Notes on Intuitionistic Fuzzy Sets
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Primary interval-valued intuitionistic fuzzy *M* group

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Abstract: The concept of interval-valued intuitionistic fuzzy *M* group is extended by introducing primary interval-valued intuitionistic fuzzy *M* group and primary interval-valued intuitionistic fuzzy anti *M* group using this concept primary interval-valued intuitionistic fuzzy *M* group and primary interval-valued intuitionistic fuzzy anti *M* group is defined and using some properties are established.

Keywords: Intuitionistic fuzzy set, Primary interval-valued intuitionistic fuzzy *M* group, Primary interval-valued intuitionistic fuzzy anti *M* group, Primary interval-valued intuitionistic fuzzy *M* group, Primary interval-valued intuitionistic fuzzy anti *M* group.
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1 Introduction

The concept of fuzzy sets was initiated by L. A. Zadeh [10], then it has become a vigorous area of research in engineering, medical science, graph theory. Rosenfeld [9] gave the idea of fuzzy subgroup. Bipolar valued fuzzy sets was introduced by K. M. Lee [6] are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [–1, 0]. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [–1, 0) indicates that elements somewhat satisfy the implicit counter property. The author W. R. Zhang [11] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. Chakrabarthy, Biswas and R. Nanda [4] investigated note on union and intersection of intuitionistic fuzzy sets. G. Prasannavengeteswari, K. Gunasekaran and S. Nandakumar [8] introduced the definition of Primary Bipolar Intuitionistic *M* Fuzzy Group and anti-*M* Fuzzy Group. A. Balasubramanian, K. L. Muruganantha Prasad, K. Arjunan [3] introduced the definition of Bipolar Interval Valued Fuzzy Subgroups of a Group. In this study some operations on primary interval-valued intuitionistic fuzzy *M* group and anti-*M* group and some properties of the same are proved.

2 Preliminaries

Definition 1. Let *G* be a non-empty set, let *A* be an interval-valued intuitionistic fuzzy set (IVIFS) in *G* and be an object of the form $A = \{\langle x, [\mu_A^-(x), \mu_A^+(x)], [\upsilon_A^-(x), \upsilon_A^+(x)]\rangle \mid x \in G\}$, where $\mu_A^+\colon G \to [0, 1]$, $\mu_A^-\colon G \to [0, 1]$ and $\upsilon_A^+\colon G \to [0, 1], \upsilon_A^-\colon G \to [0, 1]$ and $(\forall x \in G)\ (\mu_A^-(x) \le \mu_A^+(x), \upsilon_A^-(x) \le \upsilon_A^+(x), \mu_A^+(x) + \upsilon_A^+(x) \le 1)$ are called the degree of positive membership, the degree of negative membership, the degree of positive non-membership, and the degree of negative non-membership, respectively.

Definition 2. Let *G* be an *M* group and *A* be an interval-valued intuitionistic fuzzy subgroup of *G*, then *A* is called a primary interval-valued intuitionistic fuzzy *M* group of *G*. If for all $x, y \in G$ and $m \in M$, then either $\mu_A^+(mxy) \le \mu_A^+(x^p)$ and $\upsilon_A^+(mxy) \ge \upsilon_A^+(x^p)$, for some $p \in Z_+$ or else $\mu_A^+(mxy) \le \mu_A^+(y^q)$ and $\upsilon_A^+(mxy) \ge \upsilon_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \ge \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) \le \upsilon_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \ge \mu_A^-(y^q)$ and $\upsilon_A^-(mxy) \le \upsilon_A^-(y^q)$, for some $q \in Z_+$.

Example 1.

$$\mu_A^+(x) = \begin{cases} 0.7\ if\ x = 1 \\ 0.6\ if x = -1 \\ 0.4\ \ if\ x = i, -i \end{cases} \quad \upsilon_A^+(x) = \begin{cases} 0.2\ if\ x = 1 \\ 0.3\ if x = -1 \\ 0.5\ \ if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} 0.6\ \ if\ x = 1 \\ 0.5\ \ if x = -1 \\ 0.3\ \ if\ x = i, -i \end{cases} \quad \upsilon_A^-(x) = \begin{cases} 0.1\ if\ x = 1 \\ 0.2\ \ if x = -1 \\ 0.5\ \ if\ x = i, -i \end{cases}$$

