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Intuitionistic fuzzy soft generalized superconnectedness

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Abstract: We introduce a new notion intuitionistic fuzzy soft superconnectedness. The main purpose of this paper is to study generalized intuitionistic fuzzy soft superconnected spaces.
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1 Introduction

Shabir and Naz [7] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also studied some of basic concepts of soft topological spaces. In the present study, we introduce some new concepts in intuitionistic fuzzy soft topological spaces such as intuitionistic fuzzy soft generalized superconnected. We also give characterizations and properties of this notion.

2 Preliminary

In this section we will recall the intuitionistic fuzzy soft concept, the intuitionistic fuzzy soft toplogy, and intuitionistic fuzzy soft mapping.

First we denote $L^* = \{(x, y) \in [0, 1]^2,\ x + y \leq 1\}$.

Definition 1. *[6] Let X be an universe, and E be a set of parametres. Let $expX$ denote the power set of X. A pair (F, E) is called a soft set over X if F is a mapping given by $F : E \longrightarrow expX$.*

Remark 1. *A soft set F over X is a valued function from E to $expX$. We can write*

$$E = \Big\{(e, F(e)), e \in E\Big\}.$$

Note that if $F(e) = \emptyset$, then the element $(e, F(e))$ does not appear in F.

We denote $\mathbb{S}$ the set of all soft sets over X.

Definition 2. *[1] An intuitionistic fuzzy set $\langle u, v\rangle$ on X is a mapping $\langle u, v\rangle : X \longrightarrow L^*$, represente the degre of memebership and degre of non-memebership.*

Definition 3. *[1] Let $\langle u, v\rangle \in (L^*)^X$, then some basic set operations for intuitionistic fuzzy sets are defined as follows.*

1. $\langle u, v\rangle \subset \langle u', v'\rangle \Longleftrightarrow u(x) \le u'(x)\ \&\ v(x) \ge v'(x),\ \forall x \in X$
2. $\langle u, v\rangle = \langle u', v'\rangle \Longleftrightarrow u(x) = u'(x)\ \&\ v(x) = v'(x),\ \forall x \in X$
3. $\langle \delta, \eta\rangle = \langle u, v\rangle \cup \langle u', v'\rangle \Longleftrightarrow \delta(x) = u(x) \vee u'(x)\ \&\ \eta(x) = v(x) \wedge v'(x),\ \forall x \in X$
4. $\langle \delta, \eta\rangle = \langle u, v\rangle \cap \langle u', v'\rangle \Longleftrightarrow \delta(x) = u(x) \wedge u'(x)\ \&\ \eta(x) = v(x) \vee v'(x),\ \forall x \in X$
5. $\langle \delta, \eta\rangle = \langle u, v\rangle^c \Longleftrightarrow \langle \delta, \eta\rangle = \langle v, u\rangle$
6. *If $\langle \delta, \eta\rangle = \langle 0, 1\rangle$, then $\langle \delta, \eta\rangle$ called empty intuitionistic fuzzy set, denote $\widetilde{\emptyset}$.*
7. *If $\langle \delta, \eta\rangle = \langle 1, 0\rangle$, then $\langle \delta, \eta\rangle$ called universal intuitionistic fuzzy set, denote $\widetilde{X}$.*

According to above definition we will redefine an intuitionistic fuzzy soft set and it's operations.

Definition 4. *[5] Let X be an initial universal set and let E be set of parameters. Let $I\!\!F(X)$ denotes the set of all intuitionistic fuzzy sets of X. A pair $(\widetilde{F}, E)$ is called an intuitionistic fuzzy soft set over X if F is a mapping given by $\widetilde{F} : E \longrightarrow I\!\!F(X)$. We write an Intuitionistic fuzzy soft set shortly as IF soft set.*

