Notes on Intuitionistic Fuzzy Sets
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Interpreting the results of InterCriteria Analysis: Pareto principle at work

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Abstract: The present short note aims to propose a new, alternative, way to interpret the results of the intuitionistic fuzzy sets-based method for multicriteria decision support named InterCriteria Analysis. Given an $m \times n$ dataset of multiple (m) objects evaluated numerically against multiple (n) criteria, the ICA method generates an $n \times n$ table of intuitionistic fuzzy pairs $\langle \mu_{i,j}, \nu_{i,j} \rangle,\ i, j \in 1, 2, \ldots, n$ where the given pair indicates the extent of relation between the corresponding pair of criteria C_i, C_j. Traditionally, the interpretation of these intuitionistic fuzzy pairs regarding the extent of positive or negative dependence between two criteria (or, respectively, the lack of such) requires that two threshold values, both in the $[0, 1]$ interval too, are used. Now we propose to use only one such threshold value belonging to the $[0, 1]$ interval, for instance a minimal threshold of the degree of membership, while the other threshold would be essentially related to the size of the subset of intercriteria pairs being shortlisted for interpretation, rather than their degree of non-membership. We justify that the proposed approach, inspired by the Pareto Principle, in certain cases yields better results than the traditionally used one.

Keywords: Intuitionistic fuzzy sets, InterCriteria Analysis, Pareto Principle, Power law.
2020 Mathematics Subject Classification: 03E72.

1 Introduction

The present paper proposes a novel way for interpretation of the results from the application of the intuitionistic fuzzy sets (IFS, [1]) based method of InterCriteria Analysis (ICA), proposed in 2014, [2]. This new way is inspired by the famous Pareto Principle known from its application mainly in the areas of business and management.

InterCriteria Analysis is a method for finding pairwise relations among a set of criteria, done on the basis of the pairwise comparisons of the evaluations of multiple objects against these criteria and interpretation in terms of IFS. For each pair of criteria, ICA employs an intuitionistic fuzzy pair (see [3]) of numbers in the unit interval $[0, 1]$ that together serve as a measure of the relation between both criteria, calculated in a way principally different from other correlation methods and one rendering account of the inherent uncertainty of the real-life multicriteria problem. When the multicriteria problem involves some more costly, time-consuming, resource-consuming or unfavourable criteria in any other sense, one of the goals of ICA application may be the detection of reliably high similarity between these unfavourable criteria with some of the other – cheaper, faster or easer to measure – criteria, so that as a result well-grounded and objective criteria reduction may be performed. An alternative use of ICA is when the decision maker's goal is to research the discriminative properties of the criteria, rank them internally, or outline which of them are most strongly related, without the aim to eliminate any of them.

On the basis of input in the form of an $m \times n$ table of the numerical evaluations of m objects against n criteria, the result of ICA application is an $n \times n$ table, containing the intuitionistic fuzzy pairs representing in numbers from the $[0, 1]$ interval the levels of relation between each pair of criteria. Naturally, along the main diagonal the values are all $\langle 1, 0\rangle$ representing the intuitionistic fuzzy interpretation of *Truth*. In the resultant ICA table, the cell contents for criteria $\langle C_i, C_j\rangle$ and for criteria $\langle C_j, C_i\rangle$ are identical. The detailed presentation of the ICA method is given in [2] and the aspects of the computational complexity and influence of numerical precision on the results are discussed in [6], while the influence of the number of objects covered is studied in [16]. A recent bibliography of the papers with theoretical and application importance for the development of the ICA research is given in [7]. Software applications implementing the ICA algorithm have been developed by Mavrov and Ikonomov, and provided freely downloadable for the interested reader at `https://intercriteria.net/software/`, and described in details in several papers [10, 11, 13, 14].

