Notes on Intuitionistic Fuzzy Sets
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Analysis of hub parameters in fuzzy hypergraphs extending to intuitionistic fuzzy threshold hypergraphs: Applications in designing transport networks in amusement parks using hub hyperpaths

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Abstract: A hypergraph is a generalization of a graph where an edge can connect any number of vertices. In this paper, many different aspects of fuzzy hypergraphs and their applications are examined. The concepts of the hub hyperpath, hub set, and hub number of fuzzy hypergraphs are defined and analyzed. Specifically, the hub number in complete fuzzy hypergraphs has been analyzed and provided an example as an application part in transportation systems. Furthermore, these concepts have been expanded to Intuitionistic Fuzzy Hypergraphs (IFHGs), defining the hub hyperpath, hub set, and hub number. Additionally, the problem of designing an efficient electric transport system for moving between game stations in a large amusement park has been investigated using hub analysis within the framework of an Intuitionistic Fuzzy Threshold Hypergraph (IFTHG) environment.
Keywords: Hub analysis, Hub number of fuzzy hypergraph, Intuitionistic fuzzy hypergraph

(IFHG), Intuitionistic fuzzy threshold hypergraphs (IFTHG), Transport network.
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1 Introduction

In recent years, several academic works have demonstrated the increased interest in the study of hypergraphs and its applications. Claude Berge's seminal work "Hypergraphs" [1] explores fundamental concepts and extends the mathematical framework of hypergraphs, making it a cornerstone in the field. Similarly, Alain Bretto's "Hypergraph Theory" [2] provides a comprehensive foundation on the subject, highlighting its theoretical concepts and variety of applications. The integration of fuzzy into hypergraph theory is examined by Lee-Kwang Hyung and Lee Keon Myung [3], who discuss fuzzy hypergraphs and their partitioning, emphasizing the utility of fuzzy sets in handling uncertainty within hypergraphs .

The exploration of intuitionistic fuzzy hypergraphs by R. Parvathi et al [7] provides a detailed examination of these structures, underscoring their significance in modeling and analysis. Moreover, C. Radhika and R. Parvathi [8] discussed intuitionistic fuzzification functions, highlighting their importance in enhancing the versatility of hypergraphs for various applications. K. K. Myithili and C. Nandhini [6] introduced the work on Intuitionistic Fuzzy Threshold Hypergraphs (IFTHGs) for further advanced concepts.

Research into specific parameters of graphs, such as the hub number, is explored by Matt Walsh [9], who investigates the structural properties and implications of hub numbers in graphs. Then, the study by Shadi Ibrahim Khalaf and collaborators [4] on the hub and global hub numbers of graphs contributes to a deeper understanding of these metrics, further enriching the body of knowledge on hypergraph properties and applications. Building on this, K. K. Myithili and C. Nandhini [5] explore into the hub parameters of hypergraphs, offering novel insights and extending the existing knowledge in the field.

Together, these works illustrate the dynamic and evolving nature of hypergraph theory, its integration with fuzzy and intuitionistic fuzzy environments, with its broad applicability in solving complex problems across different domains.

In Section 2, the basic definitions necessary for understanding the subsequent concepts are discussed. Section 3 introduces and defines the hub set, hub hyperpath and hub number of fuzzy hypergraphs, as well as complete fuzzy hypergraphs. This section also includes a numerical example illustrating the application of these concepts in a transportation system. In Section 4, these definitions have been extended to Intuitionistic Fuzzy Hypergraphs (IFHG), providing examples to elucidate the hub set, hub hyperpath, hub number in this context. Finally, Section 5 focuses on Intuitionistic Fuzzy Threshold Hypergraphs (IFTHG), demonstrating the hub set, hub hyperpath, hub number, and their applications in addressing the problem of constructing a transport electric system for moving between game stations in a large amusement park using hyperpaths.

2 Basic definitions

In this section, we reviewed over some basic definitions associated to our main concept.

Definition 2.1. [9] A *hub set in a graph* G is a set S of vertices in G, such that any two vertices outside S are connected by a path whose all internal vertices lie in S and the *hub number of* G is the minimum cardinality of a hub set in G.

Definition 2.2. [2] A *hypergraph* is a pair $\mathbb{H} = (U, \mathcal{E})$, where:

- U is a set of elements called vertices,
- $\mathcal{E}$ is a set of non-empty subsets of U called hyperedges.

Therefore, $\mathcal{E}$ is a subset of $\rho(U)\backslash\{\emptyset\}$, where $\rho(U)$ is power set of U.

Definition 2.3. [2] Let $\mathbb{H} = (U, \mathcal{E})$ be a hypergraph without isolated vertex; a *hyperpath* $\mathbb{H}_{\mathbb{P}}$ in $\mathbb{H}$ from u_i to u_j is a vertex-hyperedge alternative sequence:

$$u_i = u_1 \quad e_1 \quad u_2 \quad e_2 \quad \cdots \quad u_n \quad e_n \quad u_{n+1} = u_j$$

for all $u_i \in e_{i-1}, e_i$, such that

- $u_i = u_1, u_2, \ldots, u_n, u_{n+1} = u_j$ are distinct vertices with the possibility that $u_i = u_j$;
- $e_1, e_2, \ldots, e_n$ are distinct hyperedges.

Definition 2.4. [5] Let $\mathbb{H} = (U, \mathcal{E})$ be a hypergraph and $\mathcal{S} \subseteq U$ with any two vertices $u_i, u_j \in U$. Then the *hub-hyperpath* $\mathbb{H}_{\mathbb{P}}$ in $\mathbb{H}$ from u_i to u_j is an alternative sequence of vertex and hyperedge say, $u_1 \quad e_1 \quad u_2 \quad e_2 \quad \ldots \quad u_n \quad e_n \quad u_{n+1}$, where $u_i = u_1$ & $u_j = u_{n+1}$ such that

- $u_i = u_1$, u_2, …,u_n, $u_{n+1} = u_j$ are distinct vertices
- e_1, e_2, …,e_n are distinct hyperedges with $u_i \in e_r$, $u_j \in e_s$; and $e_r \cap e_s \neq \emptyset$ with $r \neq s$
- the hub-hyperpath between u_i & u_j is a path with $\mathbb{H}_{P(U)}\backslash\{u_i, u_j\} \in \mathcal{S}$

Note 2.1. The degenerate case arises when,

(i) u_i and u_j belongs to same hyperedge, and

(ii) two end vertices are same, i.e., $u_i = u_j$.

In the above two cases, the hub-hyperpath does not exist.

Definition 2.5. [5] A hypergraph $\mathbb{H} = (U, \mathcal{E})$, where U is a set of all vertices and $\mathcal{E}$ is a set of non-empty subsets of U, is called hyperedges. Then a *hub set in a hypergraph* is said to be a set $\mathcal{S} \subseteq U$, if there exists a hub-hyperpath $\mathbb{H}_P$ which connects any two vertices $u_i, u_j \in U$ such that $u_i, u_j \notin H_S$ and $\mathbb{H}_{P(U)}\backslash\{u_i, u_j\} \in \mathcal{S}$.

