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Alpha generalized Homeomorphism in Intuitionistic Fuzzy Topological Spaces

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Abstract: This paper introduces intuitionistic fuzzy alpha generalized homeomorphism and intuitionistic fuzzy M - alpha generalized homeomorphism in intuitionistic fuzzy topological spaces. Also they are related to the fundamental concepts of intuitionistic continuous mappings and intuitionistic fuzzy open mappings.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy alpha generalized closed set, intuitionistic fuzzy alpha generalized continuous mapping, intuitionistic fuzzy alpha generalized homeomorphism, intuitionistic fuzzy M-alpha generalized homeomorphism.

AMS Classification code: 54A40, 03E72

1 Introduction

The concept of fuzzy sets was introduced by Zadeh [12] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy alpha generalized homeomorphism and intuitionistic fuzzy M-alpha generalized homeomorphism. Also they are related to the fundamental concepts of intuitionistic fuzzy continuous mappings and intuitionistic fuzzy open mappings. We provide some characterizations of intuitionistic fuzzy alpha generalized homeomorphism.

2 Preliminaries

Definition 2.1: [1] An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$$

where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form

$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \{\langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle\}$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1 \rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3:[3] An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$int(A) = \cup\{G / G$ is an IFOS in X and $G \subseteq A\}$,

$cl(A) = \cap\{K / K$ is an IFCS in X and $A \subseteq K\}$.

Definition 2.5:[7] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) *intuitionistic fuzzy α-open set* (IFαOS in short) if $A \subseteq int(cl(int(A)))$,

(ii) *intuitionistic fuzzy α-closed set* (IFαCS in short) if $cl(int(cl(A)) \subseteq A$.

The family of all IFCS (resp. IFαCS, IFOS, IFαOS) of an IFTS (X, τ) is denoted by IFC(X) (resp. IFαC(X), IFO(X), IFαO(X)).

Definition 2.6:[10] Let A be an IFS in an IFTS (X, τ). Then

$\alpha int(A) = \cup\{G / G$ is an IFαOS in X and $G \subseteq A\}$,

$\alpha cl(A) = \cap\{K / K$ is an IFαCS in X and $A \subseteq K\}$.

$\alpha gcl(A) = \cap\{M / M$ is an IFαGCS in X and $A \subseteq M\}$.

Result 2.7:[10] Let A be an IFS in (X, τ). Then

(i) $\alpha cl(A) = A \cup cl(int(cl(A)))$

(ii) $\alpha int(A) = A \cap int(cl(int(A)))$

Definition 2.8:[10] An IFS A in an IFTS (X, τ) is an

(i) *intuitionistic fuzzy generalized closed set* (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X.
(ii) *intuitionistic fuzzy alpha generalized closed set* (IFαGCS in short) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X.

Result 2.9:[10] Every IFCS, IFαCS is an IFαGCS but the converses may not be true in general.

Definition 2.10:[7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be
(i) *intuitionistic fuzzy continuous* (IF continuous in short) if $f^{-1}(B) \in$ IFO(X) for every B ∈ σ.
(ii) *intuitionistic fuzzy α-continuous* (IFα continuous in short) if $f^{-1}(B) \in$ IFαO(X) for every B ∈ σ.

Result 2.11: [7] Every IF continuous mapping is an IFα-continuous mapping but the converse may not be true in general.

Definition 2.12: [9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be
(i) *intuitionistic fuzzy generalized continuous* (IFG continuous in short) if $f^{-1}(B) \in$ IFGC(X) for every IFCS B in Y.
(ii) *intuitionistic fuzzy α-generalized continuous* (IFαG continuous in short) if $f^{1}(B) \in$ IFαGC(X) for every IFCS B in Y.

Definition 2.13:[9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy generalized alpha open mapping* (IFαG open mapping in short) if f(A) ∈ IFαGOS(X) for every IFOS A in X.

Definition 2.14:[9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then the map f is said to be an *intuitionistic fuzzy alpha generalized irresolute* (IFαG irresolute in short) if $f^{-1}(B) \in$ IFαGCS(X) for every IFαGCS B in Y.

