Notes on Intuitionistic Fuzzy Sets
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A new geometrical interpretation of the intuitionistic fuzzy pairs

Krassimir Atanassov

Dept. of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria,
and
Intelligent Systems Laboratory
Prof. Asen Zlatarov University, Burgas-8010, Bulgaria
e-mail: krat@bas.bg

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Abstract: A new type of a geometrical interpretation of the intuitionistic fuzzy pairs and of the operations and operators over them, is introduced.
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AMS Classification 03E72.

During the last 33 years, a lot of geometrical interpretations of the intuitionistic fuzzy objects (elements of intuitionistic fuzzy sets or logical propositions, variables and formulas) were introduced (see, e.g., [1, 2]). Here, we discuss a new geometrical interpretations of these objects.

Let p be a proposition, variable or formula. Then, we can define the truth-valued function V, so that $V(p) = \langle \mu(p), \nu(p) \rangle$, where $\mu(p)$ and $\nu(p)$ are the degrees of validity and non-validity of p. Let E be an universe, $A \subseteq E$ and $p \in E$. Then $\mu_A(p)$ and $\nu_A(p)$ are the degrees of membership and non-membership of p to A.

More generally, if p is some of the above discussed objects, it generates the Intuitionistic Fuzzy Pair (IFP) $\langle \mu(p), \nu(p) \rangle$, so that

$$\mu(p), \nu(p) \in [0, 1] \text{ and } \mu(p) + \nu(p) \leq 1.$$

The new geometrical interpretation is shown on Fig. 1, where p is an ITP. In it, the horisontal section and the two vertical sections have lenght of 1. The section determines the two boundary points that have ordinates with lenghts $\nu(p)$ and $\mu(p)$.

In this new interpretation, the point, marked by $\bullet$ in Fig. 2, is very important. It has coordinates $\langle \frac{1}{2}, \frac{1}{2} \rangle$. When some section passes through it, then this section corresponds to a IFP that is a fuzzy point.

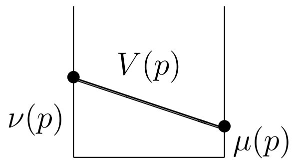


Figure 1

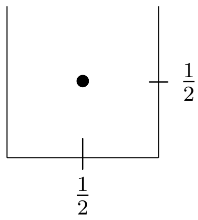


Figure 2

The three basic constants T, F and U with evaluations $\langle 1, 0 \rangle$, $\langle 0, 1 \rangle$ and $\langle 0, 0 \rangle$ are shown on Figures 3, 4 and 5, respectively.

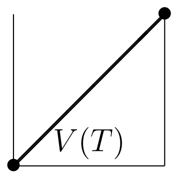


Figure 3

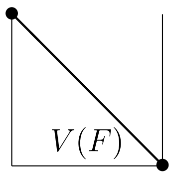


Figure 4

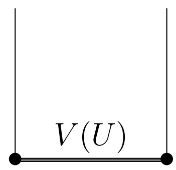


Figure 5

If p is an intuitionistic fuzzy tautology, then its geometrical interpretation has the form from Figure 6, while if p is an intuitionistic fuzzy sure, then its interpretation has the form from Figure 7.

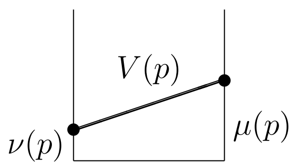


Figure 6

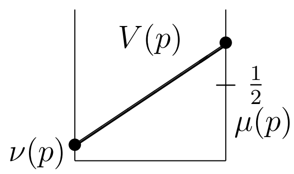


Figure 7

If p and q are two IFPs, then their conjunction and discunction are shown on Figures 8 and 9, while the negation of p is shown on Figure 10. These three operations have, respectively, the forms:

$$V(p \vee q) = \langle \max(\mu(p), \mu(q)), \min(\nu(p), \nu(q)) \rangle,$$

$$V(p \wedge q) = \langle \min(\mu(p), \mu(q)), \max(\nu(p), \nu(q)) \rangle,$$

$$V(\neg_1 p) = \langle \nu(p), \mu(p) \rangle.$$

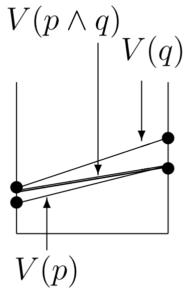


Figure 8

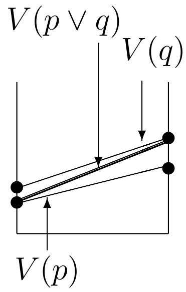


Figure 9

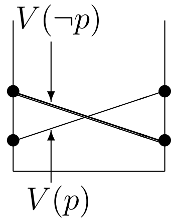


Figure 10

The operation implication has different forms. For example, the standard implication

$$V(p \to q) = \langle \max(\nu(p), \mu(q)), \min(\nu(p), \mu(q)) \rangle$$

has the interpretations from Figures 11 and 12.