2 Problem statement

In the relatively new field of InterCriteria Analysis, dating back to 2014, there have been numerous unexplored or not fully explored issues, including one issue that the authors consider of crucial importance, namely, how to interpret and objectively utilize the ICA results. Which positive consonance is strong enough to allow definitive decisions, where is the boundary between the strong and the weak consonance, or between the consonance and the dissonance. What was defined once conveniently in terms of parameters may have been perfectly fit for the purposes of definition and have intuitively incorporated the broad idea of the method, but may turn out completely void of sense if we lack the very algorithm of determining the particular values of these parameters, serving as a measure, a touch-stone, a litmus test for the outputs of the ICA algorithm.

Since the launch of ICA, the interpretation of the method's results has been done by comparing the resultant ICA pairs' values against user-defined or case-based determined threshold values, again numbers in the $[0, 1]$ interval, which are a threshold for the IF membership and a threshold for the IF non-membership, respectively. Depending on the specific problem formulation, the decision maker's purpose may be to either define the highest positive consonances among the criteria pairs (which seems to be the usual case), or the highest negative consonance, or the highest dissonance. Detailed overview of the approaches to defining the threshold values against which the ICA output is assessed (see [9]) exhibits these thresholds only as numbers in the unit interval, to which the InterCriteria pairs also naturally belong. The significance of this issue is easy to understand: depending on how rigorous or relaxed are the thresholds determined, a way different conclusions and hence actions by the decision maker would be taken, and a rather different picture of the world would be seen.

Here in the present paper, we propose a completely new approach to criteria ranking, which borrows inspiration from the famous Pareto Principle, [12, 15]. In a general formulation, this rule states that 80% of consequences come from 20% of causes, or that 80% of the work in a company are shouldered by only 20% of its employees, or, as the popular business management adage phrases it, 80% of sales come from 20% of clients. For us, Pareto Principle serves as an inspiration not only because the particular $80/20$ ratio in this "power law" seems to have proven effective in various contexts, but also in the less obvious yet more significant consideration that what is taken as thresholds in ICA need not be two complementary measures that are homogeneous in their nature, like the threshold for the membership and the threshold for the non-membership function, but can be two measures of completely different, heterogeneous nature.

As a side note, well suited here, the use of heterogeneous measures in shortlisting the best elements of an intuitionistic fuzzy set has been already explored by the first author in a previous research [4], again in an attempt to explore the diversity of approaches towards objectively selecting the best elements of an intuitionistic fuzzy set, in that case maintaining the highest possible membership to non-membership ratio, even in the presence of some inherent uncertainty.

3 Approaches to the solution

Our interest to validate if the Pareto Principle holds true when applied to the InterCriteria Analysis, requires us to check if in some previous research on ICA application the $80/20$ ratio would exhibit itself in the results and would yield comparable results.

For the "80" part of the power law, that is the part respective to the threshold for the IF membership, we can choose among two approaches. First we can take the threshold value be equal to the absolute value of 0.8. As this threshold value for the IF membership is the same as the one we are working with in the classical ICA setting, we will still denote it by the established parameter α. Then the other threshold value, the one related to the number of elements of the IFS, we will denote differently, with the parameter P, staying for *"Pareto"*, $P = 0.2 \cdot \frac{n \cdot (n-1)}{2}$.

Alternatively, in case of an IFS that is skewed in a certain area within the triangle, or when we would like to more finely tune our estimations, we can opt to work not with the whole triangle and the whole $[0, 1]$ interval for the μ function, but with just that part of the triangle that contains the IFS, respectively, the interval $[\min(\mu_A(x)), \max(\mu_A(x))]$. We then normalize that interval to the $[0, 1]$ and find the threshold value α relative to the bounds of the interval $[\min(\mu_A(x)), \max(\mu_A(x))]$, hence,

$$\alpha = \min(\mu_A(x)) + 0.8 \cdot [\max(\mu_A(x)) - \min(\mu_A(x))]$$
$$P = 0.2 \cdot \frac{n \cdot (n-1)}{2}$$

As it has been noted in [15], *"The term $80/20$ is only a shorthand for the general principle at work. In individual cases, the distribution could be nearer to $90/10$ or $70/30$. There is also no need for the two numbers to add up to the number 100, as they are measures of different things"*.

