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Weakly generalized separation axioms in intuitionistic fuzzy topological spaces

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Abstract: The purpose of this paper is to introduce and investigate several types of new separation axioms in intuitionistic fuzzy topological spaces. After giving some characterizations of ${}_{wg}T_g$, ${}_{wg}T_\alpha$ and ${}_{wg}T_{\alpha g}$ separation axioms in intuitionistic fuzzy topological spaces, we give interrelations between several types of separation axioms and some counter examples.
Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy weakly generalized closed set, intuitionistic fuzzy ${}_{wg}T_g$ spaces, intuitionistic fuzzy ${}_{wg}T_\alpha$ spaces and intuitionistic fuzzy ${}_{wg}T_{\alpha g}$ spaces.
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1 Introduction

Fuzzy set (FS) as proposed by Zadeh [9] in 1965 is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang [2] in 1968 there have been several generalizations of notions of fuzzy sets and fuzzy topology.

By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1983 which appeals more accurately to uncertainty quantification and provides the opportunity to precisely model the problem, based on the existing knowledge and observations. In 1997, Çoker [3] introduced the concept of intuitionistic fuzzy topological space.

We introduce intuitionistic fuzzy ${}_{wg}T_g$ spaces, intuitionistic fuzzy ${}_{wg}T_\alpha$ spaces and intuitionistic fuzzy ${}_{wg}T_{\alpha g}$ spaces to study the application of intuitionistic fuzzy weakly generalized closed sets and obtained some characterizations and several preservation theorems of such spaces.

2 Preliminaries

Definition 2.1: [1] Let X be a nonempty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$, where the functions $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be IFSs of the forms $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$,

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

For the sake of simplicity, the notation $A = \langle x, \mu_A, \nu_A\rangle$ shall be used instead of the notation $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$ are the *empty set* and the *whole set* of X, respectively.

Definition 2.3: [3] An *intuitionistic fuzzy topology* (IFT in short) on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

(a) $0_\sim, 1_\sim \in \tau$,

(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

$$\text{int}(A) = \cup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$

$$\text{cl}(A) = \cap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Note that for any IFS A in (X, τ), we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$.

Definition 2.5: [5] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy weakly generalized closed set* (IFWGCS in short) if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

The family of all IFWGCSs of an IFTS (X, τ) is denoted by IFWGC(X).

Definition 2.6: [5] An IFS A is said to be an *intuitionistic fuzzy weakly generalized open set* (IFWGOS in short) in (X, τ) if the complement A^c is an IFWGCS in (X, τ).

The family of all IFWGOSs of an IFTS (X, τ) is denoted by IFWGO(X).

Result 2.7: [5] Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFWGCS but the converses need not be true in general.

Definition 2.8: [6] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy weakly generalized interior* and *intuitionistic fuzzy weakly generalized closure* are defined by

$$\text{wgint}(A) = \cup \{G \mid G \text{ is an IFWGOS in } X \text{ and } G \subseteq A\},$$
$$\text{wgcl}(A) = \cap \{K \mid K \text{ is an IFWGCS in } X \text{ and } A \subseteq K\}.$$

Definition 2.9: [5] An IFTS (X, τ) is said to be an *intuitionistic fuzzy* $_{wg}T_{1/2}$ *space* (IF$_{wg}$T$_{1/2}$ space in short) if every IFWGCS in X is an IFCS in X.

Definition 2.10: [5] An IFTS (X, τ) is said to be an *intuitionistic fuzzy* $_{wg}T_p$ *space* (IF$_{wg}$T$_p$ space in short) if every IFWGCS in X is an IFPCS in X.

Definition 2.11: An IFTS (X, τ) is said to be:

(i) intuitionistic fuzzy $T_{1/2}$ space (IF$T_{1/2}$ space in short) [4] if every IFGCS in X is an IFCS in X,

(ii) intuitionistic fuzzy $_{\alpha a}T_{1/2}$ space (IF$_{\alpha a}T_{1/2}$ space in short) [7] if every IFαGCS in X is an IFCS in X,

(iii) intuitionistic fuzzy $_{\alpha b}T_{1/2}$ space (IF$_{\alpha b}T_{1/2}$ space in short) [7] if every IFαGCS in X is an IFGCS in X,

(iv) intuitionistic fuzzy $_{\alpha}T_{1/2}$ space (IF$_{\alpha}T_{1/2}$ space in short) [7] if every IFαGCS in X is an IFαCS in X,

(v) intuitionistic fuzzy $_{g}T_{1/2}$ space (IF$_{g}T_{1/2}$ space in short) [8] if every IFGSCS in X is an IFGCS in X.