Definition 3. Let *G* be an *M* group and *A* be an interval-valued intuitionistic anti fuzzy subgroup of *G*, then *A* is called a primary interval-valued intuitionistic fuzzy anti *M* group of *G*. if for all $x, y \in G$ and $m \in M$, then either $\mu_A^+(mxy) \geq \mu_A^+(x^p)$ and $\upsilon_A^+(mxy) \leq \upsilon_A^+(x^p)$, for some $p \in Z_+$or else $\mu_A^+(mxy) \geq \mu_A^+(y^q)$ and $\upsilon_A^+(mxy) \leq \upsilon_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \leq \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) \geq \upsilon_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \leq \mu_A^-(y^q)$ and $\upsilon_A^-(mxy) \geq \upsilon_A^-(y^q)$, for some $q \in Z_+$

Example 2.

$$\mu_A^+(x) = \begin{cases} 0.4 \;\; if\; x = 1 \\ 0.6 \;\; if x = -1 \\ 0.7 \;\; if\; x = i, -i \end{cases} \quad \upsilon_A^+(x) = \begin{cases} 0.5 \;\; if\; x = 1 \\ 0.3 \;\; if x = -1 \\ 0.2 \;\; if\; x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} 0.3 \;\; if\; x = 1 \\ 0.5 \;\; if x = -1 \\ 0.4 \;\; if\; x = i, -i \end{cases} \quad \upsilon_A^-(x) = \begin{cases} 0.4 \;\; if\; x = 1 \\ 0.2 \;\; if x = -1 \\ 0.1 \;\; if\; x = i, -i \end{cases}$$

Definition 4 ([1, 2, 5]). Let *A* be an interval-valued intuitionistic fuzzy set of the universal set *E*, then the Necessity operator is defined by

$$\square A^+ = *_4 \square A = \{\langle x, \mu_A^+(x), 1 - \mu_A^+(x)\rangle \,|\, x \in E\},$$

$$\square A^- = *_1 \square A = \{\langle x, \mu_A^-(x), 1 - \mu_A^-(x)\rangle \,|\, x \in E\}.$$

Definition 5 ([1, 2, 5]). Let *A* be an interval-valued intuitionistic fuzzy set of the universal set *E*, then the Possibility operator is defined by

$$\lozenge A^+ = *_4 \lozenge A = \{\langle x, 1 - \upsilon_A^+(x), \upsilon_A^+(x)\rangle \,|\, x \in E\},$$

$$\lozenge A^- = *_1 \lozenge A = \{\langle x, 1 - \upsilon_A^-(x), \upsilon_A^-(x)\rangle \,|\, x \in E\}.$$

Definition 6. Let *G* be an *M* group and *A* be an interval-valued intuitionistic fuzzy subgroup of *G*, then *A* is called a primary interval-valued intuitionistic fuzzy *M* group of *G*.

If for all $x, y \in G$ and $m \in M$, then either $\mu_A^+(mxy) = \inf M_A(mxy) \leq \inf M_A(x^p) = \mu_A^+(x^p)$ and $\upsilon_A^+(mxy) = \sup N_A(mxy) \geq \sup N_A(x^p) = \upsilon_A^+(x^p)$, for some $p \in Z_+$ or else $\mu_A^+(mxy) = \inf M_A(mxy) \leq \inf M_A(y^q) = \mu_A^+(y^q)$ and $\upsilon_A^+(mxy) = \sup N_A(mxy) \geq \sup N_A(y^q) = \upsilon_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) = \sup M_A(mxy) \geq \sup M_A(x^p) = \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) = \inf N_A(mxy) \leq \inf N_A(x^p) = \upsilon_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) = \sup M_A(mxy) \geq \sup M_A(y^q) = \mu_A^-(y^q)$ and $\upsilon_A^-(mxy) = \inf N_A(mxy) \leq \inf N_A(y^q) = \upsilon_A^-(y^q)$, for some $q \in Z_+$.

Definition 7. Let *G* be an *M* group and *A* be an interval-valued intuitionistic fuzzy subgroup of *G*, then *A* is called a primary interval-valued intuitionistic fuzzy anti *M* group of *G*.

