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HOW TWO MATRICES CAN BE COMPARED?

Lilija C. Atanassova

105 School, G. Kalchev 7, Sofia-1172, BULGARIA

It is well known that two matrices with equal dimensions are equal, if all their corresponding elements are equal (see, e.g., [1]). A more complex question is the one of the relation "inequal". For example, we can assert that $A \le B$, if every A-element is less or equal than its corresponding B-elements. Another way for comparison of square matrices is based on comparison of their determinants, using the relations: $\det(A) < \det(B)$, $\det(A) \ge \det(B)$, etc. for the given matrices A and B. In this case the matrices must be square, and conditionally, we can write that

$$A\ R\ B \text{ iff } \det(A)\ R\ \det(B),$$

where $R \in \{<, \le, =, \ge, >\}$. In this case, while

$$\begin{Vmatrix} 2 & 2 \\ 2 & 2 \end{Vmatrix} < \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix},$$

in the first case the opposite relation will be valid.

Below we introduce new interpretation of the relation u ("unequal"), which is based on the concept intuitionistic fuzzy set (see [2]). Let the matrices A and B be given and let they have equal dimensions, e.g., $m \times n$ for the natural numbers m and n. Let α be the number of all elements of A which are less than the corresponding elements of B and let β be the number of all elements of A which are greater than the corresponding elements of B. Therefore, we can juxtapose to A and B the couple $\langle \alpha, \beta \rangle$ and therefore, we can define the intuitionistic fuzzy relation u by the intuitionistic fuzzy estimation finction V which will estimate the truth-values of the relations between A and B in the following form:

$$V(A\ u\ B) \equiv \langle \mu, \gamma \rangle = \langle \frac{\alpha}{m.n}, \frac{\beta}{m.n} \rangle.$$

Obviously, $0 \le \mu, \gamma \le 1$ and $0 \le \mu + \gamma \le 1$ and therefore, this estimation is intuitionistic fuzzy one. Analogically,

$$V(A \neg u\ B) = \langle \gamma, \mu \rangle,$$

has also an intuitionistic fuzzy value.

This interpretation of the order relation can be transformed for the index matrices [3] without changes, too.

We shall assert that the assertion A is an Intuitionistic Fuzzy Tautology (IFT), if $V(A) = \langle \mu, \gamma \rangle$ and $\mu \geq \gamma$; and it is an Intuitionistic fuzzy Save (IS), if $\mu \geq 1/2$ (cf. [4]).

We shall note that for u the following properties are valid:

$$V(A\ u\ A) = \langle 0, 0 \rangle.$$

Therefore, u is a reflexive relation in the sense of the IFT and it is a non-reflexive relation in the sense of the IS.

$$V(A\ u\ B) = \neg V(B\ u\ A),$$

where $\neg\langle p, q \rangle = \langle q, p \rangle$ for every $p, q \in [0, 1]$ and $p + q \leq 1$; Therefore, u is an asymmetric relation.

If A, B and C are three (m x n)-matrices, then from

$$V(A\ u\ B) = \langle \mu, \gamma \rangle$$

follows that

$$V((A + C)\ u\ (B + C)) = \langle \mu, \gamma \rangle;$$

and if α is a positive real number then

$$V(\alpha.A\ u\ \alpha.B) = \langle \mu, \gamma \rangle$$

and if α is a negative real number then

$$V(\alpha.A\ u\ \alpha.B) = \langle \gamma, \mu \rangle.$$

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