Definition 2.15: Let f be a bijection mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be
(i) *intuitionistic fuzzy homeomorphism* (IF homeomorphism in short) if f and f^{1} are IF continuous mappings.
(ii) *intuitionistic fuzzy alpha homeomorphism* (IFα homeomorphism in short) if f and f^{-1} are IFα continuous mappings.
(iii) *intuitionistic fuzzy generalized homeomorphism* (IFG homeomorphism in short) if f and f^{-1} are IFG continuous mappings.

Result 2.16:[9] Every IF continuous mapping, IFα continuous mapping is an IFαG continuous but the converse may not be true in general.

Definition 2.17:[9] An IFTS (X, τ) is said to be an intuitionistic $_{\alpha a}T_{1/2}$ (in short $IF_{\alpha a}T_{1/2}$) space if every IFαGCS in X is an IFCS in X.

Definition 2.18:[9] An IFTS (X, τ) is said to be an intuitionistic $_{\alpha b}T_{1/2}$ (in short $IF_{\alpha b}T_{1/2}$) space if every IFαGCS in X is an IFGCS in X.

Definition 2.19: An IFTS (X, τ) is said to be an intuitionistic $_{\alpha d}T_{1/2}$ (in short $IF_{\alpha d}T_{1/2}$) space if every IFαGCS in X is an IFαCS in X.

3 Intuitionistic fuzzy alpha generalized homeomorphisms

In this section we introduce intuitionistic fuzzy alpha generalized homeomorphism and study some of its properties.

Definition 3.1: A bijection mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy alpha generalized homeomorphism* (IFαG homeomorphism in short) if f and f^{-1} are IFαG continuous mappings.

Example 3.2: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.2, 0.2), (0.6, 0.7) ⟩, T_2 = ⟨ y, (0.4, 0.7), (0.4, 0.2) ⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = { $0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a bijection mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFαG continuous mapping and f^{-1} is also an IFαG continuous mapping. Therefore f is an IFαG homeomorphism.

Theorem 3.3: Every IF homeomorphism is an IFαG homeomorphism but not conversely.
Proof: Let f: (X, τ) → (Y, σ) be an IF homeomorphism. Then f and f^{-1} are IF continuous mappings. This implies f and f^{-1} are IFαG continuous mappings. That is the mapping f is IFαG homeomorphism.

Example 3.4: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.3, 0.2), (0.6, 0.7) ⟩, T_2 = ⟨ y, (0.5, 0.4), (0.4, 0.2) ⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = { $0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a bijection mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFαG homeomorphism but not an IF homeomorphism since f and f^{-1} are not an IF continuous mappings.

Theorem 3.5: Every IFα homeomorphism is an IFαG homeomorphism but not conversely.
Proof: Let f: (X, τ) → (Y, σ) be an IFα homeomorphism. Then f and f^{-1} are IFα continuous mappings. This implies f and f^{-1} are IFαG continuous mappings. That is the mapping f is an IFαG homeomorphism.

Example 3.6: Let X = { a, b }, Y = { u, v } and T_1 = ⟨ x, (0.5, 0.4), (0.5, 0.6) ⟩, T_2 = ⟨ y, (0.2, 0.2), (0.7, 0.8) ⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFαG homeomorphism. Consider an IFCS A = ⟨ x, (0.7, 0.8), (0.2, 0.2) ⟩ in Y. Then f^{-1}(A) = ⟨ y, (0.7, 0.8), (0.2, 0.2) ⟩ is not an IFαCS in X. This implies f is not an IFα continuous mapping. Hence f is not an IFα homeomorphism.

Theorem 3.7: Let f: (X, τ) → (Y, σ) be an IFαG homeomorphism, then f is an IF homeomorphism if X and Y are $IF_{\alpha a}T_{1/2}$ space.

Proof: Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IFαGCS in X, by hypothesis. Since X is an $IF_{\alpha a}T_{1/2}$ space, $f^{-1}(B)$ is an IFCS in X. Hence f is an IF continuous mapping. By hypothesis f^{-1}: (Y, σ) → (X, τ) is a IFαG continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}(A) = f(A)$ is an IFαGCS in Y, by hypothesis. Since Y is an $IF_{\alpha a}T_{1/2}$ space, f(A) is an IFCS in Y. Hence f^{-1} is an IF continuous mapping. Therefore the mapping f is an IF homeomorphism.