If for all $x, y \in G$ and $m \in M$, then either $\mu_A^+(mxy) = \inf M_A(mxy) \geq \inf M_A(x^p) = \mu_A^+(x^p)$ and $\upsilon_A^+(mxy) = \sup N_A(mxy) \leq \sup N_A(x^p) = \upsilon_A^+(x^p)$, for some $p \in Z_+$ or else $\mu_A^+(mxy) = \inf M_A(mxy) \geq \inf M_A(y^q) = \mu_A^+(y^q)$ and $\upsilon_A^+(mxy) = \sup N_A(mxy) \leq \sup N_A(y^q) = \upsilon_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) = \sup M_A(mxy) \leq \sup M_A(x^p) = \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) = \inf N_A(mxy) \geq \inf N_A(x^p) = \upsilon_A^-(x^p)$, for some

$p \in Z_+$ or else $\mu_A^-(mxy) = \sup M_A(mxy) \leq \sup M_A(y^q) = \mu_A^-(y^q)$ and $\upsilon_A^-(mxy) = \inf N_A(mxy) \geq \inf N_A(y^q) = \upsilon_A^-(y^q)$, for some $q \in Z_+$

3 Some operations on primary interval-valued intuitionistic fuzzy *M* group and primary interval-valued intuitionistic fuzzy anti *M* group

Theorem 1. *If A is a primary interval-valued intuitionistic fuzzy M group of G, then* $\bar{\bar{A}} = A$ *is a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$. Now

$$\begin{aligned} \mu_{\bar{\bar{A}}}^+(mxy) &= \upsilon_{\bar{A}}^+(mxy) \\ &= \mu_A^+(mxy) \\ &= \inf M_A(mxy) \\ &\leq \inf M_A(x^p) \\ &= \mu_A^+(x^p). \end{aligned}$$

Therefore, $\mu_{\bar{\bar{A}}}^+(mxy) \leq \mu_A^+(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned} \upsilon_{\bar{\bar{A}}}^+(mxy) &= \mu_{\bar{A}}^+(mxy) \\ &= \upsilon_A^+(mxy) \\ &= \sup N_A(mxy) \\ &\geq \sup N_A(x^p) \\ &= \upsilon_A^+(x^p). \end{aligned}$$

Therefore, $\upsilon_{\bar{\bar{A}}}^+(mxy) \geq \upsilon_A^+(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned} \mu_{\bar{\bar{A}}}^-(mxy) &= \upsilon_{\bar{A}}^-(mxy) \\ &= \mu_A^-(mxy) \\ &= \sup M_A(mxy) \\ &\geq \sup M_A(x^p) \\ &= \mu_A^-(x^p). \end{aligned}$$

Therefore, $\mu_{\bar{\bar{A}}}^-(mxy) \geq \mu_A^-(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned} \upsilon_{\bar{\bar{A}}}^-(mxy) &= \mu_{\bar{A}}^-(mxy) \\ &= \upsilon_A^-(mxy) \\ &= \inf N_A(mxy) \\ &\leq \inf N_A(x^p) \\ &= \upsilon_A^-(x^p). \end{aligned}$$

Therefore, $\upsilon_{\bar{\bar{A}}}^-(mxy) \leq \upsilon_A^-(x^p)$, for some $p \in Z_+$. Therefore $\bar{\bar{A}} = A$ is a primary interval-valued intuitionistic fuzzy *M* group of *G*. □

Theorem 2. *Intersection of any two primary interval-valued intuitionistic fuzzy M group is again a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Let A and B be two primary interval-valued intuitionistic fuzzy M group of G. Consider $x, y \in A$ and $x, y \in B$ and $m \in M$. Now

$$\begin{aligned} \mu^+_{A\cap B}(mxy) &= \min\big(\mu^+_A(mxy), \mu^+_B(mxy)\big) \\ &= \min(\inf M_A(mxy), \inf M_B(mxy)) \\ &\le \min(\inf M_A(x^p), \inf M_B(x^p)) \\ &= \min(\mu^+_A(x^p), \mu^+_B(x^p)) \\ &= \mu^+_{A\cap B}(x^p). \end{aligned}$$

Therefore, $\mu^+_{A\cap B}(mxy) \le \mu^+_{A\cap B}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned} \upsilon^+_{A\cap B}(mxy) &= \max(\upsilon^+_A(mxy), \upsilon^+_B(mxy)) \\ &= \max(\sup N_A(mxy), \sup N_B(mxy)) \\ &\ge \max(\sup N_A(x^p), \sup N_B(x^p)) \\ &= \max(\upsilon^+_A(x^p), \upsilon^+_B(x^p)) \\ &= \upsilon^+_{A\cap B}(x^p). \end{aligned}$$

Therefore, $\upsilon^+_{A\cap B}(mxy) \ge \upsilon^+_{A\cap B}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned} \mu^-_{A\cap B}(mxy) &= \max(\mu^-_A(mxy), \mu^-_B(mxy)) \\ &= \max(\sup M_A(mxy), \sup M_B(mxy)) \\ &\ge \max(\sup M_A(x^p), \sup M_B(x^p)) \\ &= \max(\mu^-_A(x^p), \mu^-_B(x^p)) \\ &= \mu^-_{A\cap B}(x^p) \end{aligned}$$