Definition 2.6. [5] The *hub number of a hypergraph* $\mathbb{H}$, denoted by $h(\mathbb{H})$, is the minimum cardinality of a hub set in $\mathbb{H}$.

Notations

S	- Hub set
$\mathbb{H}_{\mathbb{F}}$	- Fuzzy hypergraph
P_{FH}	- Fuzzy hub-hyperpath
$P_{FH}(U)$	- Vertices in P_{FH}
$h(\mathbb{H}_{\mathbb{F}})$	- Hub number of $\mathbb{H}_{\mathbb{F}}$
$\mathbb{H}_{\mathbb{CF}}$	- Complete fuzzy hypergraph
$\mathbb{H}_{\mathbb{IF}}$	- Intuitionistic fuzzy hypergraph
P_{IFH}	- Intuitionistic fuzzy hub-hyperpath
$P_{IFH}(U)$	- Vertices in $\mathbb{H}_{\mathbb{IF}}$
$h(\mathbb{H}_{\mathbb{IF}})$	- Hub number of $\mathbb{H}_{\mathbb{IF}}$
$\mathbb{H}_{\mathbb{G}}$	- Intuitionistic fuzzy thershold hypergraph
P_{IFTH}	- Intuitionistic fuzzy hub-hyperpath
$P_{IFTH}(U)$	- Vertices in P_{IFTH}
$h(\mathbb{H}_{\mathbb{G}})$	- Hub number of $\mathbb{H}_{\mathbb{G}}$

3 Hub number of a fuzzy hypergraph

Definition 3.1. The hub set in a fuzzy hypergraph $H = (U, \mathcal{E})$ is defined as follows:
(i) $U = \{u_1, u_2, \ldots, u_n\}$: a finite set of vertices
(ii) $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_m\}$: a family of fuzzy subsets of U
(iii) $\mathcal{E}_j = \{(u_i, \mu_j(u_i))|\mu_j(u_i) > 0\}$, $j = 1, 2, \ldots, m$
(iv) $\mathcal{E}_j \neq \emptyset$, $j = 1, 2, \ldots, m$
(v) $\bigcup_j supp(\mathcal{E}_j) = U$, $j = 1, 2, \ldots, m$.
(vi) the hub set in a fuzzy hypergraph is said to be a set $S \subseteq U$, there exists a fuzzy hub hyperpath(P_{FH}) which connects any two vertices $u_i, u_j \in U$ such that $P_{FH}(U)\backslash\{u_i, u_j\} \in S$ where $u_i, u_j \notin S$.

Here, the hyperedges $\mathcal{E}_j$ are fuzzy sets of vertices. The degree of membership of vertex u_i to hyperedge $\mathcal{E}_j$ is defined by $\mu_j(u_i)$. The other sets $(U, \mathcal{E})$ are crisp sets.

Definition 3.2. Let $\mathbb{H}_{\mathbb{F}} = (U, \mathcal{E})$ is a fuzzy hypergraph and $S \subseteq U$ with any two vertices $u_i, u_j \in U$. Then the fuzzy hub-hyperpath P_{FH} in $\mathbb{H}_{\mathbb{F}}$ from u_i to u_j is an alternative sequence of vertex and hyperedge say,

$$u_1 \quad \mathcal{E}_1 \quad u_2 \quad \mathcal{E}_2 \quad \cdots \quad u_n \quad \mathcal{E}_n \quad u_{n+1}$$

where $u_i = u_1$ and $u_j = u_{n+1}$ such that

- $u_i = u_1, u_2, \ldots, u_n, u_{n+1} = u_j$ are distinct vertices
- $\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_n$ are distinct hyperedges with $u_i \in \mathcal{E}_r$, $u_j \in \mathcal{E}_s$ and $\mathcal{E}_r \cap \mathcal{E}_s = \emptyset$ with $r \neq s$
- the fuzzy hub-hyperpath between u_i and u_j is a hyperpath with $P_{FH}(U)\backslash\{u_i, u_j\} \in S$

Note 3.1. A fuzzy hub hyperpath incorporates the degree of uncertainty (fuzziness) in the connections between vertices, whereas in non-fuzzy hypergraph the connections are strictly defined.

Definition 3.3. The hub number of $\mathbb{H}_{\mathbb{F}}$ is denoted by $h(\mathbb{H}_{\mathbb{F}})$ is the minimum cardinality of a hub set in $\mathbb{H}_{\mathbb{F}}$.

Example 3.1. Let $H = (U, \mathcal{E})$ be a fuzzy hypergraph with vertices $U = \{u_1, u_2, \ldots, u_9\}$ and hyperedges $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$.

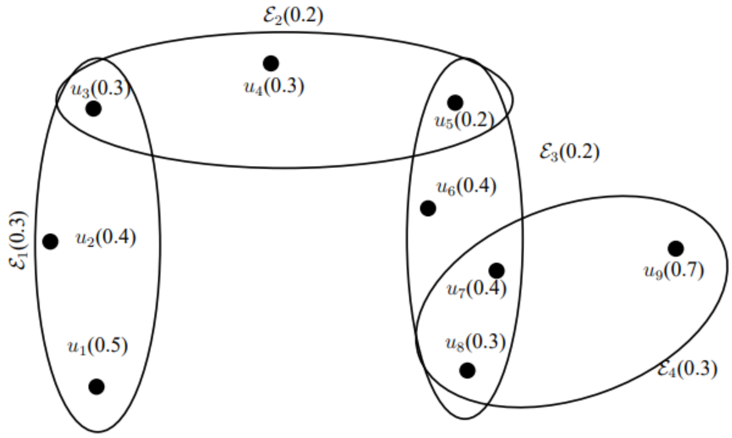

Figure 1. Fuzzy hypergraph

Then, the possible fuzzy hub-hyperpaths between u_1 and u_9 are
P_{FH} 1: $u_1 \quad \mathcal{E}_1 \quad u_3 \quad \mathcal{E}_2 \quad u_5 \quad \mathcal{E}_3 \quad u_7 \quad \mathcal{E}_4 \quad u_9$
P_{FH} 2: $u_1 \quad \mathcal{E}_1 \quad u_3 \quad \mathcal{E}_2 \quad u_5 \quad \mathcal{E}_3 \quad u_8 \quad \mathcal{E}_4 \quad u_9$
Therefore, the hub sets are $\{u_3, u_5, u_7\}$, $\{u_3, u_5, u_8\}$.
The hub number of above fuzzy hypergraph is $h(\mathbb{H}_{\mathbb{F}}) = 0.3 + 0.2 + 0.3 = 0.8$.

Definition 3.4. A complete fuzzy hypergraph $\mathbb{H}_{\mathbb{CF}}$ is a fuzzy hypergraph with $\mathbb{H}_{\mathbb{F}} = (U, \mathcal{E} = \rho(U)\backslash\{\emptyset\})$, where $\rho(U)$ is a power set of U.

Example 3.2. Here the vertices are $U = \{u_1, u_2, u_3\}$ and the membership values of the vertices are $\mu(u_1) = 0.5, \mu(u_2) = 0.4, \mu(u_3) = 0.3$, and $\mathcal{E}(u, v) = \mu(u_i) \wedge \mu(u_{i+1})$, $\forall u_i, u_{i+1} \in U$ and hub number of Figure 2 is zero, i.e., $\mathbb{H}_{\mathbb{CF}} = 0$.

