Notes on Intuitionistic Fuzzy Sets
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Intuitionistic fuzzy histograms in grid-based clustering

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Abstract: In the present paper a distribution of points is organized in the form of histograms like the result from grid-based clustering algorithm. It is supposed that the output can be received by anyone grid-based clustering method. The result is transformed in the form of histogram and the modal operators from the intuitionistic fuzzy sets theory are applied.
Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy histograms, Intuitionistic fuzzy operators.
AMS Classification: 03E72.

1 Introduction

In the present paper, intuitionistic fuzzy histograms are constructed. They can be used for analysis and visualization the points of distribution in the place. Over the intuitionistic fuzzy histograms can be applied the operators from intuitionistic fuzzy sets. This concept is introduced in [5]. On Fig. 3 is shown a 2D place with different type of points which are used for clustering. Some of most popular grid-based clustering algorithms are STING (a STatistical INformation Grid approach) by Wang, Yang and Muntz (1997) [7], WaveCluster by Sheikholeslami, Chatterjee, and Zhang (VLDB'98) [6] and CLIQUE by Agrawal, et al. (SIGMOD'98) [1].

2 Basic remarks on intuitionistic fuzzy histograms

The result of clustering analysis is presented in the form of a three clusters, one group of different points and empty cells (Fig. 2, Fig. 3). This is an output given by anyone grid-based clustering algorithm (we need to apply operators over the output of grid-based clustering analysis). The operators from intuitionistic fuzzy sets over the result can be applied, [2, 4]. Their geometric interpretation is presented on the Fig. 1.

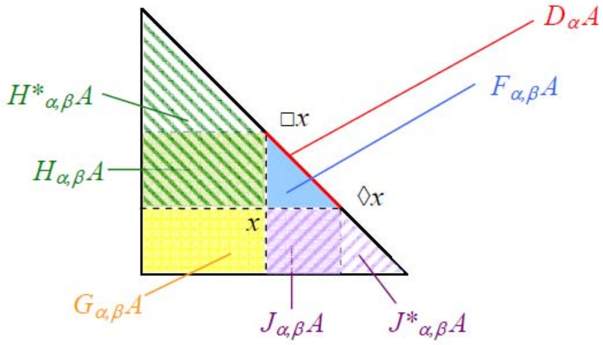

Figure 1. The geometric interpretation of the operators over intuitionistic fuzzy sets

The degree of membership in each column is filled with the color for cluster, the degree of uncertainty is shown by the gray squares, and the degree of non-membership is presented by white squares (Fig. 4, Fig. 5, Fig. 6).

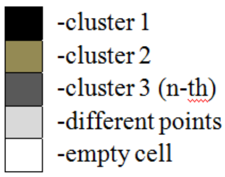

Figure 2. Legend

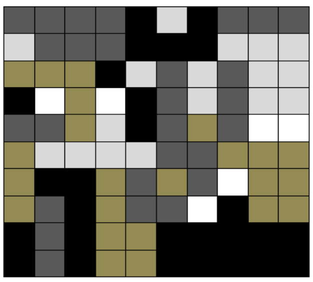

Figure 3. Grid-Based Clustering

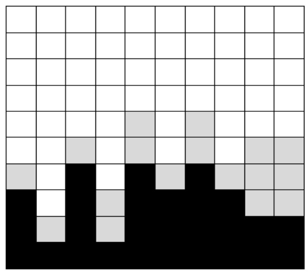

Figure 4. Histogram 1: Cluster 1

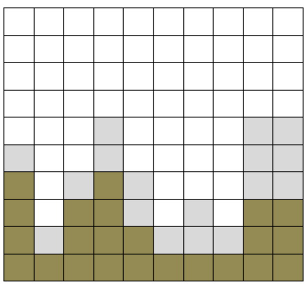

Figure 5. Histogram 2: Cluster 2

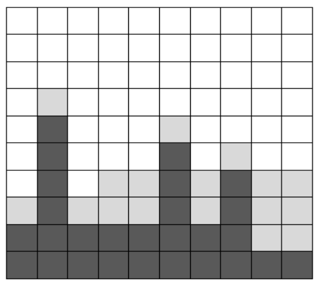

Figure 6. Histogram 3: Cluster 3

2.1 On intuitionistic fuzzy histograms in grid-based clustering

The concepts of *N*-histogram, *A*-histogram and *P*-histogram are introduced in [3]. Operator "necessity" is applied over the input data and it counts the gray squares for white. The obtained histogram is called *N*-histogram. If we have two grey squares, containing different points from all clusters (π), then the *A*-histogram will determine that the first square belongs to the cluster, and will fill it in with its colour (μ), while the second square will be coloured in white as an empty cell (ν). The histogram over which the operator "possibility" has been applied is called *P*-histogram.