Therefore, $\mu^-_{A\cap B}(mxy) \ge \mu^-_{A\cap B}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned} \upsilon^-_{A\cap B}(mxy) &= \min(\upsilon^-_A(mxy), \upsilon^-_B(mxy)) \\ &= \min(\inf N_A(mxy), \inf N_B(mxy)) \\ &= \min(\inf N_A(x^p), \inf N_B(x^p)) \\ &\le \min(\upsilon^-_A(x^p), \upsilon^-_B(x^p)) \\ &= \upsilon^-_{A\cap B}(x^p) \end{aligned}$$

Therefore, $\upsilon^-_{A\cap B}(mxy) \le \upsilon^-_{A\cap B}(x^p)$, for some $p \in Z_+$.
Therefore, $A \cap B$ is a primary interval-valued intuitionistic fuzzy M group of G. □

Theorem 3. *Union of any two primary interval-valued intuitionistic fuzzy M group is also a primary interval-valued intuitionistic fuzzy M fuzzy group if either is contained in the other.*

Proof. Let A and B be two primary interval-valued intuitionistic fuzzy M group of G. To prove that $A \cup B$ is a primary interval-valued intuitionistic fuzzy M group of G, if $A \subseteq B$ or $B \subseteq A$. Consider $x, y \in A$ and $x, y \in B$ and $m \in M$. Now

$$\begin{aligned} \mu^+_{A\cup B}(mxy) &= \max\big(\mu^+_A(mxy), \mu^+_B(mxy)\big) \\ &= \max(\inf M_A(mxy), \inf M_B(mxy)) \\ &\le \max(\inf M_A(x^p), \inf M_B(x^p)) \\ &= \max(\mu^+_A(x^p), \mu^+_B(x^p)) \\ &= \mu^+_{A\cup B}(x^p). \end{aligned}$$

Therefore, $\mu^+_{A\cup B}(mxy) \le \mu^+_{A\cup B}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned} \upsilon^+_{A\cup B}(mxy) &= \min(\upsilon^+_A(mxy), \upsilon^+_B(mxy)) \\ &= \min(\sup N_A(mxy), \sup N_B(mxy)) \\ &\ge \min(\sup N_A(x^p), \sup N_B(x^p)) \end{aligned}$$

$$= \min(\upsilon_A^+(x^p), \upsilon_B^+(x^p))$$
$$= \upsilon_{A\cup B}^+(x^p).$$

Therefore, $\upsilon_{A\cup B}^+(mxy) \geq \upsilon_{A\cup B}^+(x^p)$, for some $p \in Z_+$. Consider

$$\mu_{A\cup B}^-(mxy) = \min(\mu_A^-(mxy), \mu_B^-(mxy))$$
$$= \min(\sup M_A(mxy), \sup M_B(mxy))$$
$$\geq \min(\sup M_A(x^p), \sup M_B(x^p))$$
$$= \min(\mu_A^-(x^p), \mu_B^-(x^p))$$
$$= \mu_{A\cup B}^-(x^p)$$

Therefore, $\mu_{A\cup B}^-(mxy) \geq \mu_{A\cup B}^-(x^p)$, for some $p \in Z_+$. Consider

$$\upsilon_{A\cup B}^-(mxy) = \max(\upsilon_A^-(mxy), \upsilon_B^-(mxy))$$
$$= \max(\inf N_A(mxy), \inf N_B(mxy))$$
$$= \max(\inf N_A(x^p), \inf N_B(x^p))$$
$$\leq \max(\upsilon_A^-(x^p), \upsilon_B^-(x^p))$$
$$= \upsilon_{A\cup B}^-(x^p)$$

Therefore, $\upsilon_{A\cup B}^-(mxy) \leq \upsilon_{A\cup B}^-(x^p)$, for some $p \in Z_+$.

