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ON A NEW INTUITIONISTIC FUZZY IMPLICATION OF GAINES-RESCHER'S TYPE

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Abstract

A new intutionistic fuzzy implication from a Gaines-Rescher's type is constructed. Its relation with some forms of Modus Ponens, and Klir and Yuan's axioms are studied.

1 Introduction

The concept of *"intuitionistic fuzzy propositional calculus"* has been introduced about 20 years ago (see, e.g., [1, 2]). Initially, it contained only one form of conjunction, disjunction and two forms of implication. In [3, 5] other forms of these three operations are introduced. Here, we shall introduce a new implication and will study some of its properties.

In intuitionistic fuzzy propositional calculus, if x is a variable then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b\rangle,$$

so that $a, b, a+b \in [0,1]$, where a and b are degrees of validity and of non-validity of x.

Below we shall assume that for the three variables x, y and z the equalities: $V(x) = \langle a, b\rangle, V(y) = \langle c, d\rangle, V(z) = \langle e, f\rangle$ $(a, b, c, d, e, f, a+b, c+d, e+f \in [0,1])$ hold.

For the needs of the discussion below we shall define the notion of Intuitionistic Fuzzy Tautology (IFT, see [1]) by:

$$x \text{ is an IFT if and only if for } V(x) = \langle a, b\rangle \text{ holds: } a \geq b,$$

while x will be a tautology iff $a = 1$ and $b = 0$. As in the case of ordinary logics, x is a tautology, if $V(x) = \langle 1, 0\rangle$.

For two variables x and y operations "conjunction" (&) and "disjunction" ($\vee$) are defined (see [1]) by:

$$V(x\&y) = \langle \min(a,c), \max(b,d)\rangle,$$
$$V(x \vee y) = \langle \max(a,c), \min(b,d)\rangle,$$

while, in [3] the following definitions are given

$$V(x\&y) = \langle a.c, b+d-b.d\rangle,$$

$$V(x \vee y) = \langle a+c-a.c, b.d\rangle.$$

Below we shall use only the latter definitions.

In a series of papers a lot of intuitionistic fuzzy implications were discussed. Their number is higher than 180, but some of them coincide and about 100 are different. Here we shall introduce a new implication, that does not coincide with the rest ones. It is analogous of the intuitionistic fuzzy form of Gaines-Rescher's implication, introduced in [4], which has the form:

$$V(x \to y) = \langle 1 - \mathrm{sg}(a-c), d.\mathrm{sg}(a-c)\rangle.$$

The new implication has the fom:

$$V(x \to y) = \langle 1 - \mathrm{sg}(a) + \mathrm{sg}(a).c, d.\mathrm{sg}(a)\rangle.$$

It is correct, because

$$0 \leq 1 - \mathrm{sg}(a) + \mathrm{sg}(a).c + d.\mathrm{sg}(a) = 1 - \mathrm{sg}(a) + \mathrm{sg}(a).(c+d) \leq 1 - \mathrm{sg}(a) + \mathrm{sg}(a) = 1.$$

2 Main results

We shall start with the remark, that the new operation "imlication" generates operation "negation", as follows:

$$\neg\langle a, b\rangle = \langle a, b\rangle \to \langle 0, 1\rangle = \langle 1 - \mathrm{sg}(a), \mathrm{sg}(a)\rangle,$$

i.e., this negation coincides with the negation generated by Gaines-Rescher's implication.

Therefore, this negation satisfies the following properties.

Theorem 1. The new implication
(a) satisfies Modus Ponens in the case of tautology,
(b) does not satisfy Modus Ponens in the IFT-case. **Proof:** We shall show the validity of (a). Let $a = 1, b = 0$, i.e., $\langle a, b\rangle$ be a tautology and let $\langle 1 - \text{sg}(a) + \text{sg}(a).c, d.\text{sg}(a)\rangle$ be a tautology, i.e. $1 - \text{sg}(a) + \text{sg}(a).c = 1, d.\text{sg}(a) = 0$. Therefore, $c = 1, d = 0$, i.e., $\langle c, d\rangle$ is a tautology.

(b) is not valid, because, for example,

$$a = b = 0, c = \frac{1}{3}, d = \frac{1}{2}$$

does not satisfy Modus Ponens in the IFT-case.