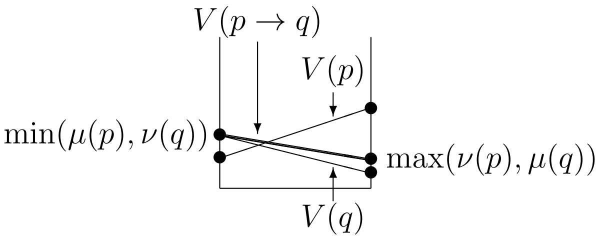


Figure 11

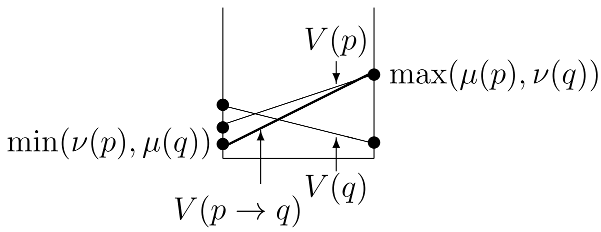


Figure 12

Standard modal operators $\Box$ and $\Diamond$ that are defined by

$$V(\Box p) = \langle \mu(p), 1 - \mu(p) \rangle,$$

$$V(\Diamond p) = \langle 1 - \nu(p), \nu(p) \rangle,$$

have the geometrical interpretations from Figures 13 and 14.

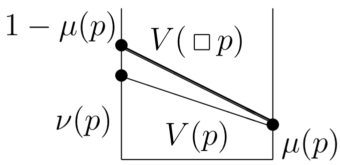


Figure 13

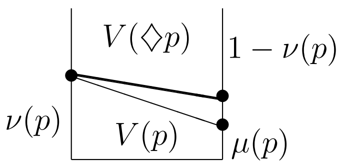


Figure 14

The extended modal operators

$V(D_\alpha(p)) = \langle \mu(p) + \alpha.(1 - \mu(p) - \nu(p)), \nu(p) + (1 - \alpha).(1 - \mu(p) - \nu(p)) \rangle,$

$V(G_{\alpha,\beta}(p)) = \langle \alpha.\mu(p), \beta.\nu(p) \rangle,$

$V(F_{\alpha,\beta}(p)) = \langle \mu(p) + \alpha.(1 - \mu(p) - \nu(p)), \nu(p) + \beta.(1 - \mu(p) - \nu(p)) \rangle,$ for $\alpha + \beta \leq 1,$

$$V(H_{\alpha,\beta}(p)) = \langle \alpha.\mu(p), \nu(p) + \beta.(1 - \mu(p) - \nu(p))\rangle,$$
$$V(H^*_{\alpha,\beta}(p)) = \langle \alpha.\mu(p), \nu(p) + \beta.(1 - \alpha.\mu(p) - \nu(p))\rangle,$$
$$V(J_{\alpha,\beta}(p)) = \langle \mu(p) + \alpha.(1 - \mu(p) - \nu(p)), \beta.\nu(p)\rangle,$$
$$V(J^*_{\alpha,\beta}(p)) = \langle \mu(p) + \alpha.(1 - \mu(p) - \beta.\nu(p)), \beta.\nu(p)\rangle$$

have the geometrical interpretations shown on Figures 15, 16, 17, 18, 19, 20 and 21.