Hence, the union of any two primary interval-valued intuitionistic fuzzy *M* groups is also an interval-valued intuitionistic fuzzy *M* group if either is contained in the other. □

Theorem 4. *If A is a primary interval-valued intuitionistic fuzzy M group of G, then □A is also a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$. Consider

$$\mu_{\square A}^+(mxy) = \mu_A^+(mxy)$$
$$= \inf M_A(mxy)$$
$$\leq \inf M_A(x^p)$$
$$= \mu_A^+(x^p)$$
$$= \mu_{\mathrm{A}}^+(x^p).$$

Therefore, $\mu_{\square A}^+(mxy) \leq \mu_{\square A}^+(x^p)$, for some $p \in Z_+$. Consider

$$\upsilon_{\square A}^+(mxy) = 1 - \mu_{\mathrm{A}}^+(mxy)$$
$$= 1 - \inf M_A(mxy)$$
$$\geq 1 - \inf M_A(x^p)$$
$$= 1 - \mu_{\mathrm{A}}^+(x^p)$$
$$= 1 - \mu_{\square A}^+(x^p)$$
$$= \upsilon_{\mathrm{A}}^+(x^p).$$

Therefore, $\upsilon_{\square A}^+(mxy) \geq \upsilon_{\square A}^+(x^p)$, for some $p \in Z_+$. Consider

$$\mu_{\square A}^-(mxy) = \mu_A^-(mxy)$$
$$= \sup M_A(mxy)$$
$$\geq \sup M_A(x^p)$$
$$= \mu_A^-(x^p)$$
$$= \mu_{\mathrm{A}}^-(x^p).$$

Therefore, $\mu_{\square A}^-(mxy) \geq \mu_{\square A}^-(x^p)$, for some $p \in Z_+$. Consider

$$\upsilon_{\square A}^-(mxy) = -1 - \mu_{\mathrm{A}}^-(mxy)$$
$$= -1 - \sup M_A(mxy)$$

$$\leq -1 - \sup M_A(x^p)$$
$$= -1 - \mu_A^-(x^p)$$
$$= -1 - \mu_{\square A}^-(x^p)$$
$$= \upsilon_A^-(x^p).$$

Therefore, $\upsilon_{\square A}^-(mxy) \leq \upsilon_{\square A}^-(x^p)$, for some $p \in Z_+$.

Therefore, $\square A$ is a primary interval-valued intuitionistic fuzzy *M* group of *G*. □

Theorem 5. *If A is a primary interval-valued intuitionistic fuzzy M group of G, then ◊A is also a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$. Consider

$$\mu_{\Diamond A}^+(mxy) = 1 - \upsilon_{\Diamond A}^+(mxy)$$
$$= 1 - \sup N_A(mxy)$$
$$\leq 1 - \sup N_A(x^p)$$
$$= 1 - \upsilon_A^+(x^p)$$
$$= 1 - \upsilon_{\Diamond A}^+(x^p)$$
$$= \mu_{\Diamond A}^+(x^p).$$

Therefore, $\mu_{\Diamond A}^+(mxy) \leq \mu_{\Diamond A}^+(x^p)$, for some $p \in Z_+$. Consider

$$\upsilon_{\Diamond A}^+(mxy) = \upsilon_A^+(mxy)$$
$$= supN_A(mxy)$$
$$\geq supN_A(x^p)$$
$$= \upsilon_A^+(x^p)$$
$$= \upsilon_{\Diamond A}^+(x^p)$$

Therefore, $\upsilon_{\Diamond A}^+(mxy) \geq \upsilon_{\Diamond A}^+(x^p)$, for some $p \in Z_+$. Consider

$$\mu_{\Diamond A}^-(mxy) = -1 - \upsilon_{\Diamond A}^-(mxy)$$
$$= -1 - infN_A(mxy)$$
$$\geq -1 - infN_A(x^p)$$
$$= -1 - \upsilon_A^-(x^p)$$
$$= -1 - \upsilon_{\Diamond A}^-(x^p)$$
$$= \mu_{\Diamond A}^-(x^p)$$

Therefore $\mu_{\Diamond A}^-(mxy) \geq \mu_{\Diamond A}^-(x^p)$, for some $p \in Z_+$. Consider

$$\upsilon_{\Diamond A}^-(mxy) = \upsilon_A^-(mxy)$$
$$= infN_A(mxy)$$
$$= infN_A(x^p)$$
$$\leq \upsilon_A^-(x^p)$$
$$= \upsilon_{\Diamond A}^-(x^p).$$

Therefore, $\upsilon_{\Diamond A}^-(mxy) \leq \upsilon_{\Diamond A}^-(x^p)$, for some $p \in Z_+$.

