ITU-T P.800.1. (07/2016). Methods for objective and subjective assessment of speech and video quality. Mean opinion score (MOS) terminology. Available online at: `https://handle.itu.int/11.1002/1000/12972`.

[23] Recommendation ITU-T P.910. (10/2023). Subjective video quality assessment methods for multimedia applications. Available online at: `https://handle.itu.int/11.1002/1000/15697`.

[24] Streijl, R., Winkler, S. & Hands, D. (2016). Mean opinion score (MOS) revisited: methods and applications, limitations and alternatives. *Multimedia Systems*, 22(2), 213–227.

[25] Theodorakopoulos, L., Theodoropoulou, A., & Halkiopoulos, C. (2025). Cognitive bias mitigation in executive decision-making: A data-driven approach integrating big data analytics, AI, and explainable systems. *Electronics*, 14, Article ID 3930.

[26] Vassilev, P. (2010). On reassessment of expert evaluations in the case of intuitionistic fuzziness. *Advanced Studies in Contemporary Mathematics*, 20(4), 569–574.