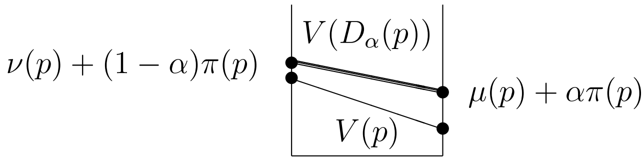
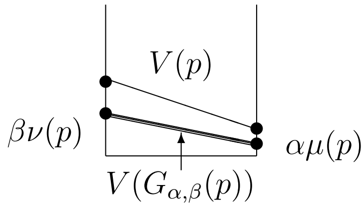


Figure 15



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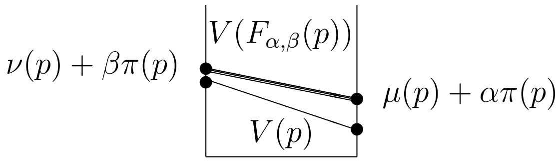


Figure 17

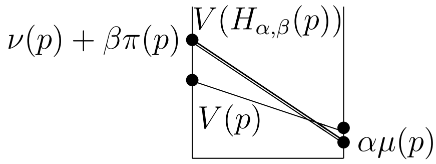
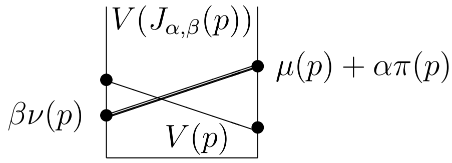


Figure 18



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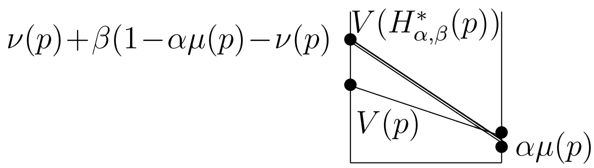

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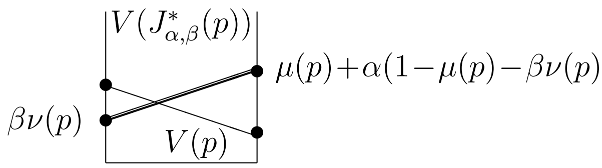

Figure 21

The modal operators from the other type are:

$$V(\boxplus A) = \left\langle \frac{\mu}{2}, \frac{\nu+1}{2} \right\rangle,$$

$$V(\boxtimes A) = \left\langle \frac{\mu+1}{2}, \frac{\nu}{2} \right\rangle,$$

$$V(\boxplus_\alpha A) = \langle \alpha\mu, \alpha\nu + 1 - \alpha\rangle,$$

$$V(\boxtimes_\alpha A) = \langle \alpha\mu + 1 - \alpha, \alpha\nu\rangle,$$

where $\alpha, \beta, \alpha + \beta \in [0, 1]$,

$$V(\boxplus_{\alpha,\beta,\gamma} A) = \langle \alpha\mu, \beta\nu + \gamma \rangle,$$

$$V(\boxtimes_{\alpha,\beta,\gamma} A) = \langle \alpha\mu + \gamma, \beta\nu \rangle,$$

where $\alpha, \beta, \gamma \in [0, 1]$ and $\max(\alpha, \beta) + \gamma \leq 1$,

$$V(E_{\alpha,\beta}(A)) = \langle \beta(\alpha\mu + 1 - \alpha), \alpha(\beta\nu + 1 - \beta) \rangle,$$

where $\alpha, \beta \in [0, 1]$,

$$V(\boxdot_{\alpha,\beta,\gamma,\delta} A) = \langle \alpha\mu + \gamma, \beta\nu + \delta \rangle,$$

where $\alpha, \beta, \gamma, \delta \in [0, 1]$ and $\max(\alpha, \beta) + \gamma + \delta \leq 1$,

$$V(\circledcirc_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} A) = \langle \alpha\mu - \varepsilon\nu + \gamma, \beta\nu - \zeta\mu + \delta \rangle,$$

where $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0, 1]$ and

$$\max(\alpha - \zeta, \beta - \varepsilon) + \gamma + \delta \leq 1,$$

$$\min(\alpha - \zeta, \beta - \varepsilon) + \gamma + \delta \geq 0.$$

The geometrical interpretations of these operators are shown on Figures 22, ..., 32.