Remark 2. *By the previous remark, we can say that, an intuitionistic fuzzy soft set $\widetilde{F}$ is a set valued function from E to $(L^*)^X$, and it can be write as*

$$\widetilde{F} = \Big\{\Big(e, \{(x, u_{\widetilde{F}(e)}(x), v_{\widetilde{F}(e)}(x))\}, x \in X\Big) :\ e \in E\Big\}$$

We denote $IF\mathbb{S}(X, E)$ set of all IF over X

Definition 5. *[5] Let $\widetilde{F}, \widetilde{G} \in \mathbb{IFS}$. Then some basic set operations of IF are defined as follows*

1. $\widetilde{F} \sqsubseteq \widetilde{G} \Longleftrightarrow \widetilde{F}(e) \subset \widetilde{G}(e),\ \forall e \in E$
2. $\widetilde{F} = \widetilde{G} \Longleftrightarrow \widetilde{F} \sqsubset \widetilde{G}\ \&\ \widetilde{G} \sqsubset \widetilde{F}$

3. $\widetilde{H} = \widetilde{F} \sqcup \widetilde{G} \Longleftrightarrow \widetilde{H}(e) = \widetilde{F}(e) \cup \widetilde{G}(e),\ \forall e \in E$

4. $\widetilde{H} = \widetilde{F} \sqcap \widetilde{G} \Longleftrightarrow \widetilde{H}(e) = \widetilde{F}(e) \cap \widetilde{G}(e),\ \forall e \in E$

5. $\widetilde{H} = \widetilde{F}^c \Longleftrightarrow \widetilde{H}(e) = (\widetilde{F}(e))^c,\ \forall e \in E$

6. *$\widetilde{F}$ is called the null intuitionistic fuzzy soft set and denoted by $\widetilde{0}_X$, if $\widetilde{F}(e) = \widetilde{\emptyset},\ \forall e \in E$*

7. *$\widetilde{F}$ is called the universal intuitionistic fuzzy soft set and denoted by $\widetilde{1}_X$, if $\widetilde{F}(e) = \widetilde{X},\ \forall e \in E$*

Theorem 1. *[5] Let $\{\widetilde{F}_i\}_i \subset IF\mathbb{S}$ and $\{\widetilde{G}_i\}_i \subset IF\mathbb{S}$. then*

1. $\widetilde{G} \sqcap \left(\sqcup_i \widetilde{F}_i\right) = \sqcup_i \widetilde{G} \cap \widetilde{F}_i$

2. $\widetilde{G} \sqcup \left(\sqcap_i \widetilde{F}_i\right) = \sqcap_i \widetilde{G} \sqcup \widetilde{F}_i$

3. $\left(\sqcup_i \widetilde{F}_i\right)^c = \sqcap_i (\widetilde{F}_i)^c$

4. $\left(\sqcap_i \widetilde{F}_i\right)^c = \sqcup_i (\widetilde{F}_i)^c$

5. $\widetilde{0}_x \sqsubseteq \widetilde{F} \sqsubseteq \widetilde{1}_X,\ \widetilde{1}_X^c = \widetilde{0}_X$

6. $\widetilde{F} \sqcup \widetilde{F}^c = \widetilde{1}_X$ & $(\widetilde{F}^c)^c$

Definition 6. *[4, 8] Let $IF\mathbb{S}(X)$ and $IF\mathbb{S}(Y)$ be the families of all intuitionistic fuzzy soft sets over X and Y, respectively. Then*

1. *A mapping $f = (\phi, \psi)$ is called a soft mapping from $IF\mathbb{S}(X, E)$ into $IF\mathbb{S}(Y, K)$, where $\phi : X \longrightarrow Y$ and $\psi : E \longrightarrow K$ are two mappings.*

2. *If $\widetilde{F}_A \in IF\mathbb{S}(X, E)$, then the image of $\widetilde{F}_A$ under the soft mapping (ϕ, ψ) is a soft set over Y denoted by $(\phi, \psi)\widetilde{F}_A$ and defined by:*

$$(\phi, \psi)\widetilde{F}_A(k) = \begin{cases} \bigcup_{e \in \psi^{-1}(k) \cap A} \phi(\widetilde{F}_A)(e)\ if\ \psi^{-1}(k) \cap A \neq \emptyset \\ \emptyset\ otherwise \end{cases} \tag{1}$$

