Second International Workshop on GNs, Sofia, June 26 – 27, 2001, 62-72

GENERALIZED NETS AND SYSTEMS THEORY. IX[1]

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Abstract: Some properties of the connections and events of the abstract systems, interpreted in terms of generalized nets, are discussed.

Keywords: connection, event, generalized net, system

Introduction

The eight papers [1-8] are devoted to representing the ways of functioning and the results of the work of the basic types of abstract (stationary, dynamical, hierarchical, etc.) systems in the sense of [9,10,11].

The results of these eight papers were collected in the book [12]. There the basic properties of the systems and the ways of their representations by Generalized Nets (GNs; see [13,14]) are discussed.

Here we shall discuss other system properties which can be formulated using expressive power of GNs. In a word, the GNs from [1-8,12] describe types of systems, but only a part of the GN-components are used for the models. Many capabilities of GN have not been used. Here we shall consider these GN-components and discuss the result of their use in systems theory.

We shall focus on these system properties which are related to the connections between system components and events. Whenever the system connections and/or the system events satisfy some property, we will say that the system also has this property.

As it is discussed in [12], an event can be represented by a token residing in a place of the GN-model. On the other hand, connections are characterized by their existence or absence, and therefore can be interpreted as token's characteristics. Both concepts (connection and event) can have GN-interpretations simultaneously, if every token obtains characteristics in all GN-places. If some token does not obtain any characteristic at some place, this will correspond to the presence of events but absence of respective connections (related to these events). And vice versa, if a token obtains some values as a current characteristic, then it can obtain these values of lots of connections simultaneously as characteristics.

By this reason, we shall discuss below the properties, related to both connections and events.

[1]This work is supported partly by the National Foundation for Scientific Research No. TK-L-3/1998.

§1. Material and ideal connections, material and ideal events

If there exists a connection C in a given system S, then there will exist at least one transition in the GN-model, which will interpret the process of the system functioning, related to connection C validity check.

There are at least two ways of accomplishing this situation. Both of them relate the realization of the connection to calculation of the characteristic function, associated with at least one output transition place. The basic difference between the two ways of interpretation is that in the first case the connection is accomplished within the frameworks of a transition, while in the second case the realization of the connection involves evaluation of the transition condition predicates.

For example, if the first transition (Z_1) of the sub-GN from Fig. 1 interprets connection C mentioned above and if its existence is determined by the possibility of token activation, then we shall test whether the transition type has a truth value "*true*" and after this, the occurrence of the connection will depend on the calculation of the characteristic function associated with place l_2.

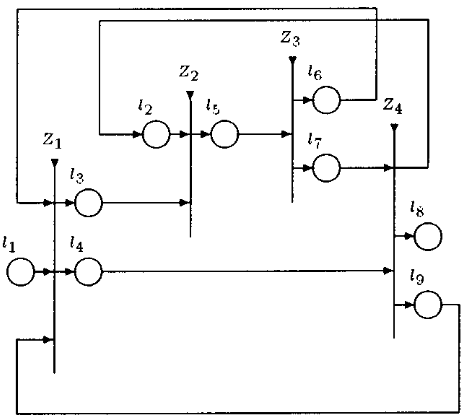

Fig. 1.

On the other hand, if the third transition (Z_3) of the same sub-GN interprets connection C mentioned above, then its existence can depend on the truth value, e.g., of predicate

$$r_{5,6} = \text{"}\overline{c}(l_2, TIME) > 0\text{"},$$

where (see, e.g., [13]) $\overline{c}(l, TIME)$ determines the number of tokens in place l at the current time-moment $TIME$. Therefore, we shall evaluate predicate $r_{5,6}$, but this is related to checking information connected with another part of the net (is outside the discussed transition). After this, as above, accomplishing connection C is due to calculation of the characteristic function associated, this time, with place l_6.

It is clear that in the first case the condition for token's transfer is implied by the satisfaction of the first transition's type, i.e., by the existence of a sufficient number of tokens in the transition inputs; while in the second case the condition for token's transfer is related to the satisfaction of a transition condition predicate, i.e., it is related only to necessary information, coming from another part of the GN-model. In other words, in the former case the reason for realizing connection C is material (expressed in existence of tokens), while in the latter case - an ideal one (determined by existence of positive information about some event in the net). For this reason, the connection accomplished in the first way can be called *"material connection"*, while the one realized in the second way we call *"ideal connection"*.