2.2 New intuitionistic fuzzy histograms in grid-based clustering

Before constructing the new intuitionistic fuzzy histograms, the *N*-histogram, the *A*-histogram and the *P*-histogram will be constructed for the case of grid-based clustering. The "necessity" operator have the following form:

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle \mid x \in E\},$$

where it gives the following pairs of values for μ and ν:

$$(\mu,\nu) = \{(0.3, 0.7); (0.1, 0.9); (0.4, 0.6); (0.1, 0.9); (0.4, 0.6); (0.3, 0.7); (0.4, 0.6); (0.3, 0.7); (0.2, 0.8); (0.2, 0.8)\},$$

and the intuitionistic fuzzy histogram is presented on the Fig. 7.

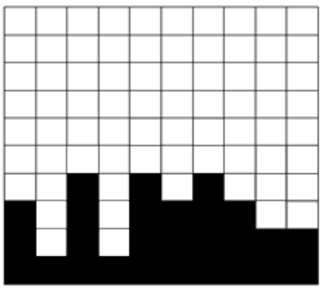

Figure 7. *N*-histogram in the case of grid-based clustering for *A* = Cluster 1

The possibility operator have the following form:

$$\Diamond A=\{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle \mid x \in E\},$$

where it gives the pairs of values for μ and ν:

$$(\mu,\nu) = \{(0.4, 0.6); (0.2, 0.8); (0.5, 0.5); (0.3, 0.7); (0.6, 0.4); (0.4, 0.6); (0.6, 0.4); (0.4, 0.6); (0.5, 0.5); (0.5,0.5)\},$$

and the intuitionistic fuzzy histogram is presented on the Fig. 8.

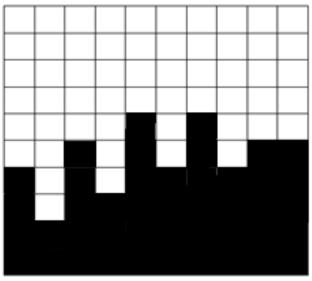

Figure 8. P-histogram in the case of grid-based clustering for A = Cluster 1

Let us consider the squares including different points and which not belongs to cluster 1 to have the possibility 50% to belongs to him and 50% to belongs to other clusters. $\alpha \in [0, 1]$ and will be fixed number.

Operator $D_\alpha(A)$ presents both the operators "◊" and "□" (Fig. 9). It presents the connection between them and changes the degree of uncertainty. It transforms intuitionistic fuzzy set in fuzzy set.

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1 - \alpha).\pi_A(x)\rangle \mid x \in E\}.$$

Let us suppose that value $\alpha = 0.5$ corresponds to a half filled square. The obtained values for μ and ν are:

$$(\mu, \nu) = \{(0.35,0.65); (0.15, 0.85); (0.45, 0.55); (0.20,0.80); (0.50,0.50); (0.35,0.65); (0.50,0.50); (0.35,0.65); (0.35,0.65); (0.35,0.65)\}.$$

The intuitionistic fuzzy histogram received from applying the operator $D_\alpha(A)$ is presented on the Fig. 9.

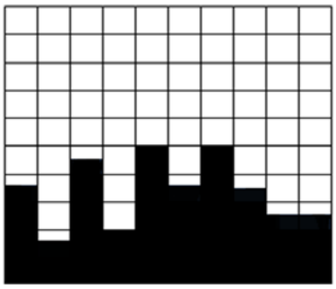

Figure 9. Operator $D_\alpha(A)$, where $\alpha = 0.5$, A = Cluster 1

Let us suppose that the squares with different points have the possibility to belong to cluster 1 – α and not to belong to cluster 1 – β. Operator $F_{\alpha,\beta}(A)$ changes the degree of uncertainty.

$$F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle \mid x \in E\},$$

where $\alpha, \beta \in [0; 1]$ are fixed numbers. In this case, there are $\alpha = 0.6$ and $\beta = 0.2$. Operator $F_{\alpha,\beta}(A)$ gives the following values for squares (Fig. 10).