Some variants of fuzzy implications (marked by $I(x, y)$) are described in book [6] by Georg Klir and Bo Yuan and the following nine axioms are discussed, where

$$I(x, y) \equiv x \to y.$$

Axiom 1 $(\forall x, y)(x \leq y \to (\forall z)(I(x, z) \geq I(y, z))$.
Axiom 2 $(\forall x, y)(x \leq y \to (\forall z)(I(z, x) \leq I(z, y))$.
Axiom 3 $(\forall y)(I(0, y) = 1)$.
Axiom 4 $(\forall y)(I(1, y) = y)$.
Axiom 5 $(\forall x)(I(x, x) = 1)$.
Axiom 6 $(\forall x, y, z)(I(x, I(y, z)) = I(y, I(x, z)))$.
Axiom 7 $(\forall x, y)(I(x, y) = 1 \text{ iff } x \leq y)$.
Axiom 8 $(\forall x, y)(I(x, y) = I(N(y), N(x)))$, where N is an operation for a negation.
Axiom 9 I is a continuous function.

Theorem 2. The new implication satisfies Axioms 1,2,3,4,6.

Proof: Let $a \leq c$ and $b \geq d$. Then for Axiom 1 we obtain:

$$V(I(x, z)) = \langle 1 - \text{sg}(a) + \text{sg}(a).e, f.\text{sg}(a)\rangle,$$

$$V(I(y, z)) = \langle 1 - \text{sg}(c) + \text{sg}(c).e, f.\text{sg}(c)\rangle,$$

and

$$1 - \text{sg}(a) + \text{sg}(a).e - (1 - \text{sg}(c) + \text{sg}(c).e = (\text{sg}(c) - \text{sg}(a))(1 - e) \geq 0,$$

and

$$f.\text{sg}(c) - f.\text{sg}(a) \geq 0.$$

Therefore, Axiom 1 is valid. Axiom 2 is checked analogically.

Axiom 3 is valid, because

$$V(I(0, y)) = \langle 1 - \text{sg}(0) + \text{sg}(0).e, f.\text{sg}(0)\rangle = \langle 1, 0\rangle.$$

Axiom 4 is checked analogically.

Here we shall mention that there are counterexamples that show that Axiom 5 is not valid for this implication.

For axiom 6 we obtain:

$$V(I(x, I(y, z))) = \langle a, b\rangle \to \langle 1 - \mathrm{sg}(c) + \mathrm{sg}(c).e, f.\mathrm{sg}(c)\rangle$$

$$= \langle 1 - \mathrm{sg}(a) + \mathrm{sg}(a) - \mathrm{sg}(a).\mathrm{sg}(c) + \mathrm{sg}(a).\mathrm{sg}(c).e, f.\mathrm{sg}(c).\mathrm{sg}(a)\rangle$$

$$= \langle 1 - \mathrm{sg}(a).\mathrm{sg}(c) + \mathrm{sg}(a).\mathrm{sg}(c).e, f.\mathrm{sg}(c).\mathrm{sg}(a)\rangle$$

$$= \langle 1 - \mathrm{sg}(c).\mathrm{sg}(a) + \mathrm{sg}(c).\mathrm{sg}(a).e, f.\mathrm{sg}(a).\mathrm{sg}(c)\rangle$$

$$= \langle 1 - \mathrm{sg}(c) + \mathrm{sg}(c) - \mathrm{sg}(c).\mathrm{sg}(a) + \mathrm{sg}(c).\mathrm{sg}(a).e, f.\mathrm{sg}(a).\mathrm{sg}(c)\rangle$$

$$= \langle c, d\rangle \to \langle 1 - \mathrm{sg}(a) + \mathrm{sg}(a).e, f.\mathrm{sg}(a)\rangle$$

$$V(I(y, I(a, z))),$$

i.e., Axiom 6 is valid.

Here we shall mention that there are counterexamples that show that Axioms 5, 8 and 9 are not valid for this implication.

3 Conclusion

In next research other new implications will be introduced and studied. All they show that intuitonistc fuzzy sets and logics correspond to the ideas of Brouwer's intuitionism.

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