The above example can be used without changes for illustrating the concepts of *"material"* and *"ideal events"*.

Thus, an event is called material if the necessary condition for its realization is related to check of transition types (i.e., it is related to existence of some GN-tokens). An event is called ideal if its realization is only related to information generated elsewhere in the net.

§2. Temporal and invariant connections, temporal and invariant events

In [13] it is pointed out that all nets which are modifications and extensions of Petri nets, are invariant with the time. Only GNs can vary with time, because they have global time-components. Therefore, the systems modelled by GNs can have as temporal (in the sense that time is accounted for by a global time-scale), as well as an invariant character. The same will be true of the connections in these systems, and hence, the connections can be called *"temporal"* or *"invariant"*.

Of course, each invariant connection can be interpreted as a temporal one, if the global time-scale starts at time-moment 0. For example, this could be the case if we wish to model some astrophysical processes related to the Big Bang, or any historical processes 2000 years ago at the beginning of the present historical time-scale.

Obviously, each event in the system can also be classified among these two cases.

The temporal and invariant connections allow for unified treatment of discrete (semi-discrete) dynamic systems and their steady state behaviour.

If the system trajectory is divided in cycles and each cycle contains a number of steps, GN with temporal connections are able to model procedure like data processing (first group of steps), identification and extrapolation ((second group of steps), advanced control algorithm synthesis (adaptation, gain scheduling, productive control) in the last group of steps.

§3. Permanent and conditional connections, permanent and conditional events

As it is noted in [13], some of the transition condition predicates are constantly *true*, while in general they are evaluated as '*true*" or "*false*" in different moments, and there are at least two moments in which these predicates have different values.

If a connection or event is to be realized in some output place (e.g., in place l_2 from the GN from Fig. 2.), then the form of this connection or event will depend on the forms of the predicates, associated with the place (in our example - it will depend on the forms of predicates $r_{1,2}$ and $r_{3,2}$). Let, for example,

$$r_{3,2} = \text{“}true\text{”}.$$

Therefore, every token located in place l_3 in the next time-moment will inevitably enter place l_2, i.e., the event represented by the respective tokens will occur *permanently.* If the tokens always obtain positive characteristics related to some connection(s) in place l_2, then the connection(s) will exist *permanently* as well.

On the other hand, as we will note below, in the general case, predicate $r_{3,2}$ will not be always evaluated as "*true*" and, therefore, tokens will not always enter place l_2. In this case we can say that the modelled event is a *conditional* one. Similarly, if the tokens that enter place l_2 do not always obtain positive characteristics, related to the fixed connection, then it will be called a *conditional* one.

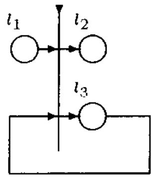

Fig. 2.

Here we should note that there is a difference between situations related to connections and events.

The event interpreted by a fixed place (e. g., by place l_2 from Fig. 2) will be a permanent one, if in each time-moment in the GN-functioning every token that may enter place l_2 does so. If it obtains there an expression of the form "*the value of the fact estimation that the fixed connection exists is true*" as a current characteristic and if this value is always "*true*", then the connection will be permanent. If some tokens cannot enter place l_2 (in our example, this is possible, if the current truth-value of predicates $r_{1,2}$ and $r_{1,3}$ are "*false*"), then the event interpreted by these tokens cannot claim to be permanent, i.e., it is a conditional one. On the other hand, the connection interpreted by the current characteristics of the tokens (in our example - the current characteristic of the tokens in place l_2) will keep its permanent status until the moment when some token in the corresponding place obtains a characteristic in the opposite sense.

The opposite case is also possible: the tokens enter regularly the fixed place (in our case - place l_2) and, therefore, the event is permanent, but these tokens obtain at some time-moments the fact of the connection not being present as a characteristic. Therefore, this connection will be conditional.

§4. Global and local connections, global and local events

We have already said that to a great extent the GN-interpretations of the events depend on the transition condition predicates, while the GN-interpretations of the connections depend on the token characteristics. When these predicates and these characteristic functions depend on some of the global GN-parameters, then we shall name the corresponding events and connections *global* ones. If these predicates and these characteristic functions depend only on local GN-parameters, then we shall name them *local* ones.

The introduction of the global and local connections and parameters allows us to treat in a uniform way the behaviour of aggregated objects like agents and holons which have been established as useful tools in structural investigations last decade. Multiagent approach gives a lot of benefits in complex heterogenous system like manufacturing, communication, computer networks, economics, etc. Emphasis on unification of global and local properties make holon based analysis and synthesis of decentralized and hierarchical system a promising method to overcome complexity and multidimensionality.