3. *If $\widetilde{G}_B \in IF\mathbb{S}(Y, K)$, then the preimage of $\widetilde{G}_B$ under the soft mapping (ϕ, ψ) is a soft set over X denoted by $(\phi, \psi)^{-1}(\widetilde{G}_B)$ and defined by:*

$$(\phi, \psi)^{-1}\widetilde{G}_B(e) = \begin{cases} \phi^{-1}\left(\widetilde{G}_B(\psi(e))\right)\ if\ \psi(e) \in B \\ \emptyset\ otherwise \end{cases} \tag{2}$$

Theorem 2. *[8] Let $f : (X, E) \longrightarrow (Y, E')$, $u : X \longrightarrow Y$ and $p : E \longrightarrow E'$ be mappings. Then for intuitionistic fuzzy soft set $(\widetilde{F}, A)$, $(\widetilde{G}, B)$ and a family of an intuitionistic soft set $(\widetilde{F}_i, A_i)$ in the intuitionistic fuzzy soft class (X, E) we have :*

(1) $f(\widetilde{0}_X) = \widetilde{0}_Y$

(2) $f(IF\mathbb{S}) \sqsubseteq IF\mathbb{S}$

(3) $f\Big((\widetilde{F}, A)) \sqcup (\widetilde{G}, B)\Big) = f(\widetilde{F}, A)) \sqcup f(\widetilde{G}, B)$*, generally* $f\Big(\sqcup_i (\widetilde{F}_i, A_i)\Big) = \sqcup_i f(\widetilde{F}_i, A_i)$

(4) $f\Big((\widetilde{F}, A)) \sqcap (\widetilde{G}, B)\Big) \sqsupseteq f(\widetilde{F}, A)) \sqcap f(\widetilde{G}, B)$*, generally* $f\Big(\sqcap_i (\widetilde{F}_i, A_i)\Big) \sqsubseteq \sqcap_i f(\widetilde{F}_i, A_i)$

(5) If $(\widetilde{F}, A) \sqsubseteq (\widetilde{G}, B)$ *then* $f(\widetilde{F}, A) \sqsubseteq f(\widetilde{G}, B)$.

Definition 7. *An intuitionistic fuzzy soft generalized topology on* X *is a family* τ *of intuitionistic fuzzy soft sets over* X *satisfying the following properties:*

1. $\widetilde{0}_X \in \tau$
2. *If* $\{\widetilde{F}_i\}_i \subset \tau$*, then* $\bigcup_i \widetilde{F}_i \in \tau$.

The triplet (X, τ, E) *is called intuitionistic fuzzy soft generalized topological space. An element of this topology is called* τ*-IFS-open.*

We denote $c_\tau(\widetilde{F})$ the smallest intuitionistic fuzzy set containing $\widetilde{F}$, and i_τ the largest intuitionistic fuzzy set containing $\widetilde{F}$.

Remark 3. *Note that* $c_\tau(\widetilde{F})$ *is the intersection of all intuitionistic fuzzy set containing* $\widetilde{F}$*, and* i_τ *is the union of all intuitionistic fuzzy soft set containing* $\widetilde{F}$.

Definition 8. *An intuitionistic fuzzy soft set* $\widetilde{G}$ *of generalized intuitionistic fuzzy soft topological space* (X, τ, E) *is called*

1. τ*-IFS-preopen if* $\widetilde{G} \sqsubset i_\tau\Big(c_\tau(\widetilde{G})\Big)$
2. τ*-IFS-semiopen if* $\widetilde{G} \sqsubset c_\tau\Big(i_\tau(\widetilde{G})\Big)$
3. τ*-*β*-IFS-open if* $\widetilde{G} \sqsubset c_\tau\Big(i_\tau(c_\tau(\widetilde{G}))\Big)$
4. τ*-*r*-IFS-open if* $\widetilde{G} = i_\tau(c_\tau(\widetilde{G}))$