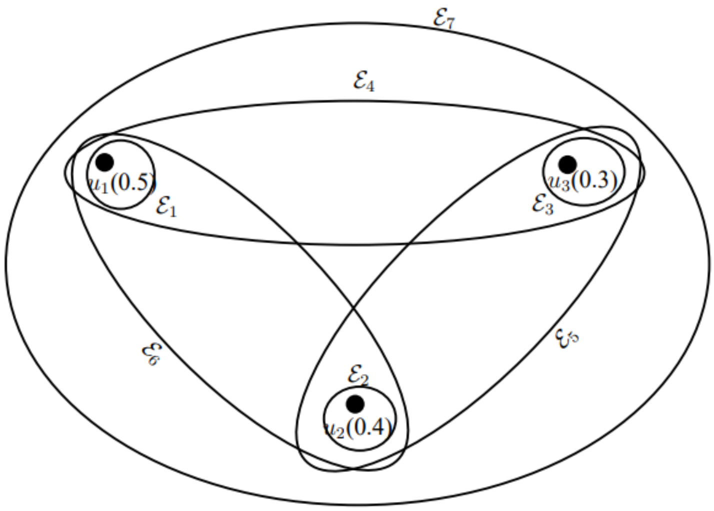

Figure 2. Complete fuzzy hypergraph

Observation 1. *If* $\mathbb{H}_{\mathbb{F}} = (U, \mathcal{E})$ *is a complete fuzzy hypergraph. Then* $h(\mathbb{H}_{\mathbb{F}}) = 0$.

Explanation. A complete fuzzy hypergraph $\mathbb{H}_{\mathbb{CF}}$ is a fuzzy hypergraph in which each pair of distinct vertices is contained in a hyperedge $\mathcal{E}$. In other words, every possible hyperedge $\mathcal{E}$ is present in $\mathbb{H}_{\mathbb{F}}$.

From which it means that, there are no intermediate vertices in fuzzy hub-hyperpath $\mathbb{H}_{\mathbb{FP}}$. Hence, every hyperedge of $\mathbb{H}_{\mathbb{F}}$ must contain every vertex in $\mathbb{H}_{\mathbb{F}}$. Then the result is obvious, if $\mathbb{H}_{\mathbb{F}} = (U, \mathcal{E})$ is a complete fuzzy hypergraph which implies $h(\mathbb{H}_{\mathbb{F}}) = 0$.

Remark. The converse part of the above observation holds when every pair of distinct vertices is connected by a hyperedge $\mathcal{E}$. Otherwise, the converse fails.

Theorem 3.1. *A fuzzy hypergraph* $\mathbb{H}_{\mathbb{F}}$ *with* $h(\mathbb{H}_{\mathbb{F}}) > 0$ *iff* $\mathbb{H}_{\mathbb{F}}$ *is not a complete fuzzy hypergraph* $\mathbb{H}_{\mathbb{CF}}$.

Proof. Necessary part: Assume $\mathbb{H}_{\mathbb{F}}$ has a hub number $h(\mathbb{H}_{\mathbb{F}}) > 0$. This means, there is at least one vertex in $\mathbb{H}_{\mathbb{F}}$ that needs to be included in a hub set S to ensure connectivity between all other pairs of vertices. If $\mathbb{H}_{\mathbb{F}}$ is a complete fuzzy hypergraph, then every possible subset of vertices would form a hyperedge. This means, for any pair of vertices u_i and u_j, there would be a direct hyperedge connecting them without the need for an intermediate vertex. In a complete fuzzy hypergraph, the smallest hub set S need to ensured its connectivity between all pairs of vertices, which results in the empty set $S = \emptyset$. Hence $h(\mathbb{H}_{\mathbb{F}}) = 0$. Which contradicts the given concept. Thus $\mathbb{H}_{\mathbb{F}}$ is not a complete fuzzy hypergraph when $h(\mathbb{H}_{\mathbb{F}}) > 0$.

Sufficient part: Assume $\mathbb{H}_{\mathbb{F}}$ is not a complete fuzzy hypergraph. This means, there exists at least one pair of vertices u_i and u_j that do not have a direct hyperedge connecting them. Because $\mathbb{H}_{\mathbb{F}}$ is not complete, there must be a pair of vertices that require intermediate vertices to ensure the connectivity through hyperedges. To ensure connectivity between each pair of vertices u_i and u_j that do not have a direct hyperedge, we need to find a set S of intermediate vertices such that for every pair u_i and u_j not in S, there exists a hyperedge containing u_i and u_j with all intermediate vertices in S.

The smallest such set S will have at least one vertex. i.e., $h(\mathbb{H}_{\mathbb{F}}) > 0$.

Hence, it has been proven that a hypergraph $\mathbb{H}_{\mathbb{F}}$ has a hub number greater than 0 if and only if $\mathbb{H}_{\mathbb{F}}$ is not a complete hypergraph. □

4 Hub number of an intuitionistic fuzzy hypergraph

Definition 4.1. The hub set in an intuitionistic fuzzy hypergraph (IFHG) is an ordered pair $H = (U, \mathcal{E})$, where:

(i) $U = \{u_1, u_2, \ldots, u_n\}$, is a finite set of intuitionistic fuzzy vertices,

(ii) $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_m\}$ is a family of crisp subsets of U,

(iii) $\mathcal{E}_j = \{(u_i, \mu_j(u_i), \nu_j(u_i)) : \mu_j(u_i), \nu_j(u_i) \geq 0 \text{ and } \mu_j(u_i) + \nu_j(u_i) \leq 1\}, \quad j = 1, 2, \ldots, m$,

(iv) $\mathcal{E}_j \neq \emptyset$, $j = 1, 2, \ldots, m$,

(v) $\bigcup_j supp(\mathcal{E}_j) = U$, $j = 1, 2, \ldots, m$ (the support of an IFS U in $\mathcal{E}$ is denoted by $supp(\mathcal{E}_j) = \{u_i | \mu_{ij}(u_i) > 0$ and $\nu_{ij}(u_i) > 0\}$.)

(vi) the hub set in an IFHG is said to be a set $S \subseteq U$, there exists an intuitionistic fuzzy hub-hyperpath(P_{IFH}) which connects for any two end vertices $u_i, u_j \in U$ such that $P_{IFH}(U) \backslash \{u_i, u_j\} \in S$, where $u_i, u_j \notin S$.

Here, the hyperedges $\mathcal{E}_j$ are crisp sets of an intuitionistic fuzzy vertices, $\mu_j(u_i)$ and $\nu_j(u_i)$ denote the degrees of membership and non-membership of vertex u_i to hyperedge $\mathcal{E}_j$. Thus, the elements of the incidence matrix of IFHG are of the form $(u_{ij}, \mu_j(u_i), \nu_j(u_i))$. The sets $(U, \mathcal{E})$ are crisp sets.