§5. Synchronous and asynchronous connections, synchronous and asynchronous events

Consider the sub-GN from Fig. 3. Let the connections C_1 and C_2 of the system be interpreted by the possibility of a tokens' transfer from transition Z_1 to transition Z_2 (i.e, from place l_2 to place l_3) and from transition Z_3 to transition Z_4 (i.e, from place l_5 to place l_6).

We will say that the connections C_1 and C_2 are *synchronous* if and only if whenever there exists a transformation of a token from transition Z_1 to transition Z_2, at the same time-moment or at another time-moment, which is at a fixed distance from the present time-moment, there exists a transformation of at least one token from transition Z_3 to transition Z_4 and vive versa. Two connections are *asynchronous* when they are not synchronous.

The same can be said about the events of the system. If we interpret them by the fact that corresponding tokens enter corresponding places, then two events will be *synchronous* if and only if whenever a token related to the first event enters a fixed place, at the same time-moment or at another time-moment, which is at a fixed distance from the present time-moment, a token related to the second event enters a corresponding fixed place and vice versa. Two events are *asynchronous* when they are not synchronous.

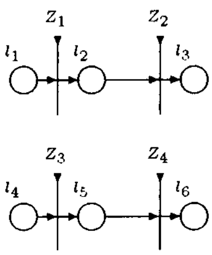

Fig. 3.

The unification of synchronous and asynchronous connections/events is a perspective way to describe and calculate the systems with discrete-event elemenst. Introducing the unified crossection of time-driven and event-driven processes based on GNs will hopefully be a fruitful field of investigation of unsolved problems in system theory.

§6. Strong and weak connections, strong and weak events

The previous property of the connections and events is very strong for them. The next properties are modifications of this property.

Event E_1 is *stronger* than event E_2 if and only if whenever event E_1 starts at the same time-moment or at another time-moment, which is at a fixed distance from the present time-moment, event E_2 also starts. However, the opposite is not always true. In this case event E_2 is said to be *weaker* than event E_1.

A similar property holds of connections, too, and for this reason we shall not discuss it.

§7. Time driven and event driven connections, time driven and event driven events

Integration of continuous and discrete event systems is still an unsettled problem. GN by a new interpretation of time driven and event driven connections as well as events are a promising approach to tackle this hard challenge. A lot of applications in manufacturing, transportation, complex systems, finances are closely connected with this problem.

§8. Specific and unspecific connections, specific and unspecific events

These properties are similar to the above ones, but now they are related to whether the corresponding tokens' characteristics and transtion condition predicates are unique to the GN-model, or they occur elsewhere in the model, respectively.

§9. Interior and exterior connections, interior and exterior events

A modification of the property above is the following, which is related to whether calculation of tokens' characteristics and transtion condition predicates requires information specific only to the involved transition or the information is related to other processes beyond that transition. With respect to this, the system connections and system events can be called *interior* and *exterior*, depending on the nature of the required information.

§10. Organized and disorganized connections, organized and disorganized events

We start with an example. Let us have the sub-GN from Fig. 4, which contains a sequence of transitions with an input and an output place. Let a fixed event be related to the occurrence of a series of sub-events as each one of them takes place in the sub-GN-place associated with it. If the transition condition predicates do not have the form of stochastic or ineffectively calculated functions (an example of an ineffectively calculated function is, e.g., the predicate "*There exists at least one star around which tere is a planet which resembles the Earth and where life similar to our one exists*") and if there are no GN-components that can stop the process of tokens' transfer (e.g., the capacity of some arc or some place is zero, some transition condition predicate is constantly "*false*", etc.) then the given event can be named *an organized* one. In the opposite case it can be called *a disorganized event.* A similar definition can be given of the connections.

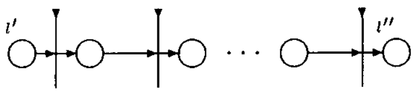

Fig. 4.

A global operator is defined over GNs (see [13]) which can replace the sub-GN from Fig. 4 with a single transition containing one input (l') and one output (l'') places. In this case the definitions above remain valid, too.

Organized and disorganized connections and event offer a tool to unify deterministic (organized) and stochastic (disorganized) properties in order to have them processed in a general way. A lot of probelms in actual complex systems are connected with obtaining efficient tools suitable for analysis and synthesis of deterministic/probabilistic events.