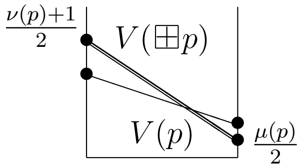

Figure 22

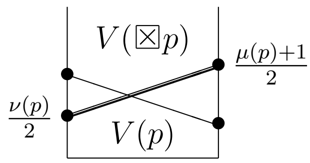

Figure 23

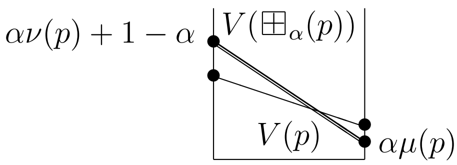

Figure 24

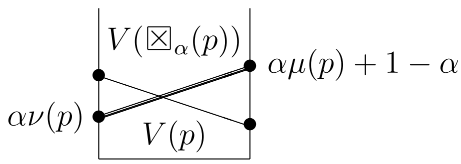

Figure 25

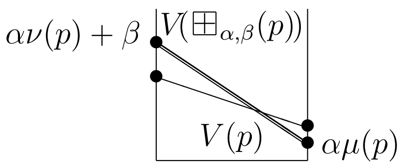

Figure 26

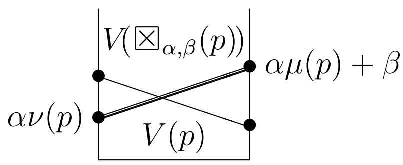

Figure 27

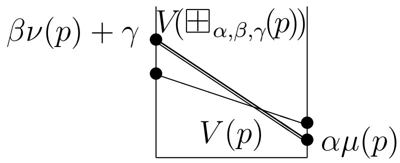

Figure 28

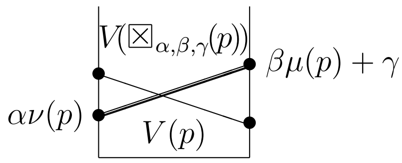

Figure 29

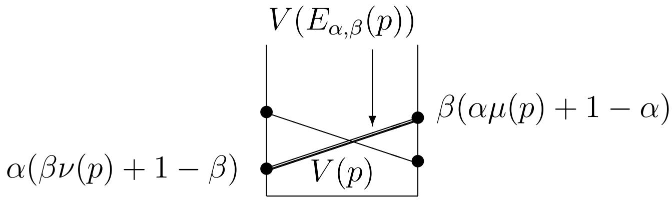

Figure 30

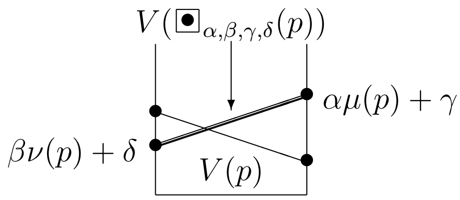

Figure 31

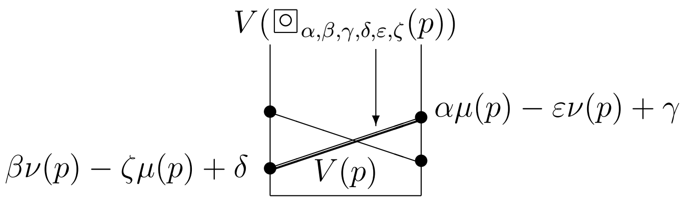

Figure 32

In [3], it is introduced the following operator from modal type, which is a modification of the above discussed operators. It had the form

$$\otimes_{\alpha,\beta,\gamma,\delta}A = \{\langle x, \alpha.\mu_A(x) + \gamma.\nu_A(x), \beta.\mu_A(x) + \delta.\nu_A(x)\rangle | x \in E\},$$

where $\alpha, \beta, \gamma, \delta \in [0,1]$ and $\alpha + \beta \leq 1, \gamma + \delta \leq 1$, and the geometrical interpretation on Figure 33.

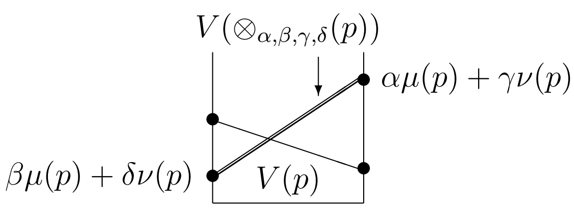

Figure 33

Acknowledgments

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References

[1] Atanassov, K. (1989) Geometrical interpretation of the elements of the intuitionistic fuzzy objects, *Preprint IM-MFAIS-1-89*, Sofia. Reprinted: *Int. J. Bioautomation*, 2016, 20(S1), S27–S42.

[2] Atanassov, K. (2012) *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin.

[3] Atanassov, K., Cuvalcioglu, G. & Atanassova, V. (2014) A new modal operator over intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets*, 20(5), 1–8.