3 Weakly generalized separation axioms in intuitionistic fuzzy topological spaces

In this section, we introduce three new spaces namely intuitionistic fuzzy $_{wg}T_g$ spaces, intuitionistic fuzzy $_{wg}T_\alpha$ spaces and intuitionistic fuzzy $_{wg}T_{\alpha g}$ spaces and study some of their properties.

Definition 3.1: An IFTS (X, τ) is called an intuitionistic fuzzy $_{wg}T_g$ space (IF$_{wg}$T$_g$ space in short) if every IFWGCS in X is an IFGCS in X.

Theorem 3.2: An IFTS (X, τ) is an IF$_{wg}$T$_g$ space if and only if IFWGO(X) = IFGO(X).

Proof: Necessity: Let A be an IFWGOS in X. Then A^c is an IFWGCS in X. By hypothesis, A^c is an IFGCS in X. Therefore A is an IFGOS in X. Hence IFWGO(X) = IFGO(X).

Sufficiency: Let A be an IFWGCS in X. Then A^c is an IFWGOS in X. By hypothesis, A^c is an IFGOS in X. Therefore A is an IFGCS in X. Hence (X, τ) is an $IF_{wg}T_g$ space. □

Theorem 3.3: Every $IF_{wg}T_{1/2}$ space in an IFTS (X, τ) is an $IF_{wg}T_g$ space but not conversely.

Proof: Let (X, τ) be an $IF_{wg}T_{1/2}$ space and let A be an IFWGCS in X. By hypothesis, A is an IFCS in X. Since every IFCS is an IFGCS, A is an IFGCS in X. Hence (X, τ) is an $IF_{wg}T_g$ space. □

Example 3.4: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \left\langle x, \left(\frac{a}{0.9}, \frac{b}{0.9}\right), \left(\frac{a}{0.1}, \frac{b}{0.1}\right)\right\rangle$. Then clearly every IFWGCS in X is an IFGCS in X. Therefore (X, τ) is an $IF_{wg}T_g$ space. Let us consider the IFS $A = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.8}, \frac{b}{0.7}\right)\right\rangle$ in X. Then A is an IFWGCS in X but not an IFCS in X. Hence (X, τ) is not an $IF_{wg}T_{1/2}$ space.

Definition 3.5: An IFTS (X, τ) is called an intuitionistic fuzzy ${}_{wg}T_\alpha$ space ($IF_{wg}T_\alpha$ space in short) space if every IFWGCS in X is an IFαCS in X.

Theorem 3.6: An IFTS (X, τ) is an $IF_{wg}T_\alpha$ space if and only if IFWGO(X) = IFαO(X).

Proof: Necessity: Let A be an IFWGOS in X. Then A^c is an IFWGCS in X. By hypothesis, A^c is an IFαCS in X. Therefore A is an IFαOS in X. Hence IFWGO(X) = IFαO(X).

Sufficiency: Let A be an IFWGCS in X. Then A^c is an IFWGOS in X. By hypothesis, A^c is an IFαOS in X. Therefore A is an IFαCS in X. Hence (X, τ) is an $IF_{wg}T_\alpha$ space. □

Theorem 3.7: Every $IF_{wg}T_{1/2}$ space in an IFTS (X, τ) is an $IF_{wg}T_\alpha$ space but not conversely.

Proof: Let (X, τ) be an $IF_{wg}T_{1/2}$ space and let A be an IFWGCS in X. By hypothesis, A is an IFCS in X. Since every IFCS is an IFαCS, A is an IFαCS in X. Hence (X, τ) is an $IF_{wg}T_\alpha$ space. □

Example 3.8: In Example 3.4, (X, τ) is an $IF_{wg}T_\alpha$ space but not an $IF_{wg}T_{1/2}$ space, since the IFS $A = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.8}, \frac{b}{0.7}\right)\right\rangle$ is an IFWGCS in X but not an IFCS in X.