§11. Periodical and aperiodical connections, periodical and aperiodical events

A fixed connection or event may be called *periodical* if it happens periodically at time. In the opposite case it will be called an *aperiodical* one. In general, this definition coincides with the standard one in the systems theory.

We must note that periodical and aperiodical properties can be related only to processes which take place in a fixed transition.

The unified approach of periodical and aperiodical connections/events treatment will find a numerous applications in the area of systems with periodic coefficients in mathematical models, in the field of production planning and scheduling and optimal sequencing in manufacturing.

§12. A GN for checking the properties described above

In [12] a GN which describes the processes in some system properties checking is proposed. Here we shall construct a similar GN for checking the properties of the systems above. The GN-construction is analogical to the previous one.

Let us have the GN from Fig. 5 and let us check the existence of the eleven properties for a fixed connection and for a fixed event. If we have to check the same property for a set of a different connections and/or for a set of b different events, then we may change place m from Fig. 5 with $c = max(a, b)$ in number places $m^1, m^2, ..., m^c$, using the procedure below with some small obvious changes.

The GN has only one transition $\overline{Z}$ of the kind:

$$\overline{Z} =< \{m, m_1, m_2, ..., m_{11}\}, \{m, m_1, m_2, ..., m_{11}\}, *, t^o, \overline{r}, \overline{M},$$

$$\wedge(m, m_1, m_2, ..., m_{11}) >,$$

where

$$\overline{r} = \begin{array}{c|ccccc} & m & m_1 & m_2 & \dots & m_{11} \\ \hline m & true & false & false & \dots & false \\ m_1 & false & true & false & \dots & false \\ m_2 & false & false & true & \dots & false \\ \dots & \dots & \dots & \dots & \dots & \dots \\ m_{11} & false & false & false & \dots & true \end{array}$$

and

$$\overline{M} = \begin{array}{c|ccccc} & m & m_1 & m_2 & \dots & m_{11} \\ \hline m & 1 & 0 & 0 & \dots & 0 \\ m_1 & 0 & 1 & 0 & \dots & 0 \\ m_2 & 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ m_{11} & 0 & 0 & 0 & \dots & 1 \end{array}.$$

Let token α reside in place m, so that in the first time-moment it has initial characteristic

$$\text{“}\underbrace{\langle\langle 1,1\rangle,\ \langle 1,1\rangle,\ \dots,\ \langle 1,1\rangle\rangle}_{11 \text{ times}}\text{”}$$

and token γ_i be situated in place m_i $(1 \leq i \leq 11)$ at the first time-moment with no initial characteristic, where the number i corresponds to the i–th of the above properties.

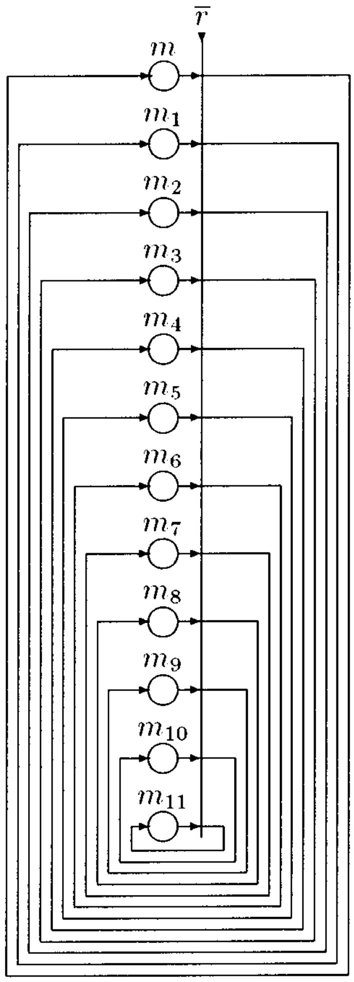

Fig. 5.

Let Pr(“*expression*”) is a predicate evaluating the truth-value of the expression “*expression*”, i.e.,

$$Pr(\text{“}expression\text{”}) = \begin{cases} 1, & \text{the expression is true} \\ 0, & \text{otherwise} \end{cases}$$

We will use the following predicate properties:

$$Pr(p\&q) = min(Pr(p), Pr(q)),$$

$$Pr(p \vee q) = max(Pr(p), Pr(q)),$$

$$Pr((\forall x \in S)p(x)) = \prod_{x \in S} Pr(p(x)),$$

where p, q are some predicates and S some set over which p is defined.