Definition 4.2. Let $\mathbb{H}_{\mathbb{IF}} = (U, \mathcal{E})$ be an Intuitionistic Fuzzy Hypergraph(IFHG) and $S \subseteq U$ with two end vertices $u_i, u_j \in U$. Then the intuitionistic fuzzy hub-hyperpath P_{IFH} in $\mathbb{H}_{\mathbb{G}}$ from u_i to u_j is an alternative sequence of vertex and hyperedge say,

$$u_1 \quad \mathcal{E}_1 \quad u_2 \quad \mathcal{E}_2 \quad \cdots \quad u_n \quad \mathcal{E}_n \quad u_{n+1}$$

where $u_i = u_1$ and $u_j = u_{n+1}$ such that

- $u_i = u_1, u_2, \ldots, u_n, u_{n+1} = u_j$ are distinct vertices,
- $\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_n$ are distinct hyperedges with $u_i \in \mathcal{E}_r$, $u_j \in \mathcal{E}_s$ and $\mathcal{E}_r \cap \mathcal{E}_s = \emptyset$ with $r \neq s$,
- an intuitionistic fuzzy hub-hyperpath between u_i and u_j is a hyperpath with $P_{IFH}(U) \backslash \{u_i, u_j\} \in S$.

Definition 4.3. The hub number of an IFHG($\mathbb{H}_{\mathbb{IF}}$) is represented by $h(\mathbb{H}_{\mathbb{IF}})$ is the minimum cardinality of a hub set in $\mathbb{H}_{\mathbb{IF}}$.

Example 4.1. Consider an IFHG($\mathbb{H}_{\mathbb{IF}}$) $= (U, \mathcal{E})$, where $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$, $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3\}$. Then, the possible intuitionistic fuzzy hyperpaths are:

$$P_{IFH}\ 1\text{:}\ u_1 \quad \mathcal{E}_1 \quad u_2 \quad \mathcal{E}_2 \quad u_5 \quad \mathcal{E}_3 \quad u_7$$
$$P_{IFH}\ 2\text{:}\ u_1 \quad \mathcal{E}_1 \quad u_3 \quad \mathcal{E}_2 \quad u_5 \quad \mathcal{E}_3 \quad u_7$$

Therefore, the hub sets are $\{u_2, u_5\}$,$\{u_3, u_5\}$. The hub number of IFHG in Figure 3 is $h(\mathbb{H}_{\mathbb{G}}) = (0.4, 1.4)$.

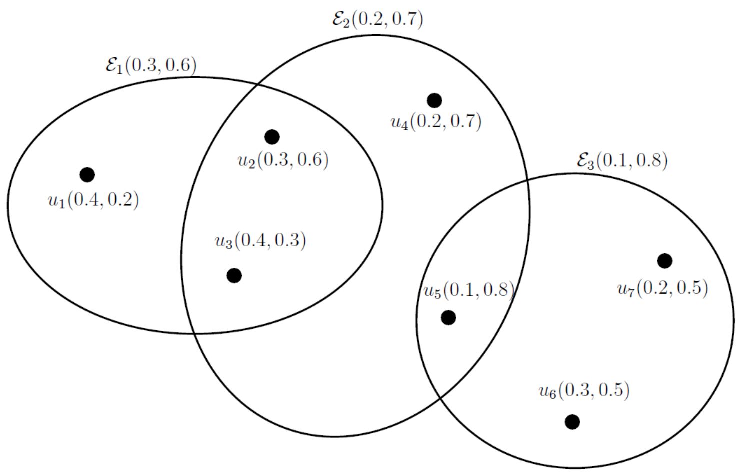

Figure 3. IFHG

Theorem 4.1. *If a set S is a hub set of IFHG, then for every pair of end vertices, $u_i, u_j \notin S$, there exists an intuitionistic fuzzy hub-hyperpath (P_{IFH}) between u_i and u_j.*

Proof. Suppose S be the hub set of an IFHG. Let u_i and u_j be two end vertices not in S. Since S is a hub set, there must exist an intuitionistic fuzzy hub-hyperpath(P_{IFH}) between u_i and u_j, as all intermediate vertices of this intuitionistic fuzzy hub-hyperpath(P_{IFH}) are from S. Thus, the theorem holds. □

Theorem 4.2. *If the hub number of an IFHG is the smallest size of a hub set S such that for every pair of end vertices, $u_i, u_j \notin S$, then there exists an intuitionistic fuzzy hub-hyperpath between them.*

Proof. To prove this, we need to satisfy the following conditions:

(i) There exists a hub set of size k for every IFHG with hub number k.

(ii) There does not exist a hub set of size less than k for any IFHG with hub number k.

Proof of (i): Let S be a hub set of size k for IFHG with hub number k. By definition, for every pair of end vertices, $u_i, u_j \notin S$, there exists an intuitionistic fuzzy hub-hyperpath between them. Thus, S satisfies the conditions of being a hub set.

Proof of (ii) Suppose there exists a hub set S of size less than k for IFHG with hub number k. This implies that there are k end vertices not in S such that there exists no intuitionistic fuzzy hub-hyperpath between them, contradicting the definition of the hub number. Therefore, there cannot exist a hub set of size less than k for IFHG with hub number k.

Combining both proofs, the hub number of an IFHG is indeed the smallest size of a hub set S satisfying the given conditions. □

5 Hub number of an intuitionistic fuzzy threshold hypergraph

Definition 5.1. *The hub set in an Intuitionistic Fuzzy Threshold Hypergraph (IFTHG) $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ is defined as follows:*

(i) $U = \{u_1, u_2, ..., u_n\}$ *is a finite set of intuitionistic fuzzy vertices,*

(ii) $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, ..., \mathcal{E}_m\}$ *is a family of crisp subsets of* U*,*

(iii) $\mathcal{E}_j = \{u_i, \mu_j(u_i), \nu_j(u_i)/0 \leq \mu_j(u_i) + \nu_j(u_i) \leq 1\}, j = 1, 2, ...m,$

(iv) $\mathcal{E}_j \neq \emptyset$, $j = 1, 2, \ldots, m,$

(v) $\bigcup_j supp(\mathcal{E}_j) = U$, $j = 1, 2, \ldots, m,$

(vi) an independent set $V \subseteq U$ has a set of all distinct combinations of a non-adjacent vertices in $\mathbb{H}_{\mathbb{G}}$ iff there exists a threshold values $s_1, s_2 > 0$ such that $\sum_{u_i \in V} \mu_j(u_i) \leq s_1$ and $\sum_{u_i \in V}(1 - \nu_j(u_i)) \leq s_2$,

(vii) the hub set in an IFTHG is said to be a set $S \subseteq U$, there exists an intuitionistic fuzzy hub-hyperpath(P_{IFTH}) which connects for any two end vertices $u_i, u_j \in U$ such that $P_{IFTH}(U) \backslash \{u_i, u_j\} \in S$, where $u_i, u_j \notin S$.