3 Intuitionistic fuzzy generalized soft superconnectedness

Definition 9. *If* $e_1, \ldots, e_n \in X$ *and* $\widetilde{G} \in IF\mathbb{S}$*, then*

$$\widetilde{G} \sqcup_{i=1}^{p} \{e_i\} = \widetilde{G}(e) \bigcup_{i=1}^{p} \{e_i\}, \quad \forall e \in X$$

Definition 10. *Let* (X, τ, E) *be an intuitionistic fuzzy soft generalized topological space and* $\widetilde{G} \in IF\mathbb{S}$.

1. $\widetilde{G}$ *is called* τ*-IFS-dense if* $c_\tau(\widetilde{G}) = X$.

2. (X, τ, E) *is called intuitionistic fuzzy soft hyperconnected if* $\widetilde{G}$ *is* τ*-dense for every* τ*-IFS-open* $\widetilde{G} \neq \widetilde{0}_X$ *of* (X, τ, E)

3. $\widetilde{G}$ *is said to be* τ*-IFS-nowhere dense if* $i_\tau(c_\tau(\widetilde{G})) = \widetilde{0}_X$

Definition 11. *Let* (X, τ, E) *be an intuitionistic fuzzy soft generalized topological space and* $\widetilde{G} \in IF\mathbb{S}$.

1. $\widetilde{G}$ *is called* τ*-scaled-IFS-dense if there is* $e_1, \ldots, e_n \in X$ *such that* $c_\tau(\widetilde{G}) = X$.

2. (X, τ, E) *is called intuitionistic fuzzy soft superconnected if* $\widetilde{G}$ *is* τ*-IFS-scaled-dense for every* τ*-IFS-open* $\widetilde{G} \neq \widetilde{0}_X$ *of* (X, τ, E).

Example 1. $X = \{a, b, c, d\}$, $E = \{e_1\}$, $\widetilde{F} = \{(e_1, \langle 0.2, 0.3\rangle), (e_1, \langle 0.4, 0.3\rangle), (e_1, \langle 0.0, 0.3\rangle), (e_1, \langle 0.1, 0.0\rangle), \}$, and $\tau = \Big\{\widetilde{0}_X, \{(e_1, \langle 0.2, 0.3\rangle)\}, \{(e_1, \langle 0.4, 0.3\rangle), (e_1, \langle 0.0, 0.3\rangle)\}, \{(e_1, \langle 0.2, 0.3\rangle), (e_1, \langle 0.0, 0.3\rangle)\}\Big\}$. Then the GIFST is superconnected.

Remark 4. *The implication*

$$(X, \tau, E) \; is \; hyperconnected \Longrightarrow (IF\mathbb{S}, \tau, E) \; is \; superconnected$$

is not reversible.

Example 2. $X = \{a, b, c, d\}$, $E = \{e_1\}$, $\widetilde{F} = \{(e_1, \langle 0.2, 0.3\rangle), (e_1, \langle 0.4, 0.3\rangle), (e_1, \langle 0.0, 0.3\rangle), (e_1, \langle 0.1, 0.0\rangle)\}$, and $\tau = \{\widetilde{0}, \{(e_1, \langle 0.2, 0.3\rangle)\}, (e_1, \langle 0.0, 0.3\rangle), (e_1, \langle 0.1, 0.0\rangle), \{(e_1, \langle 0.2, 0.3\rangle), (e_1, \langle 0.4, 0.3\rangle), (e_1, \langle 0.0, 0.3\rangle)\}\}$. Then the GIFST is superconnected not hyperconnected.