Therefore, ◊ *A* is a primary interval-valued intuitionistic fuzzy *M* group of *G*. □

Theorem 6. *If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then* $\bar{\bar{A}} = A$ *is a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$. Now,

$$\begin{aligned}\mu_{\bar{\bar{A}}}^{+}(mxy) &= \upsilon_{\bar{A}}^{+}(mxy)\\ &= \mu_{A}^{+}(mxy)\\ &= \inf M_A(mxy)\\ &\geq \inf M_A(x^p)\\ &= \mu_A^{+}(x^p).\end{aligned}$$

Therefore, $\mu_{\bar{\bar{A}}}^{+}(mxy) \geq \mu_A^{+}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned}\upsilon_{\bar{\bar{A}}}^{+}(mxy) &= \mu_{\bar{A}}^{+}(mxy)\\ &= \upsilon_{A}^{+}(mxy)\\ &= \sup N_A(mxy)\\ &\leq \sup N_A(x^p)\\ &= \upsilon_A^{+}(x^p).\end{aligned}$$

Therefore, $\upsilon_{\bar{\bar{A}}}^{+}(mxy) \leq \upsilon_A^{+}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned}\mu_{\bar{\bar{A}}}^{-}(mxy) &= \upsilon_{\bar{A}}^{-}(mxy)\\ &= \mu_{A}^{-}(mxy)\\ &= \sup M_A(mxy)\\ &\leq \sup M_A(x^p)\\ &= \mu_A^{-}(x^p).\end{aligned}$$

Therefore, $\mu_{\bar{\bar{A}}}^{-}(mxy) \leq \mu_A^{-}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned}\upsilon_{\bar{\bar{A}}}^{-}(mxy) &= \mu_{\bar{A}}^{-}(mxy)\\ &= \upsilon_{A}^{-}(mxy)\\ &= \inf N_A(mxy)\\ &\geq \inf N_A(x^p)\\ &= \upsilon_A^{-}(x^p).\end{aligned}$$

Therefore, $\upsilon_{\bar{\bar{A}}}^{-}(mxy) \geq \upsilon_A^{-}(x^p)$, for some $p \in Z_+$.

Therefore, $\bar{\bar{A}} = A$ is a primary bipolar interval valued intuitionistic anti *M*-fuzzy group of *G*. □

Theorem 7. *Intersection of any two primary interval-valued intuitionistic fuzzy anti M group is again a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Let *A* and *B* be two primary interval-valued intuitionistic fuzzy anti *M* group of *G*.
Consider $x, y \in A$ and $x, y \in B$ and $m \in M$. Now

$$\begin{aligned}\mu_{A\cap B}^{+}(mxy) &= \min\big(\mu_A^{+}(mxy), \mu_B^{+}(mxy)\big)\\ &= \min(\inf M_A(mxy), \inf M_B(mxy))\\ &\geq \min(\inf M_A(x^p), \inf M_B(x^p))\\ &= \min(\mu_A^{+}(x^p), \mu_B^{+}(x^p))\\ &= \mu_{A\cap B}^{+}(x^p).\end{aligned}$$

Therefore, $\mu_{A\cap B}^{+}(mxy) \geq \mu_{A\cap B}^{+}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned}\upsilon_{A\cap B}^{+}(mxy) &= \max(\upsilon_A^{+}(mxy), \upsilon_B^{+}(mxy))\\ &= \max(\sup N_A(mxy), \sup N_B(mxy))\\ &\leq \max(\sup N_A(x^p), \sup N_B(x^p))\end{aligned}$$

$$= \max(\upsilon_A^+(x^p), \upsilon_B^+(x^p))$$
$$= \upsilon_{A\cap B}^+(x^p).$$
Therefore, $\upsilon_{A\cap B}^+(mxy) \leq \upsilon_{A\cap B}^+(x^p)$, for some $p \in Z_+$. Consider
$$\begin{aligned}\mu_{A\cap B}^-(mxy) &= \max\left(\mu_A^-(mxy), \mu_B^-(mxy)\right)\\ &= \max(\sup M_A(mxy), \sup M_B(mxy))\\ &\leq \max(\sup M_A(x^p), \sup M_B(x^p))\\ &= \max(\mu_A^-(x^p), \mu_B^-(x^p))\\ &= \mu_{A\cap B}^-(x^p).\end{aligned}$$
Therefore, $\mu_{A\cap B}^-(mxy) \leq \mu_{A\cap B}^-(x^p)$, for some $p \in Z_+$. Consider
$$\begin{aligned}\upsilon_{A\cap B}^-(mxy) &= \min(\upsilon_A^-(mxy), \upsilon_B^-(mxy))\\ &= \min(\inf N_A(mxy), \inf N_B(mxy))\\ &= \min(\inf N_A(x^p), \inf N_B(x^p))\\ &\geq \min(\upsilon_A^-(x^p), \upsilon_B^-(x^p))\\ &= \upsilon_{A\cap B}^-(x^p).\end{aligned}$$
Therefore, $\upsilon_{A\cap B}^-(mxy) \geq \upsilon_{A\cap B}^-(x^p)$, for some $p \in Z_+$.
Therefore, $A \cap B$ is a primary interval-valued intuitionistic fuzzy anti M group of G. □