Definition 5.2. Let $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an Intuitionistic Fuzzy Threshold Hypergraph (IFTHG) and $S \subseteq U$ with any two vertices $u_i, u_j \in U$. Then the intuitionistic fuzzy hub-hyperpath P_{IFTH} in $\mathbb{H}_{\mathbb{G}}$ from u_i to u_j is an alternative sequence of vertex and hyperedge say,

$$u_1 \quad \mathcal{E}_1 \quad u_2 \quad \mathcal{E}_2 \quad \cdots \quad u_n \quad \mathcal{E}_n \quad u_{n+1}$$

where $u_i = u_1$ and $u_j = u_{n+1}$ such that

- $u_i = u_1, u_2, \ldots, u_n, u_{n+1} = u_j$ are distinct vertices,
- $\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_n$ are distinct hyperedges with $u_i \in \mathcal{E}_r$, $u_j \in \mathcal{E}_s$ and $\mathcal{E}_r \cap \mathcal{E}_s = \emptyset$ with $r \neq s$,
- an intuitionistic fuzzy hub-hyperpath between u_i and u_j is a hyperpath with $P_{IFTH}(U) \setminus \{u_i, u_j\} \in S$.

Definition 5.3. The hub number of an IFTHG $\mathbb{H}_{\mathbb{G}}$, denoted by $h(\mathbb{H}_{\mathbb{G}})$, is the minimum cardinality of a hub set in $\mathbb{H}_{\mathbb{G}}$.

Example 5.1. Consider an IFTHG $\mathbb{H}_{\mathbb{G}}$ with $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ vertices, $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3\}$. IF hyperedges with threshold value $\langle 0.3, 0.6 \rangle$. Then, the possible intuitionistic fuzzy hyperpath is:

$$P_{IFTH} : u_1 \quad \mathcal{E}_1 \quad u_6 \quad \mathcal{E}_3 \quad u_5$$

Therefore, the hub set is $\{u_6\}$. The hub number of above IFTHG is $h(\mathbb{H}_{\mathbb{G}}) = \langle 0.3, 0.6 \rangle$.

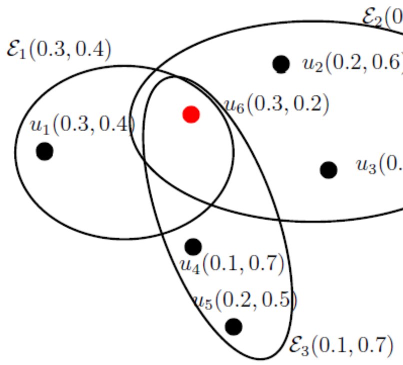

Figure 4. IFTHG

Theorem 5.1. *There is an intuitionistic fuzzy hub-hyperpath between for every pair of end vertices, $u_i, u_j \notin S$ if and only if the hub number of an IFTHG $h(\mathbb{H}_{\mathbb{G}})$ is the smallest size of a hub set S.*

Observation 2. *Every connected IFTHG has a hub set.*

Explanation. Consider any connected IFTHG. By the definition, "Consider $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an IFTHG on a non-empty set U is *connected* if every two distinct vertices in $\mathbb{H}_{\mathbb{G}}$ are linked by an intuitionistic fuzzy threshold hyperpath". We need to verify that every connected IFTHG possesses a hubset. Choose any vertex u_j in connected IFTHG $\mathbb{H}_{\mathbb{G}}$ as the initial hub set S. From the definition, for any pair of vertices, $u_i, u_k \notin S$, since $\mathbb{H}_{\mathbb{G}}$ is connected, there exists an intuitionistic fuzzy hub-hyperpath (P_{IFTH}) between u_i and u_k. Take any such intuitionistic fuzzy hub-hyperpath(P_{IFTH}) and add all its intermediate vertices to S. Repeat this process for all pairs of vertices not in S until no more vertices can be added. The resulting set S is a hub set of $\mathbb{H}_{\mathbb{G}}$.

5.1 Application

Imagine a wide amusement park featuring 51 exciting games and a food court. The map of the amusement park is illustrated in the Figure 5 below.

The actual problem is that a fast electric transport network can be developed and its stations placed in an amusement park so that a person just needs to go to an adjacent station, board the transport system, and then walk to the required games in order to travel between two nonadjacent games (which are not stations).

The data regarding the preferences of games rated on a scale of 0 to 10, among various age groups has been tabulated below for clarity and analysis (Table 1).

Table 1. Preference level of games

S.No.	Game	Preference level	S.No.	Game	Preference level
1	Free fall tower	8.5	27	Centrox	8.25
2	Child bumper car	8.25	28	Video plaza	7.75
3	Octopus	7.5	29	Disco fly	7.5
4	Super tele combat	7.75	30	Crazy horses	8
5	Trampolin	8.25	31	Boating	9.25
6	Alpen blitz	8.5	32	Simulator	8.5
7	Roller coaster	9	33	Cable car	8.25
8	Enterprise	8.75	34	Fun house	7.5
9	Pirata	7.5	35	Mirror house	8.75
10	Kids taxi	8	36	Dark house	9
11	Mini avio	7.75	37	Go kart	7.5
12	Tora tora	8.25	38	Battery car	8.25
13	Swing weight	9	39	Battery scooter	7.75
14	Ventura river	7.5	40	Bumper boat	8
15	Carousel	8.75	41	Queens express train	8.25
16	Space journey	7.5	42	Himalayan water ride	7.75
17	Mini wheel	8.25	43	Swimming pool	9
18	Orchestra	7.75	44	Frog slide	7.75
19	Mini jet	8.75	45	Mush room	8.25
20	Dragon fly	8.5	46	Play system	8
21	Children play games	9	47	Covered twisters	8.25
22	Mini train	7.5	48	3 Lane slides	8.5
23	Dry pool	8.25	49	Free fall slide	7.75
24	Adult bumper car	8.75	50	Half twister	7.5
25	Cliff hanger	9	51	American wave pool	7.75
26	Thumbrole	8.75	52	Food court	7.5

Figure 5. Amusement park (Image source: https://www.queenslandamusementpark.com/map.htm)

The tabulated data from Table 1 is represented graphically below in Figure 6.

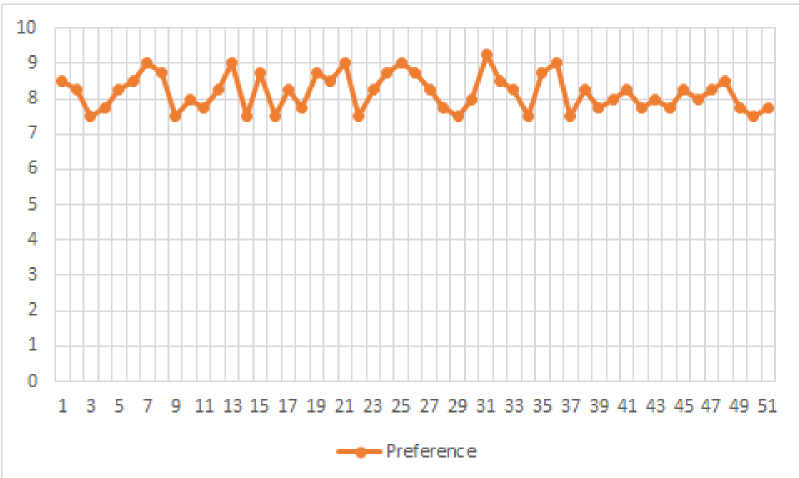

Figure 6. Data

Fuzzification of intuitionistic fuzzy data is the process of converting crisp data into intuitionistic fuzzy sets, allowing for more accurate and flexible representation of uncertainty and ambiguity. And intuitionistic fuzzification functions [8] involve the formulation of membership and non-membership functions of an Intuitionistic Fuzzy Set (IFS). The Intuitionistic Fuzzy Triangular Function (IFTriF), is specified by three parameters, a lower limit a, an upper limit c, and a value b, where $a \leq b \leq c$.