Let the GN-characteristic function be defined so that it will give for the $i-$th place m_i the current characteristic

$$x^i_{cu} = \text{“}\langle Pr(i - \text{th property holds for the given connection}),$$

$$Pr(i - \text{th property holds for the given event})\rangle\text{”}$$

and for place m the current characteristic

$$x^\alpha_{cu} = \text{“}\langle\langle pr_1(pr_1(x^\alpha_{cu-1})).pr_1(x^1_{cu}), pr_1(pr_2(x^\alpha_{cu-1})).pr_2(x^1_{cu})\rangle,$$

$$\langle pr_2(pr_1(x^\alpha_{cu-1})).pr_1(x^2_{cu}), pr_2(pr_2(x^\alpha_{cu-1})).pr_2(x^2_{cu})\rangle, \dots,$$

$$\langle pr_{11}(pr_1(x^\alpha_{cu-1})).pr_1(x^{11}_{cu}), pr_{11}(pr_2(x^\alpha_{cu-1})).pr_2(x^{11}_{cu})\rangle\rangle\text{”},$$

where cu is the number of the current token's characteristic ($cu \geq 0$) and $pr_i X$ is the $i-$th component of the $n-$dimensional vector X.

The transition $\overline{Z}$ is activated at every time-moment. Therefore, we get information about the validity of the above eleven properties for the fixed connection and/or events for each time-steps of the GN-model's functioning. Hence, the characteristic of token α will contain information about the moments, when some of the connections or events fail to have some property (at that time-moment the value of the respective component of the $\alpha-$characteristic will start being 0; and if there is no such moment, i.e., if some property always holds for some connection or for some event, then the respective component of the final $\alpha-$characteristic will be 1 and this will indicate that the fixed connection or the fixed event have the fixed property.

As we note above, the GN can be modified for a great number of connections and/or events.

Conclusion

The defined properties raise the possibility for more precise studying of a given (abstract) system. Some of them have analogues in the systems theory, but here all of them are formulated in the GN-terms. Of course, other properties can be defined, too. Here we have not specified the forms of the connections and events, because our discussion is related to a large group of connections and events, which can be modelled by GNs.

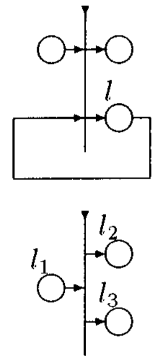

Fig. 6.

Finally, we shall give an example of a connection between two GN-transitions (for example, let them be the transitions Z_1 and Z_2 from Fig. 1). We can name the connection between them:

• a *forward-correct* one, if the logical expression of the predicates related to the token's transfer from the first transition to the second transition is always equal to "*true*" (in our example, if:

$$(r_{1,3} \& r_{3,5} \& r_{5,7} \& (r_{7,9} \vee r_{7,9})) \vee (r_{1,4} \& (r_{4,8} \vee r_{4,9})) = \text{"}true\text{"});$$

• an *backward-correct* one, if the logical expression of the predicates related to the token's transfer from the second transition to the first transition is always equal to "*true*" (in our example, if:

$$((r_{4,9} \vee r_{7,9}) \& r_{9,4}) \vee ((r_{4,2} \vee r_{7,2}) \& r_{2,5}) \& r_{5,6} \& (r_{6,3} \vee r_{6,4}) = \text{"}true\text{"});$$

• an *uncertain* one, if the logical expressions of the predicates related to the token's transfer from the first transition to the second transition and from the second transition to the first transition are not equal to "*true*".

The last case may be called *stochastical connection* , when at least one of the predicates is a stochastical function.

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A new series of research, devoted to GNs and systems theory will begin with the present research. It will go beyond the boundaries of the traditional concept of system. The concepts of catastrophe and chaos and their relations with systems and events will be discussed in the next papers, too.

Our point of view on systems theory will be essentially closer to the ideas, e.g., from [15], than these, e.g., from [16], [17], i.e., than more than 90 % of the books on systems theory. Our aspect of research will allow us to apply the GN-interpretation of systems theory approach simultaneously to areas of artificial intelligence, human body, ecology and others. Searching the joint points of these areas will be essentially easier, because all models will be described by uniform means. Therefore, the approach, discussed in [14] can be transferred here and this is fully natural – finally, for example, every expert *system* is a *system* from abstract systems theory point of view.

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