Definition 12. *An intuitionistic fuzzy soft set of a generalized soft topological space* (X, τ, E) *is said to be* τ*-IFS-nowhere scaled-dense if there is* $x_1, \ldots, x_p \in X$ *such that*

$$i_\tau\left(c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\})\right) = \widetilde{0}_X.$$

Theorem 3. *Let* (X, τ, E) *be a GIFST where* $c_\tau(\widetilde{0}_X) = \widetilde{0}_X$ *the following properties are equivalent:*

1. (X, τ, E) *is GIFS superconnected.*

2. $\widetilde{G}$ *is* τ*-IFS-scaled-dense or* τ*-IFS-nowher-dense for every IFS* $\widetilde{G}$.

3. *For nonempety* τ*-IFS-open* $\widetilde{G}$ *and* $\widetilde{H}$*, there is* $x_1, \ldots, x_p \in X$ *such that*

$$\left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right) \sqcap H \neq \widetilde{0}_X.$$

Proof. 1) $\Rightarrow$ 2) Let (X, τ, E) be a GIFS superconnected and $\widetilde{G} \sqsubset X$. Suppose that $\widetilde{G}$ is not τ-IFS-nowher-dense, we have

$$c_\tau\left(X \setminus (c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}))\right) = X \setminus i_\tau\left(c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\})\right) \neq X.$$

This implies from (1) that for $i_\tau\left(c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\})\right) \neq \widetilde{0}_X$, $c_\tau\left(i_\tau(c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}))\right) = X$. Since

$$X = c_\tau\left(i_\tau(c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}))\right) \sqsubset c_\tau\left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right).$$

Then $c_\tau\left(\widetilde{G} \sqcup_{i=1}^{p} \tilde{x}_i\right) = X$. Hence $\widetilde{G}$ is aτ-IFS-scaled-dense set.

2) $\Rightarrow$ 3) Suppose that for some nonempty τ-IFS-open $\widetilde{G}$ and $\widetilde{H}$, for every $x_1, \dots, x_p \in X$, $\left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right) \sqcap H = \widetilde{0}$, we have $c_\tau\left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\} \sqcap H\right) \sqsubset c_\tau\left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right) \sqcap c_\tau(H) \sqsubset c_\tau\left((\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) \sqcap H\right) = c_\tau(\widetilde{0}_X) = \widetilde{0}_X$.

Hence $c_\tau\left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right) \sqcap \widetilde{H} = \widetilde{0}_X$. Thus $\widetilde{G}$ is not a τ-IFS-scaled-dense set. Since $\widetilde{G}$ is τ-IFS-open, then

$$\widetilde{0}_X \neq \widetilde{G} \sqsubset \widetilde{G} \sqcup_{i=1}^{p} \{x_i\} \sqsubset i_\tau\left(c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\})\right).$$

Then $\widetilde{G}$ is not a τ-IFS-nowhere dense set. This is a contradiction. Thus

$$(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) \sqcap \widetilde{H} \neq \widetilde{0}_X$$

for every nonempty τ-IFS-open subsets $\widetilde{G}$ and $\widetilde{H}$ of (X, τ, E).

3) $\Rightarrow$ 1) Let $(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) \sqcap \widetilde{H} \neq \widetilde{0}_X$ for every nonempty τ-IFS-open subsets $\widetilde{G}$ and $\widetilde{H}$ of (X, τ, E). Suppose that (X, τ, E) is not superconnected. This implies that there exists a nonempty τ-IFS-open subsets $\widetilde{A}$ of (X, τ, E) such that $\widetilde{A}$ is not τ-IFS-scaled-dense in (X, τ, E). i.e., for every $x_1, \dots, x_p \in X$, $c_\tau(\widetilde{A} \sqcup_{i=1}^{p} \{x_i\}) \neq X$. We have $X \setminus c_\tau(\widetilde{A} \sqcup_{i=1}^{p} \{x_i\}) \neq \widetilde{0}_X$.