Theorem 3.8: Let f: (X, τ) → (Y, σ) be an IFαG homeomorphism, then f is an IFG homeomorphism if X and Y are $IF_{\alpha b}T_{1/2}$ space.

Proof: Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IFαGCS in X, by hypothesis. Since X is an $IF_{\alpha b}T_{1/2}$ space, $f^{-1}(B)$ is an IFGCS in X. Hence f is an IFG continuous mapping. By hypothesis f^{-1}: (Y, σ) → (X, τ) is a IFαG continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}(A) = f(A)$ is an IFαGCS in Y, by hypothesis. Since Y is an $IF_{\alpha b}T_{1/2}$ space, f(A) is an IFGCS in X. Hence f^{-1} is an IFG continuous mapping. Therefore the mapping f is an IFG homeomorphism.

Theorem 3.9: Let f: (X, τ) → (Y, σ) be a bijective mapping. If f is an IFαG continuous mapping, then the following are equivalent.

(i) f is an IFαG closed mapping
(ii) f is an IFαG open mapping
(iii)f is an IFαG homeomorphism.

Proof: (i) →(ii): Let f: (X, τ) → (Y, σ) be a bijective mapping and let f is an IFαG closed mapping. This implies f^{-1}: (Y, σ) → (X, τ) is IFαG continuous mapping. That is every IFOS in X is an IFαGOS in Y. Hence f^{-1} is an IFαG open mapping.

Proof: (ii) →(iii): Let f: (X, τ) → (Y, σ) be a bijective mapping and let f is an IFαG open mapping. This implies f^{-1}: (Y, σ) → (X, τ) is IFαG continuous mapping. Hence f and f^{-1} are IFαG continuous mappings. That is f is an IFαG homeomorphism.

(iii) →(i): Let f is an IFαG homeomorphism. That is f and f^{-1} are IFαG continuous mappings. Since every IFCS in X is an IFαGCS in Y, f is an IFαG closed mapping.

Remark 3.10: The composition of two IFαG homeomorphisms need not be an IFαG homeomorphism in general.

Example 3.11: Let X = {a, b}, Y = {c, d} and Z = {u, v}. Let T_1 = ⟨x, (0.8, 0.6), (0.2, 0.4)⟩, T_2 = ⟨ y, (0.6, 0.1), (0.4, 0.3) ⟩ and T_3 = ⟨ z, (0.4, 0.4), (0.6, 0.2) ⟩. Then τ = {$0_\sim$, T_1, $1_\sim$}, σ = { $0_\sim$, T_2, $1_\sim$} and Ω = { $0_\sim$, T_3, $1_\sim$} are IFTs on X,Y and Z respectively. Define a bijection mapping f: (X, τ) → (Y, σ) by f(a) = c , f(b) = d and g: (Y, σ) → (Z, Ω) by f(c) = u , f(d) = v. Then f and f^{-1} are IFαG continuous mappings. Also g and g^{-1} are IFαG continuous mappings. Hence f and g are IFαG homeomorphisms. But the composition g.f: X → Z is not an IFαG homeomorphism since g.f is not an IFαG continuous mapping.

4 Intuitionistic fuzzy M - alpha generalized homeomorphisms

Definition 4.1: A bijection mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy M-alpha generalized homeomorphism* (IFMαG homeomorphism in short) if f and f^{-1} are IFαG irresolute mappings.

Theorem 4.2: Every IFMαG homeomorphism is an IFαG homeomorphism but not conversely.
Proof: Let f: (X, τ) → (Y, σ) be an IFMαG homeomorphism. Let B be IFCS in Y. This implies B is an IFαGCS in Y. By hypothesis $f^{-1}(B)$ is an IFαGCS in X. Hence f is an IFαG continuous mapping. Similarly we can prove f^{-1} is an IFαG continuous mapping. Hence f and f^{-1} are IFαG continuous mappings. This implies the mapping f is an IFαG homeomorphism.

Example 4.3: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.4, 0.3), (0.6, 0.7) ⟩, T_2 = ⟨ y, (0.2, 0.1), (0.4, 0.5) ⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = { $0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a bijection mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFαG homeomorphism. Let us consider an IFS G= ⟨ y, (0.3, 0.2), (0.7, 0.7) ⟩ in Y. Clearly G is an IFαGCS in Y. But $f^{-1}(G)$ is not an IFαGCS in X. That is f is not an IFαG irresolute mapping. Hence f is not an IFMαG homeomorphism.