Theorem 8. *Union of any two primary interval-valued intuitionistic fuzzy anti M group is also a primary interval-valued intuitionistic fuzzy anti M group if either is contained in the other.*
Proof. Let A and B be two primary interval-valued intuitionistic fuzzy anti M group of G. To prove that $A \cup B$ is a primary interval-valued intuitionistic fuzzy anti M group of G, if $A \subseteq B$ or $B \subseteq A$.
Consider $x, y \in A$ and $x, y \in B$ and $m \in M$. Now
$$\begin{aligned}\mu_{A\cup B}^+(mxy) &= \max\left(\mu_A^+(mxy), \mu_B^+(mxy)\right)\\ &= \max(\inf M_A(mxy), \inf M_B(mxy))\\ &\geq \max(\inf M_A(x^p), \inf M_B(x^p))\\ &= \max(\mu_A^+(x^p), \mu_B^+(x^p))\\ &= \mu_{A\cup B}^+(x^p).\end{aligned}$$
Therefore, $\mu_{A\cup B}^+(mxy) \geq \mu_{A\cup B}^+(x^p)$, for some $p \in Z_+$. Consider
$$\begin{aligned}\upsilon_{A\cup B}^+(mxy) &= \min(\upsilon_A^+(mxy), \upsilon_B^+(mxy))\\ &= \min(\sup N_A(mxy), \sup N_B(mxy))\\ &\leq \min(\sup N_A(x^p), \sup N_B(x^p))\\ &= \min(\upsilon_A^+(x^p), \upsilon_B^+(x^p))\\ &= \upsilon_{A\cup B}^+(x^p).\end{aligned}$$
Therefore, $\upsilon_{A\cup B}^+(mxy) \leq \upsilon_{A\cup B}^+(x^p)$, for some $p \in Z_+$. Consider
$$\begin{aligned}\mu_{A\cup B}^-(mxy) &= \min(\mu_A^-(mxy), \mu_B^-(mxy))\\ &= \min(\sup M_A(mxy), \sup M_B(mxy))\\ &\leq \min(\sup M_A(x^p), \sup M_B(x^p))\\ &= \min(\mu_A^-(x^p), \mu_B^-(x^p))\\ &= \mu_{A\cup B}^-(x^p).\end{aligned}$$
Therefore, $\mu_{A\cup B}^-(mxy) \leq \mu_{A\cup B}^-(x^p)$, for some $p \in Z_+$. Consider
$$\begin{aligned}\upsilon_{A\cup B}^-(mxy) &= \max(\upsilon_A^-(mxy), \upsilon_B^-(mxy))\\ &= \max(\inf N_A(mxy), \inf N_B(mxy))\end{aligned}$$

$$= \max(\inf N_A(x^p), \inf N_B(x^p))$$
$$\geq \max(\upsilon_A^-(x^p), \upsilon_B^-(x^p))$$
$$= \upsilon_{A\cup B}^-(x^p).$$

Therefore $\upsilon_{A\cup B}^-(mxy) \geq \upsilon_{A\cup B}^-(x^p)$, for some $p \in Z_+$.

Hence, the union of any two primary interval-valued intuitionistic fuzzy anti *M* group is also a primary interval-valued intuitionistic fuzzy anti *M* group if either is contained in the other. □

Theorem 9. *If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then □A is also a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$. Consider

$$\mu_{\square \mathrm{A}}^+(mxy) = \mu_A^+(mxy)$$
$$= \inf M_A(mxy)$$
$$\geq \inf M_A(x^p)$$
$$= \mu_A^+(x^p)$$
$$= \mu_{\mathrm{A}}^+(x^p).$$

Therefore, $\mu_{\square \mathrm{A}}^+(mxy) \geq \mu_{\square \mathrm{A}}^+(x^p)$, for some $p \in Z_+$. Consider

$$\upsilon_{\square \mathrm{A}}^+(mxy) = 1 - \mu_{\mathrm{A}}^+(mxy)$$
$$= 1 - \inf M_A(mxy)$$
$$\leq 1 - \inf M_A(x^p)$$
$$= 1 - \mu_{\mathrm{A}}^+(x^p)$$
$$= 1 - \mu_{\square \mathrm{A}}^+(x^p)$$
$$= \upsilon_{\mathrm{A}}^+(x^p).$$