Intuitionistic fuzzy triangular membership function takes the form

$$\mu_j(x) = \begin{cases} 0; & x = a \\ (\frac{x-a}{b-a}) - \epsilon; & a < x \leq b \\ (\frac{c-x}{c-b}) - \epsilon; & b \leq x < c \\ 0; & x = c \end{cases}$$

The corresponding, intuitionistic fuzzy triangular non-membership function is of the form

$$\nu_j(x) = \begin{cases} 1 - \epsilon; & x = a \\ 1 - (\frac{x-a}{b-a}); & a < x \leq b \\ 1 - (\frac{c-x}{c-b}); & b \leq x < c \\ 1 - \epsilon; & x = c \end{cases}$$

In this model, data on game preferences and dislikes are rated on a scale from 0 to 10. Subsequently, triangular functions representing membership and non-membership are specified by three parameters: $a = 0, b = 5$, and $c = 10$. The functions are defined as follows:

$$\mu_j(x) = \begin{cases} 0; & x = 0 \\ (\frac{x-0}{5-0}) - \epsilon; & 0 < x \leq 5 \\ (\frac{10-x}{10-5}) - \epsilon; & 5 \leq x < 10 \\ 0; & x = 10 \end{cases}$$

$$\nu_j(x) = \begin{cases} 1 - \epsilon; & x = 0 \\ 1 - (\frac{x-0}{5-0}); & 0 < x \leq 5 \\ 1 - (\frac{10-x}{10-5}); & 5 \leq x < 10 \\ 1 - \epsilon; & x = 10 \end{cases}$$

The original preference data on a scale from 0 to 10 has been transformed into membership and non-membership values on a scale from 0 to 1.

This change in scale helps in interpreting the data in terms of fuzzy logic, where values closer to 1 indicate higher degrees of membership or non-membership.

While the original data suggests a relatively stable preference, the fuzzified data reveals more about the underlying distribution of preferences in terms of membership and non-membership.

This detailed view can help in better understanding the dynamics of preferences and making more informed decisions based on the degree of association with the criteria.

Due to high preference for playing a game, the interpretation of the intuitionistic fuzzy membership value denotes availability of that particular game and non-membership denotes the non-availability, respectively. This has been designed according to threshold values.

Table 2. Membership and non-membership values of availability and non-availability of the amusement part games (according to the preference)

S.No.	Game	$\mu(u_i)$	$\nu(u_i)$	S.No.	Game	$\mu(u_i)$	$\nu(u_i)$
1	Free fall tower	0.2	0.7	27	Centrox	0.25	0.65
2	Child bumper car	0.25	0.65	28	Video plaza	0.35	0.55
3	Octopus	0.4	0.5	29	Disco fly	0.4	0.5
4	Super tele combat	0.35	0.55	30	Crazy horses	0.3	0.6
5	Trampolin	0.25	0.65	31	Boating	0.05	0.85
6	Alpen blitz	0.2	0.7	32	Simulator	0.2	0.7
7	Roller coaster	0.1	0.8	33	Cable car	0.25	0.65
8	Enterprise	0.15	0.75	34	Fun house	0.4	0.5
9	Pirata	0.4	0.5	35	Mirror house	0.15	0.75
10	Kids taxi	0.3	0.6	36	Dark house	0.1	0.8
11	Mini avio	0.35	0.55	37	Go kart	0.4	0.5
12	Tora tora	0.25	0.65	38	Battery car	0.25	0.65
13	Swing weight	0.1	0.8	39	Battery scooter	0.35	0.55
14	Ventura river	0.4	0.5	40	Bumper boat	0.3	0.6
15	Carousel	0.15	0.75	41	Queens express train	0.25	0.65
16	Space journey	0.4	0.5	42	Himalayan water ride	0.35	0.55
17	Mini wheel	0.25	0.65	43	Swimming pool	0.1	0.8
18	Orchestra	0.35	0.55	44	Frog slide	0.35	0.55
19	Mini jet	0.15	0.75	45	Mush room	0.25	0.65
20	Dragon fly	0.2	0.7	46	Play system	0.3	0.6
21	Children play games	0.1	0.8	47	Covered twisters	0.25	0.65
22	Mini train	0.4	0.5	48	3 Lane slides	0.2	0.7
23	Dry pool	0.25	0.65	49	Free fall slide	0.35	0.55
24	Adult bumper car	0.15	0.75	50	Half twister	0.4	0.5
25	Cliff hanger	0.1	0.8	51	American wave pool	0.35	0.55
26	Thumbrole	0.15	0.75	52	Food court	0.4	0.5

In this model, a wide amusement park with 51 games is illustrated by an IFTHG. Here, games represents vertices $U = \{u_1, u_2, ..., u_{51}\}$ and a hyperedges $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4, \mathcal{E}_5\}$ indicates group of games which is categorized by its types like Normal & Gentle Rides, Water Games, Thrill and Adventure Rides, Thematic Experiences and Skill Games.

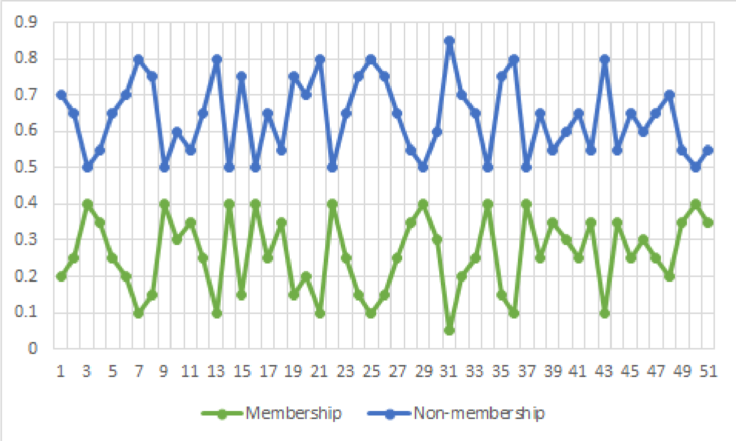

Figure 7. Fuzzified data

There are often cases of uncertainty, such as maintenance issues, scheduling constraints, insufficient timing and player preferences for playing or not playing a particular game. Entry and exit do not belong to the set. Threshold values of IFTHG denote the preference values of cafe.