This implies that $X \setminus c_\tau(\widetilde{A} \sqcup_{i=1}^{p} \{x_i\}) \sqcap \widetilde{A}$ and $\widetilde{A}$ are nonempty τ-IFS-open subsets of (X, τ, E) such that $X \setminus c_\tau(\widetilde{A} \sqcup_{i=1}^{p} \{x_i\}) \sqcap \widetilde{A} = \widetilde{0}_X$. This a contradiction. Consequently the GIFS (X, τ, E) is superconnected. $\square$

Definition 13. *A GIFST (X, τ, E) is called irreducible if for each nonempty $\widetilde{U}$, $\widetilde{V} \in \tau \setminus \widetilde{0}_X$, we have $\widetilde{U} \sqcap \widetilde{V} = \widetilde{0}_X$.*

Definition 14. *A GIFST (X, τ, E) is called scaled-irreducible if for each nonempty $\widetilde{U}$, $\widetilde{V} \in \tau \setminus \widetilde{0}_X$, there exists $x_1, \dots, x_p \in X$, we have $(U \sqcup_{i=}^{p} \{x_i\}) \sqcap V \neq \widetilde{0}_X$.*

Definition 15. *The τ-IFS-semiclosure (resp. τ-IFS-preclosure) of a subset $\widetilde{G}$ of a GIFST (X, τ, E), denoted by $c_\sigma(\widetilde{G})$ (resp. $c_\pi(\widetilde{G})$), is defined by the intersection of all τ-IFS-semiclosed (respectively, τ-IFS-preclosed) sets of (X, τ, E) containing $\widetilde{G}$.*

Theorem 4. *Let (X, τ, E) be a GIFST where $c_\tau(\widetilde{0}) = \widetilde{0}_X$. Then the following properties are equivalent:*

1. *(X, τ, E) is intuitionistic fuzzy soft superconnected,*

2. *$\widetilde{G}$ is τ-IFS-scaled-dense for every τ-IFS-preopen subset $\widetilde{0}_X \neq \widetilde{G} \sqsubset X$,*

3. *For every τ-IFS-preopen subsets $\widetilde{0} \neq \widetilde{G} \sqsubset X$ there exists $x_1, \ldots, x_p \in X$ such that,*

$$c_\sigma(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) = X,$$

4. *For every τ-IFS-semiopen subsets $\widetilde{0} \neq \widetilde{G} \sqsubset X$ there exists $x_1, \ldots, x_p \in X$ such that,*

$$c_\pi(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) = X.$$

Proof. 1) $\Rightarrow$ 2) Let (X, τ, E) be an intuitionistic fuzzy soft superconnected GIFST. Suppose that $\widetilde{G}$ is a nonempty τ-IFS-preopen subset of (X, τ, E). This implies that

$$\widetilde{0}_X \neq \widetilde{G} \subset i_\tau(c_\tau(\widetilde{G})) \sqsubset i_\tau\Big(c_\tau\Big(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\Big)\Big),$$

where $x_1, \ldots, x_p \in X$. Thus, $\widetilde{G}$ is not τ-nowhere scaled-dense. Then by theorem 3, $\widetilde{G}$ is τ-IFS-scaled-dense.
2) $\Rightarrow$ 3) Suppose that there exists a nonempty τ-preopen set $\widetilde{G}$ such that for every $x_1, \ldots, x_p \in X$ we have $c_\sigma(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) \neq X$. Then there exists a nonempty τ-IFS-semiopen set $\widetilde{A}$ such that $\widetilde{A} \sqcap (\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) = \widetilde{0}_X$. Thus $i_\tau(\widetilde{A}) \sqcap (\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) = \widetilde{0}_X$. Hence

$$\widetilde{0}_X = i_\tau(\widetilde{A}) \sqcap c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) = i_\tau(\widetilde{A}),$$

by 2) $\widetilde{A}$ is a nonempty τ-IFS-semiopen set, we have $\widetilde{A} \sqsubset c_\tau(i_\tau(\widetilde{A})) = c_\tau(\widetilde{0}_X) = \widetilde{0}_X$. This is a contradiction.
3) $\Rightarrow$ 4) Suppose that there exists a nonempty τ-IFS-semiopen set $\widetilde{G}$ such that $c_\pi(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) \neq X$ Then there exists a nonempty τ-IFS-preopen set $\widetilde{A}$ such that

$$\Big(\widetilde{A} \sqcup_{i=1}^{p} \{x_i\}\Big) \sqcap \widetilde{G} = \widetilde{0}_X.$$