Therefore, $\upsilon_{\square \mathrm{A}}^+(mxy) \leq \upsilon_{\square \mathrm{A}}^+(x^p)$, for some $p \in Z_+$. Consider

$$\mu_{\square \mathrm{A}}^-(mxy) = \mu_A^-(mxy)$$
$$= \sup M_A(mxy)$$
$$\leq \sup M_A(x^p)$$
$$= \mu_A^-(x^p)$$
$$= \mu_{\mathrm{A}}^-(x^p).$$

Therefore, $\mu_{\square \mathrm{A}}^-(mxy) \leq \mu_{\square \mathrm{A}}^-(x^p)$, for some $p \in Z_+$. Consider

$$\upsilon_{\square \mathrm{A}}^-(mxy) = -1 - \mu_{\mathrm{A}}^-(mxy)$$
$$= -1 - \sup M_A(mxy)$$
$$\geq -1 - \sup M_A(x^p)$$
$$= -1 - \mu_A^-(x^p)$$
$$= -1 - \mu_{\square \mathrm{A}}^-(x^p)$$
$$= \upsilon_{\mathrm{A}}^-(x^p).$$

Therefore, $\upsilon_{\square A}^-(mxy) \geq \upsilon_{\square A}^-(x^p)$, for some $p \in Z_+$.

Therefore, □*A* is a primary interval-valued intuitionistic fuzzy anti *M* group of *G*. □

Theorem 10. *If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then ◊A is also a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$. Consider

$$\begin{aligned} \mu^{+}_{\lozenge \mathrm{A}}(mxy) &= 1 - \upsilon^{+}_{\lozenge A}(mxy) \\ &= 1 - \sup N_A(mxy) \\ &\geq 1 - \sup N_A(x^p) \\ &= 1 - \upsilon^{+}_{A}(x^p) \\ &= 1 - \upsilon^{+}_{\lozenge A}(x^p) \\ &= \mu^{+}_{\lozenge \mathrm{A}}(x^p). \end{aligned}$$

Therefore, $\mu^{+}_{\lozenge \mathrm{A}}(mxy) \geq \mu^{+}_{\lozenge \mathrm{A}}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned} \upsilon^{+}_{\lozenge \mathrm{A}}(mxy) &= \upsilon^{+}_{A}(mxy) \\ &= \sup N_A(mxy) \\ &\leq \sup N_A(x^p) \\ &= \upsilon^{+}_{A}(x^p) \\ &= \upsilon^{+}_{\lozenge \mathrm{A}}(x^p). \end{aligned}$$

Therefore, $\upsilon^{+}_{\lozenge \mathrm{A}}(mxy) \leq \upsilon^{+}_{\lozenge \mathrm{A}}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned} \mu^{-}_{\lozenge \mathrm{A}}(mxy) &= -1 - \upsilon^{-}_{\lozenge \mathrm{A}}(mxy) \\ &= -1 - \inf N_A(mxy) \\ &\leq -1 - \inf N_A(x^p) \\ &= -1 - \upsilon^{-}_{\mathrm{A}}(x^p) \\ &= -1 - \upsilon^{-}_{\lozenge \mathrm{A}}(x^p) \\ &= \mu^{-}_{\lozenge \mathrm{A}}(x^p). \end{aligned}$$

Therefore, $\mu^{-}_{\lozenge \mathrm{A}}(mxy) \leq \mu^{-}_{\lozenge \mathrm{A}}(x^p)$, for some $p \in Z_+$. Consider

$$\begin{aligned} \upsilon^{-}_{\lozenge \mathrm{A}}(mxy) &= \upsilon^{-}_{A}(mxy) \\ &= \inf N_A(mxy) \\ &= \inf N_A(x^p) \\ &\geq \upsilon^{-}_{A}(x^p) \\ &= \upsilon^{-}_{\lozenge \mathrm{A}}(x^p). \end{aligned}$$

Therefore, $\upsilon^{-}_{\lozenge \mathrm{A}}(mxy) \geq \upsilon^{-}_{\lozenge \mathrm{A}}(x^p)$, for some $p \in Z_+$.
Therefore, $\lozenge A$ is a primary interval-valued intuitionistic fuzzy anti *M* group of *G*. □

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