4 Numerical examples

So, is it true that the Pareto Principle manifests in the results of InterCriteria Analysis and in our previous discussions of datasets and problems addressable with this approach?

For example, in [5], the calculated ICA pairs of the ten subindicators (criteria) in Pillar 12 "Innovation capability" (Table 5) are sorted by their distance from the IF "Truth" (the point $\langle 1, 0 \rangle$). The top nine of them (20% of the elements of the resultant IFS) all have their μ degree greater than 0.8 (precisely, $\mu_{C_{12.04}, C_{12.09}} = 0.812$ and above) and the tenth best ICA pair has $\mu = 0.796$. As the whole set's μ-degrees range from 0.450 to 0.899, this means in the alternative formulation of the threshold α after normalization of the interval, that it should be equal to 0.8092, again true.

Again in [5], the 80 calculated ICA pairs of the total 18 subindicators in Pillar "11 Business dynamism" and Pillar 12 "Innovation capability" (Table 6), produce for the Pareto parameter $P = 0.2 \cdot 40 = 16$. Then the 16^{th} pair top down when sorted in descending order by the distance from the IF "Truth", yields that element's membership degree is $\mu_{C_{11.03}, C_{12.04}} = 0.772$, which is close to 0.8, and if $\alpha = 0.8$, the shortlisted ICA pairs are eight, that is the top 10%, so the power law is more like $80/10$. Alternatively, in the case of the normed interval $[\min(\mu_A(x)), \max(\mu_A(x))]$ which in this case is $[0.360, 0.913]$, then $\alpha = 0.8024$, yielding again the same eight top correlating ICA pairs of criteria.

In our next example, the set of 120 intercriteria pairs (that is, the IFS) has none of its elements featuring a membership function greater than 0.8. In a paper analysing with ICA the most problematic factors for doing business in the European Union in 2017–2018, [8], the ICA pair of highest positive consonance is the one between criteria "Poor work ethic in national labor force" and “Inadequately educated workforce” and its IF values are $\langle 0.767, 0.217\rangle$. This leaves no choice but to proceed to normalization of the interval $[\min(\mu_A(x)), \max(\mu_A(x))] = [0.262, 0.767]$ and thus determine the parameter $\alpha = 0.666$. There are only five elements of the set with their $\mu > \alpha = 0.666$, this makes it like the top 4% percent of the elements, so the Pareto power law looks more like $80/4$. Otherwise, when the parameter $P = 0.2$ is set, the 24 elements of the set have their membership values evaluated to at least 0.563, which on its own turn gives that the Pareto ratio is transformed to $60/20$.

Further analysis is worth carrying whether a “skewed” Pareto law is applicable in such a case, or the law is generally inapplicable, as well as to what is the underlying explanation in this case.

5 Conclusion

Embedded in the very first paper which defined and launched ICA in 2014 [2] was one of the keystones which now we are aiming to safely remove from the construction, and that is the idea that the membership and the non-membership parts of ICA – two numbers in the $[0, 1]$ interval – need be mandatorily, by definition compared against two threshold values, which are – predictably – again two numbers in the $[0, 1]$ interval. Now we will consider an alternative scenario where using the formulation of the Pareto Principle, we will consider that one of the original thresholds, that for the IF elements' membership is preserved, while a second threshold of a different nature is implemented, and it refers to the number of shortlisted elements of the set, or, more accurately, the share of those shortlisted to the whole size of the IFS.

While we remark that the Pareto principle literally refers to the $80/20$ ratio, we demonstrate with some general considerations and examples that the power law holds true with approximately similar numbers. This is envisaged to serve as a well working heuristic for the experts applying InterCriteria analysis about how practically to set customized threshold values, tailored to the particular problem formulations and datasets that are approached with the method of InterCriteria analysis.

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