Theorem 4.4: If the mapping f: X→Y is an IFMαG homeomorphism, then $\alpha gcl(f^{-1}(B)) \subseteq f^{-1}(\alpha cl(B))$ for every IFS B in Y.

Proof: Let B be an IFS in Y. Then αcl(B) is an IFαCS in Y. This implies αcl(B) is an IFαGCS in Y. Since the mapping f is an IFαG irresolute mapping, $f^{-1}(\alpha cl(B))$ is an IFαGCS in X. This implies $\alpha gcl(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B))$. Now $\alpha gcl(f^{-1}(B)) \subseteq \alpha gcl(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B))$. Hence $\alpha gcl(f^{-1}(B)) \subseteq f^{-1}(\alpha cl(B))$ for every IFS B in Y.

Theorem 4.5: If f: X → Y is an IFMαG homeomorphism, then $\alpha cl(f^{-1}(B)) = f^{-1}(\alpha cl(B))$ for every IFS B in Y.

Proof: Since f is an IFMαG homeomorphism, f is an IFαG irresolute mapping. Consider an IFS B in Y. Clearly αcl(B) is an IFαGCS in Y. This implies αcl(B) is an IFαGCS in Y.. By hypothesis $f^{-1}(\alpha cl(B))$ is an IFαGCS in X. Since $f^{-1}(B) \subseteq f^{-1}(\alpha cl(B))$, $\alpha cl(f^{-1}(B)) \subseteq \alpha cl(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B)))$. This implies $\alpha cl(f^{-1}(B)) \subseteq f^{-1}(\alpha cl(B))$.

Since f is an IFMαG homeomorphism, f^{-1}: Y → X is an IFαG irresolute mapping. Consider an IFS $f^{-1}(B)$ in X. Clearly $\alpha cl(f^{-1}(B))$ is an IFαGCS in X. Hence $\alpha cl(f^{-1}(B))$ is an IFαGCS in X. This implies $(f^{-1})^{-1}(\alpha cl(f^{-1}(B))) = f(\alpha cl(f^{-1}(B)))$ is an IFαGCS in Y. Clearly $B = (f^{-1})^{-1}(f^{-1}(B)) \subseteq (f^{-1})^{-1}(\alpha cl(f^{-1}(B))) = f(\alpha cl(f^{-1}(B)))$,. Therefore $\alpha cl(B) \subseteq \alpha cl(f(\alpha cl(f^{-1}(B)))) = f(\alpha cl(f^{-1}(B))$, since f^{-1} is an IFαG irresolute mapping. Hence $f^{-1}(\alpha cl(B)) \subseteq f^{-1}(f(\alpha cl(f^{-1}(B)) = \alpha cl(f^{-1}(B))$. That is $f^{-1}(\alpha cl(B)) \subseteq \alpha cl(f^{-1}(B))$. This implies $\alpha cl(f^{-1}(B)) = f^{-1}(\alpha cl(B))$.

Theorem 4.6: If f: X → Y is an IFMαG homeomorphism, then αcl(f(B)) = f(αcl(B)) for every IFS B in X.

Proof: Since f is an IFMαG homeomorphism, f^{-1} is an IFαG homeomorphism. Let us consider an IFS B in X. By theorem(4.6) αcl(f(B)) = f(αcl(B)) for every IFS B in X.

Remark 4.7: The composition of two IFMαG homeomorphisms is IFMαG homeomorphism in general.

Proof: Let f: X → Y and g: Y → Z be any two IFMαG homeomorphisms. Let A be an IFαGCS in Z. Then by hypothesis, $g^{-1}(A)$ is an IFαGCS in Y. Then by hypothesis, $f^{-1}(g^{-1}(A))$ is an IFαGCS in X. Hence $(g.f)^{-1}$ is an IFαG irresolute mapping. Now let B be an IFαGCS in X. Then by hypothesis, f(B) is an IFαGCS in Y. Then by hypothesis g(f(B)) is an IFαGCS in Z. This implies g.f is an IFαG irresolute mapping. Hence g.f is an IFMαG homeomorphism. That is the composition of two IFMαG homeomorphisms is an IFMαG homeomorphism in general.

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