Thus $\Big(\widetilde{A} \sqcup_{i=1}^{p} \{x_i\}\Big) \sqcap i_\tau(\widetilde{G}) = \widetilde{0}_X$. Hence, $\widetilde{0}_X = c_\tau\Big(\widetilde{A} \sqcup_{i=1}^{p} \{x_i\}\Big) \sqcap i_\tau(\widetilde{G}) \sqsupset c_\sigma\Big(\widetilde{A} \sqcup_{i=1}^{p} \{x_i\}\Big) \sqcap i_\tau(\widetilde{G}) = X \sqcap i_\tau(\widetilde{G}) = i_\tau(\widetilde{G})$. Since $\widetilde{G}$ is τ-IFS-semiopen set, then $\widetilde{G} \sqsubset c_\tau\Big(i_\tau(\widetilde{G})\Big) = c_\tau(\widetilde{0}_X) = \widetilde{0}_X$, this is a contradiction.
4) $\Rightarrow$ 1) Let $\widetilde{G}$ be a nonempty τ-open set of (X, τ, E). Since $\widetilde{G}$ is τ-IFS-semiopen, by 4), there exists $x_1, \ldots, x_p \in X$ such that $c_\tau\Big(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\Big) \sqsupset c_\pi\Big(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\Big) = X$. Consequently, (X, τ, E) is a GIFST soft superconnected. $\square$

By [2] it is easy to show the following lemma.

Lemma 1. *We have*

$$c_\sigma(\widetilde{G}) = \widetilde{G} \sqcup i_\tau\Big(c_\tau(\widetilde{G})\Big).$$

Corollary 1. *Let (X, τ, E) be a GIFST where $c_\tau(\widetilde{0}_X) = \widetilde{0}_X$. Then the following properties are equivalent:*

1. (X, τ, E) *is soft superconnected,*

2. $\widetilde{G}$ *is* τ*-IFS-scaled-dense for every* τ*-*β*-IFS-open subset* $\widetilde{0}_X \neq \widetilde{G} \sqsubset X$*,*

3. For every τ*-*β*-IFS-open subsets* $\widetilde{0}_X \neq \widetilde{G} \sqsubset X$ *there exists* $x_1, \ldots, x_p \in X$ *such that,*

$$c_\sigma(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) = X.$$

Proof. 1) $\Rightarrow$ 2) Let (X, τ, E) be a GIFST soft superconnected. Assume that $\widetilde{G}$ is a nonempty τ-β-IFS-open subset of (X, τ, E), it follows that $\widetilde{0}_X \neq i_\tau(c_\tau(\widetilde{G})) \sqsubset i_\tau\left(c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\})\right)$. Then

$$X = c_\tau\left(i_\tau\left(c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\})\right)\right) = c_\tau\left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right).$$

2) $\Rightarrow$ 3) Let $\widetilde{G}$ be any nonempty τ-β-IFS-open subset of (X, τ, E). By Lemma 1, we have $c_\sigma\left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right) = \left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right) \sqcup i_\tau\left(c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\})\right) = \widetilde{G} \sqcup_{i=1}^{p} \{x_i\} \sqcup i_\tau(X) = X$.
Then $c_\sigma\left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right) = X$.
3) $\Rightarrow$ 1) Let $\widetilde{G}$ be a nonempty τ-open set of (X, τ, E). It follows from 3) that $c_\sigma\left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right) = \left(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}\right) \sqcup i_\tau\left(c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\})\right) = X$.
Thus, $c_\tau(\widetilde{G} \sqcup_{i=1}^{p} \{x_i\}) = X$. Consequently, (X, τ, E) is a GIFST soft superconnected. □

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