(μ, ν) = (0.36, 0.62); (0.16, 0.82); (0.46, 0.52); (0.22, 0.74); (0.52, 0.44); (0.36, 0.62); (0.52, 0.44); (0.36, 0.62); (0.38, 0.56) ; (0.38, 0.56)

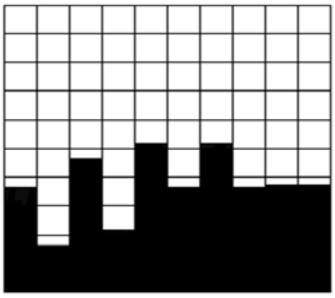

Figure 10. Operator $F_{\alpha,\beta}(A)$, where $\alpha = 0.6$, $\beta = 0.2$, A = Cluster 1

Operator $G_{\alpha,\beta}(A)$ has no analogues in the ordinary modal logic.

$$G_{\alpha,\beta}(A) = \{\langle x,\ \alpha.\mu_A(x), \beta.\nu_A(x)\rangle \mid x \in E\}$$

where $\alpha, \beta \in [0; 1]$ are fixed numbers. For $\alpha = 0.6$ and $\beta = 0.2$, operator $G_{\alpha,\beta}(A)$ is presented by the following values for the histogram on the Fig. 11.

(μ, ν) = (0.18, 0.12); (0.06,0.16); (0.24, 0.10); (0.06, 0.14); (0.24, 0.08); (0.18, 0.12); (0.24, 0.08);(0.18, 0.12); (0.12, 0.10); (0.12, 0.10).

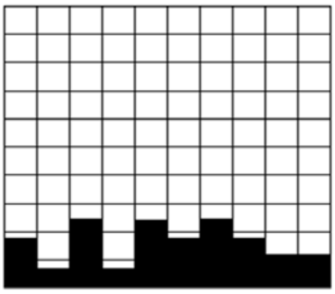

Figure 11. Operator $G_{\alpha,\beta}(A)$ where $\alpha = 0.6$, $\beta = 0.2$, A = Cluster 1

In the following text the operator $H_{\alpha,\beta}(A)$ is applied over intuitionistic fuzzy histogram.

$$H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x),\ \nu_A(x) + \beta.\pi_A(x)\rangle \mid x \in E\},$$

$\alpha, \beta \in [0,1]$ are fixed numbers. Let $\alpha = 0.6$ and $\beta = 0.2$. Operator $H_{\alpha,\beta}(A)$ is presented on the histogram in Fig. 12:

(μ, ν) = (0.18, 0.62); (0.06, 0.82); (0.24, 0.52); (0.06, 0.74);(0.24, 0.44); (0.18, 0.62); (0.24, 0.44); (0.18, 0.62); (0.12, 0.56); (0.12, 0.56).

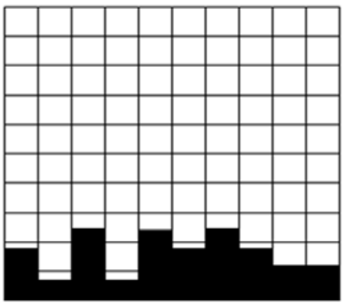

Figure 12. Operator $H_{\alpha,\beta}$(A) where $\alpha = 0.6$, $\beta = 0.2$ for A = Cluster 1

Operator $H^*_{\alpha,\beta}(A)$ is applied over intuitionistic fuzzy histogram:

$$H^*_{\alpha,\beta}(A) = \{\langle x,\ \alpha.\mu_A(x),\ \nu_A(x) + \beta(1 - \alpha.\mu_A(x) - \nu_A(x))\rangle \mid x \in E\},$$

where α, $\beta \in [0, 1]$ are fixed numbers. Let us suppose that $\alpha = 0.6$ and $\beta = 0.2$. Operator $H^*_{\alpha,\beta}(A)$ is shown by the following pairs of values for μ and ν on the histogram in Fig. 13:

(μ, ν) = (0.18, 0.644); (0.06, 0.828); (0.24, 0.552); (0.06, 0.748); (0.24, 0.472);
(0.18, 0.644); (0.24, 0.472); (0.18, 0.644); (0.12, 0.576); (0.12, 0.576).