In this model, the hyperedges are considered as follows

- $\mathcal{E}_1 = \{u_2, u_3, u_5, u_{10}, u_{11}, u_{15}, u_{16}, u_{17}, u_{18}, u_{19}, u_{21}, u_{22}, u_{23}, u_{24}, u_{28}, u_{31}, u_{32}, u_{33}, u_{34}, u_{35}, u_{38}, u_{39}, u_{40}, u_{41}, u_{45}, u_{46}, u_{51}\}$
- $\mathcal{E}_2 = \{u_{14}, u_{31}, u_{40}, u_{42}, u_{43}, u_{44}, u_{45}, u_{47}, u_{48}, u_{49}, u_{50}, u_{51}\}$
- $\mathcal{E}_3 = \{u_1, u_4, u_6, u_7, u_8, u_9, u_{12}, u_{13}, u_{20}, u_{25}, u_{26}, u_{27}, u_{29}, u_{30}, u_{36}, u_{37}\}$
- $\mathcal{E}_4 = \{u_8, u_9, u_{16}, u_{18}, u_{28}, u_{32}, u_{35}, u_{36}, u_{46}\}$
- $\mathcal{E}_5 = \{u_5, u_{14}, u_{21}, u_{28}, u_{29}, u_{30}\}$

The following IFTHG denotes the pictorial representation of Figure 8.

The values of the vertices and hyperedges are:
$u_1(0.2, 0.7)$, $u_2(0.25, 0.65)$, $u_3(0.4, 0.5)$, $u_4(0.35, 0.55)$, $u_5(0.25, 0.65)$, $u_6(0.2, 0.7)$, $u_7(0.1, 0.8)$, $u_8(0.15, 0.75)$, $u_9(0.4, 0.5)$, $u_{10}(0.3, 0.6)$, $u_{11}(0.35, 0.55)$, $u_{12}(0.25, 0.65)$, $u_{13}(0.1, 0.8)$, $u_{14}(0.4, 0.5)$, $u_{15}(0.15, 0.75)$, $u_{16}(0.4, 0.5)$, $u_{17}(0.25, 0.65)$, $u_{18}(0.35, 0.55)$, $u_{19}(0.15, 0.75)$, $u_{20}(0.2, 0.7)$, $u_{21}(0.1, 0.8)$, $u_{22}(0.4, 0.5)$, $u_{23}(0.25, 0.65)$, $u_{24}(0.15, 0.75)$, $u_{25}(0.1, 0.8)$, $u_{26}(0.15, 0.75)$, $u_{27}(0.25, 0.65)$, $u_{28}(0.35, 0.55)$, $u_{29}(0.4, 0.5)$, $u_{30}(0.3, 0.6)$, $u_{31}(0.05, 0.85)$, $u_{32}(0.2, 0.7)$, $u_{33}(0.25, 0.65)$, $u_{34}(0.4, 0.5)$, $u_{35}(0.15, 0.75)$, $u_{36}(0.1, 0.8)$, $u_{37}(0.4, 0.5)$, $u_{38}(0.25, 0.65)$, $u_{39}(0.35, 0.55)$, $u_{40}(0.3, 0.6)$, $u_{41}(0.25, 0.65)$, $u_{42}(0.35, 0.55)$, $u_{43}(0.1, 0.8)$, $u_{44}(0.35, 0.55)$, $u_{45}(0.25, 0.65)$, $u_{46}(0.3, 0.6)$, $u_{47}(0.25, 0.65)$, $u_{48}(0.2, 0.7)$, $u_{49}(0.35, 0.55)$, $u_{50}(0.4, 0.5)$, $u_{51}(0.35, 0.55)$, $u_{52}(0.4, 0.5)$

and

$$\mathcal{E}_1(0.1, 0.8), \mathcal{E}_2(0.05, 0.85), \mathcal{E}_3(0.1, 0.8), \mathcal{E}_4(0.1, 0.8), \mathcal{E}_5(0.1, 0.8).$$

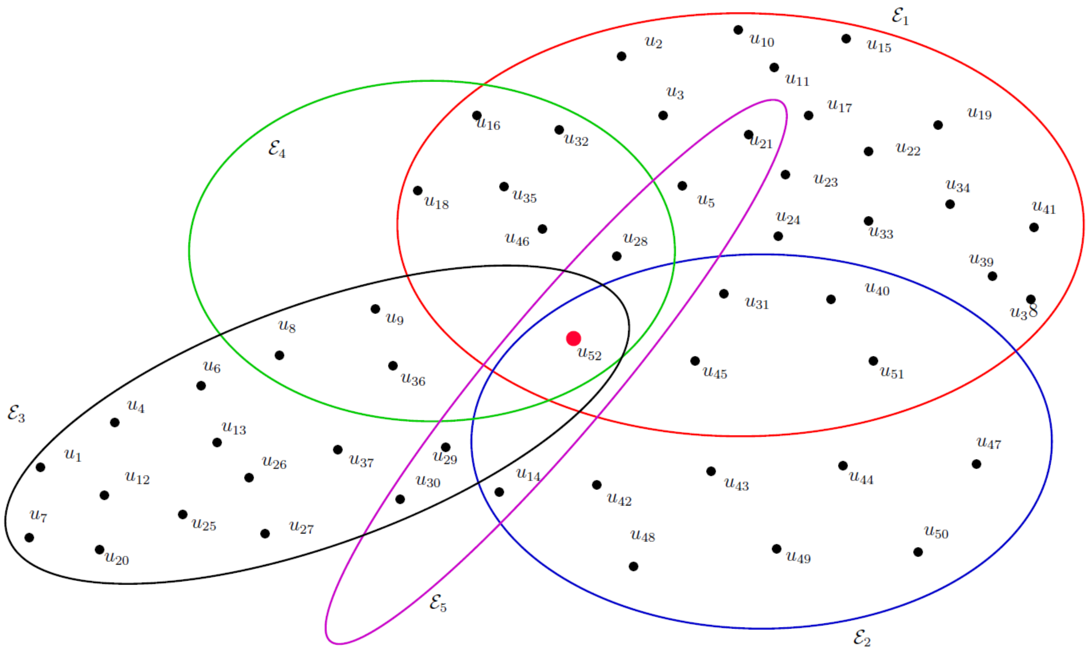

Figure 8. Amusement park

The possible intuitionistic fuzzy threshold hyperpaths are:

1. $u_1 \to \mathcal{E}_3 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
2. $u_1 \to \mathcal{E}_3 \to u_{29} \to \mathcal{E}_5 \to u_{14} \to \mathcal{E}_2 \to u_{51}$
3. $u_1 \to \mathcal{E}_3 \to u_{30} \to \mathcal{E}_5 \to u_{14} \to \mathcal{E}_2 \to u_{51}$
4. $u_1 \to \mathcal{E}_3 \to u_8 \to \mathcal{E}_4 \to u_{16} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
5. $u_1 \to \mathcal{E}_3 \to u_8 \to \mathcal{E}_4 \to u_{18} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
6. $u_1 \to \mathcal{E}_3 \to u_8 \to \mathcal{E}_4 \to u_{32} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
7. $u_1 \to \mathcal{E}_3 \to u_8 \to \mathcal{E}_4 \to u_{35} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
8. $u_1 \to \mathcal{E}_3 \to u_8 \to \mathcal{E}_4 \to u_{46} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
9. $u_1 \to \mathcal{E}_3 \to u_8 \to \mathcal{E}_4 \to u_{28} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
10. $u_1 \to \mathcal{E}_3 \to u_8 \to \mathcal{E}_4 \to u_{28} \to \mathcal{E}_1 \to u_{51}$
11. $u_1 \to \mathcal{E}_3 \to u_9 \to \mathcal{E}_4 \to u_{16} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
12. $u_1 \to \mathcal{E}_3 \to u_9 \to \mathcal{E}_4 \to u_{18} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
13. $u_1 \to \mathcal{E}_3 \to u_9 \to \mathcal{E}_4 \to u_{32} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
14. $u_1 \to \mathcal{E}_3 \to u_9 \to \mathcal{E}_4 \to u_{35} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
15. $u_1 \to \mathcal{E}_3 \to u_9 \to \mathcal{E}_4 \to u_{46} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
16. $u_1 \to \mathcal{E}_3 \to u_9 \to \mathcal{E}_4 \to u_{28} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
17. $u_1 \to \mathcal{E}_3 \to u_9 \to \mathcal{E}_4 \to u_{28} \to \mathcal{E}_1 \to u_{51}$
18. $u_1 \to \mathcal{E}_3 \to u_{36} \to \mathcal{E}_4 \to u_{16} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
19. $u_1 \to \mathcal{E}_3 \to u_{36} \to \mathcal{E}_4 \to u_{18} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
20. $u_1 \to \mathcal{E}_3 \to u_{36} \to \mathcal{E}_4 \to u_{32} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
21. $u_1 \to \mathcal{E}_3 \to u_{36} \to \mathcal{E}_4 \to u_{35} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
22. $u_1 \to \mathcal{E}_3 \to u_{36} \to \mathcal{E}_4 \to u_{46} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$
23. $u_1 \to \mathcal{E}_3 \to u_{36} \to \mathcal{E}_4 \to u_{28} \to \mathcal{E}_1 \to u_{52} \to \mathcal{E}_2 \to u_{51}$

24. $u_1 \to \mathcal{E}_3 \to u_{36} \to \mathcal{E}_4 \to u_{28} \to \mathcal{E}_1 \to u_{51}$.

From these results, the hub sets are, $\{u_{52}\}$, $\{u_{29}, u_{14}\}$, $\{u_{30}, u_{14}\}$, $\{u_8, u_{16}, u_{52}\}$, $\{u_8, u_{18}, u_{52}\}$, $\{u_8, u_{32}, u_{52}\}$, $\{u_8, u_{35}, u_{52}\}$, $\{u_8, u_{46}, u_{52}\}$, $\{u_8, u_{28}, u_{52}\}$, $\{u_8, u_{28}\}$, $\{u_9, u_{16}, u_{52}\}$, $\{u_9, u_{18}, u_{52}\}$, $\{u_9, u_{32}, u_{52}\}$, $\{u_9, u_{35}, u_{52}\}$, $\{u_9, u_{46}, u_{52}\}$, $\{u_9, u_{28}, u_{52}\}$, $\{u_9, u_{28}\}$, $\{u_{36}, u_{16}, u_{52}\}$, $\{u_{36}, u_{18}, u_{52}\}$, $\{u_{36}, u_{32}, u_{52}\}$, $\{u_{36}, u_{35}, u_{52}\}$, $\{u_{36}, u_{46}, u_{52}\}$, $\{u_{36}, u_{28}, u_{52}\}$, $\{u_{36}, u_{28}\}$.

Therefore, hub number of this IFTHG is $\langle 0.4, 0.5\rangle$.

Each hyperedge must contain at least one electric train station to ensure accessibility. Within the same hyperedge, people can easily walk between points. In this system, u_{52} serves as a critical hub point connecting game stations 1 and 51. Consequently, the optimal route for people to move between these two game stations is $u_1 \to \mathcal{E}_3 \to u_{52} \to \mathcal{E}_2 \to u_{51}$, providing the most efficient path to reach their destination.

The actual problem is to determine the optimal locations for installing electric transport stations in a way that minimizes costs while maximizing convenience for visitors. Now, we focus on connecting the high-preference games by placing stations along key routes, optimizing the placement of stations within the hyperedges for better accessibility. Here is the algorithm to determine the optimal placement of an electric transport system for efficiently moving between different games in the amusement park.

Algorithm

Step 1: Represent the amusement park map as an IFTHG
Represent each game in the park as a vertex U_i. Define hyperedges $\mathcal{E}_j$ corresponding to the themes of games, where each hyperedge includes a subset of vertices U_i that represent games under that theme.

Step 2: Assign membership and non-membership values to vertices
Assign a membership value μ_{ij} and non-membership value ν_{ij} based on the relevance of game.

Step 3: Identify connecting key games between hyperedges
Determine the vertices (games) that are common to both hyperedges $\mathcal{E}_i$ and $\mathcal{E}_j$. These vertices are the connecting key games between the two hyperedges.

Step 4: Apply the Cartesian product to determine the exact station placement
Evaluate each key game using the Cartesian product:
Calculate the minimum membership value and maximum non-membership among the key games using

$$U_1 \times_4 U_2 \times_4 \ldots \times_4 U_n = \{\langle (u_1, u_2, \ldots, u_n), \min(\mu_1, \mu_2, \ldots, \mu_n), \max(\nu_1, \nu_2, \ldots, \nu_n)\rangle \\ \mid u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n\}$$

Step 5: Select the optimal game
Choose the key game with the highest minimum membership value and the lowest maximum non-membership value as the exact location for station placement.

From the algorithm, the connecting games between the hyperedges are:
$\mathcal{E}_1 \to \mathcal{E}_2 \implies u_{31}, u_{40}, u_{41}, u_{45}$
$\mathcal{E}_2 \to \mathcal{E}_5 \implies u_{14}$
$\mathcal{E}_3 \to \mathcal{E}_4 \implies u_8, u_9, u_{36}$
$\mathcal{E}_3 \to \mathcal{E}_5 \implies u_{29}, u_{30}$
$\mathcal{E}_1 \to \mathcal{E}_5 \implies u_{28}, u_5, u_{21}$
$\mathcal{E}_4 \to \mathcal{E}_1 \implies u_{16}, u_{18}, u_{32}, u_{35}, u_{46}, u_{28}$
Based on the preference levels, the electric stations should be placed near the following locations: $u_{31}, u_{21}, u_{14}, u_{36}, u_{30}, u_{35}, u_{52}$. This placement will help optimize accessibility and ensure that the most popular games are easily reachable via the transport network.

6 Conclusion

This paper provides a comprehensive study of the hub hyperpath, hub set, and hub number within the context of fuzzy hypergraphs and intuitionistic fuzzy hypergraphs. By defining these concepts and illustrating their applications, particularly in transportation systems, we demonstrate the practical relevance and potential of fuzzy hypergraphs in real-world scenarios. The numerical example and the application in constructing a transport electric system for amusement parks highlight the utility of these theoretical constructs. Our findings contribute to the broader understanding of hypergraph theory and its extensions, opening avenues for future research in optimization and network design using fuzzy and intuitionistic fuzzy hypergraph environments. In the same way, we can find out the best way to move the next game station by calculating other games.

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