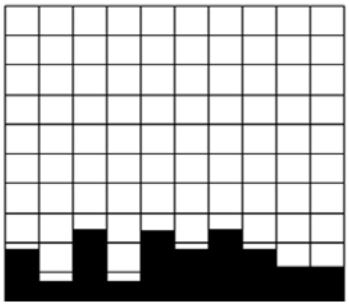

Figure 13. Operator $H^*_{\alpha,\beta}$(A) where $\alpha = 0.6$, $\beta = 0.2$ for A = Cluster 1

On Fig. 14, intuitionistic fuzzy histogtam operator $J_{\alpha,\beta}(A)$ is presented.

$$J_{\alpha,\beta}(A) = \{\langle x,\ \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x)\rangle \mid x \in E\},$$

where α, $\beta \in [0, 1]$ are fixed numbers. It is written α=0.6 and β=0.2. Operator $J_{\alpha,\beta}$(A) is given the following pairs of values for for μ and ν:

(μ, ν) = (0.36, 0.12); (0.16, 0.16); (0.46, 0.1); (0.22, 0.14); (0.52, 0.08);
(0.36, 0.12); (0.52, 0.08); (0.36, 0.12); (0.38, 0.1); (0.38, 0.1).

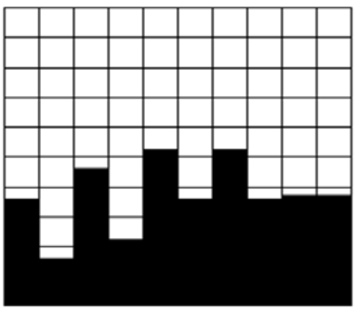

Figure 14. Operator $J_{\alpha,\beta}(A)$ where $\alpha = 0.6$, $\beta = 0.2$, A = Cluster 1

On the following intuitionistic fuzzy histogtam (Fig. 15) operator $J^*_{\alpha,\beta}(A)$ is shown:

$$J^*_{\alpha,\beta}(A) = \{\langle x,\ \mu_A(x) + \alpha.(1 - \mu_A(x) - \beta.\nu_A(x)), \beta.\nu_A(x)\rangle \mid x \in E\},$$

where α, $\beta \in [0, 1]$ are fixed numbers. It is written $\alpha = 0.6$ and $\beta = 0.2$. Operator $J^*_{\alpha,\beta}(A)$ is presented by following values on the histogram:

(μ, ν) = (0.648, 0.12); (0.544, 0.16); (0.7, 0.1); (0.568, 0.14); (0.592, 0.08); (0.648, 0.12); (0.712, 0.08); (0.648, 0.12); (0.584, 0.1); (0.584, 0.1).

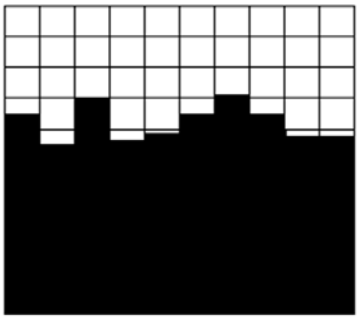

Figure 15. Operator $J^*_{\alpha,\beta}(A)$, $\alpha = 0.6$, $\beta = 0.2$, A = Cluster 1

Operator $X_{a,b,c,d,e,f}(A)$ is universal and includes as partial cases all operators of modal type, [3].

$$X_{a,b,c,d,e,f}(A) = \{\langle x, a.\nu_A(x) + b.(1 - \nu_A(x) - c.\mu_A(x)), \\ d.\mu_A(x) + e.(1 - f.\nu_A(x) - \mu_A(x))\rangle \mid x \in E\},$$

where $a, b, c, d, e, f \in [0, 1]$ are fixed numbers: $a = 0.2$, $b = 0.3$, $c = 0.1$, $d = 0.2$, $e = 0.4$, $f = 0.3$ and $a + e - e.f \leq 1$, $b + d - b.c \leq 1$.

Operator $X_{a,b,c,d,e,f}(A)$ is counted and presented with the following pairs of numbers on the histogram on Fig. 16.

(μ, ν) = (0.23, 0.26); (0.22, 0.26); (0.238, 0.26); (0.23, 0,71); (0.25, 0.27); (0.23, 0.26); (0.25, 0.27); (0.23, 0.26); (0.24, 0.03); (0.24, 0.03)

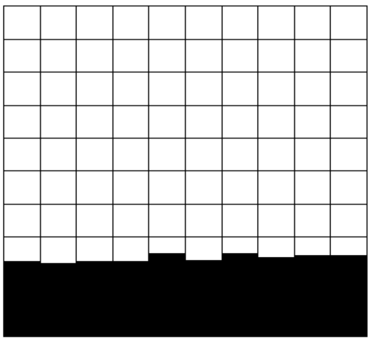

Figure 16. Operator $X_{a,b,c,d,e,f}(A)$

3 Conclusions

In the current paper, the histograms of the modal operators and their extensions from intuitionistic fuzzy sets theory are shown. In the presented example they are obtained by clustering analysis. The histograms can be used for the results visualization and the values analysis.

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