Theorem 3.9: If (X, τ) is an $IF_{wg}T_\alpha$ space then every IFWGCS in (X, τ) is an IFGαCS in (X, τ)

Proof: Let A be an IFWGCS in X. Since (X, τ) is an $IF_{wg}T_\alpha$ space, A is an IFαCS in (X, τ). Further, since every IFαCS is an IFGαCS, A is an IFGαCS in X. □

Definition 3.10: An IFTS (X, τ) is called an intuitionistic fuzzy ${}_{wg}T_{\alpha g}$ space ($IF_{wg}T_{\alpha g}$ space in short) if every IFWGCS in X is an IFαGCS in X.

Theorem 3.11: Every $IF_{wg}T_{1/2}$ space in an IFTS (X, τ) is an $IF_{wg}T_{\alpha g}$ space but not conversely.

Proof: Let (X, τ) be an $IF_{wg}T_{1/2}$ space and let A be an IFWGCS in X. By hypothesis, A is an IFCS in X. Since every IFCS is an IFαGCS, A is an IFαGCS in X. Hence (X, τ) is an $IF_{wg}T_{\alpha g}$ space. □

Example 3.12: In Example 3.4, (X, τ) is an $IF_{wg}T_{\alpha g}$ space but not an $IF_{wg}T_{1/2}$ space, since the IFS $A = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.8}, \frac{b}{0.7}\right)\right\rangle$ is an IFWGCS in X but not an IFCS in X.

Theorem 3.13 Every $IF_{wg}T_g$ space in an IFTS (X, τ) is an $IF_{wg}T_{\alpha g}$ space but not conversely.

Proof: Let (X, τ) be an $IF_{wg}T_g$ space and let A be an IFWGCS in X. By hypothesis, A is an IFGCS in X. Since every IFGCS is an IFαGCS, A is an IFαGCS in X. Hence (X, τ) is an $IF_{wg}T_{\alpha g}$ space. □

Example 3.14: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.2}\right), \left(\frac{a}{0.6}, \frac{b}{0.7}\right)\right\rangle$. Then clearly every IFWGCS in X is an IFαGCS in X. Therefore (X, τ) is an $IF_{wg}T_{\alpha g}$ space. Let us consider the IFS $A = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.2}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right)\right\rangle$ in X. Then A is an IFWGCS in X but not an IFGCS in X. Hence (X, τ) is not an $IF_{wg}T_g$ space.

Theorem 3.15: Every $IF_{wg}T_\alpha$ space in an IFTS (X, τ) is an $IF_{wg}T_{\alpha g}$ space but not conversely.

Proof: Let (X, τ) be an $IF_{wg}T_\alpha$ space and let A be an IFWGCS in X. By hypothesis, A is an IFαCS in X. Since every IFαCS is an IFαGCS, A is an IFαGCS in X. Hence (X, τ) is an $IF_{wg}T_{\alpha g}$ space. □

Example 3.16 In Example 3.14, (X, τ) is an $IF_{wg}T_{\alpha g}$ space but not an $IF_{wg}T_\alpha$ space, since the IFS $A = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.2}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right)\right\rangle$ is an IFWGCS in X but not an IFαCS in X.

Theorem 3.17: Every $IF_{wg}T_{1/2}$ space in an IFTS (X, τ) is an $IFT_{1/2}$ space.

Proof: Let (X, τ) be an $IF_{wg}T_{1/2}$ space and let A be an IFGCS in X. Since every IFGCS is an IFWGCS, A is an IFWGCS in X. By hypothesis, A is an IFCS in X. Hence (X, τ) is an $IFT_{1/2}$ space. □

Theorem 3.18: Every $IF_{wg}T_{1/2}$ space in an IFTS (X, τ) is an $IF_{\alpha a}T_{1/2}$ space.

Proof: Let (X, τ) be an $IF_{wg}T_{1/2}$ space and let A be an IFαGCS in X. Since every IFαGCS is an IFWGCS, A is an IFWGCS in X. By hypothesis, A is an IFCS in X. Hence (X, τ) is an $IF_{\alpha a}T_{1/2}$ space. □

Theorem 3.19: Every $IF_{wg}T_g$ space in an IFTS (X, τ) is an $IF_{\alpha b}T_{1/2}$ space.

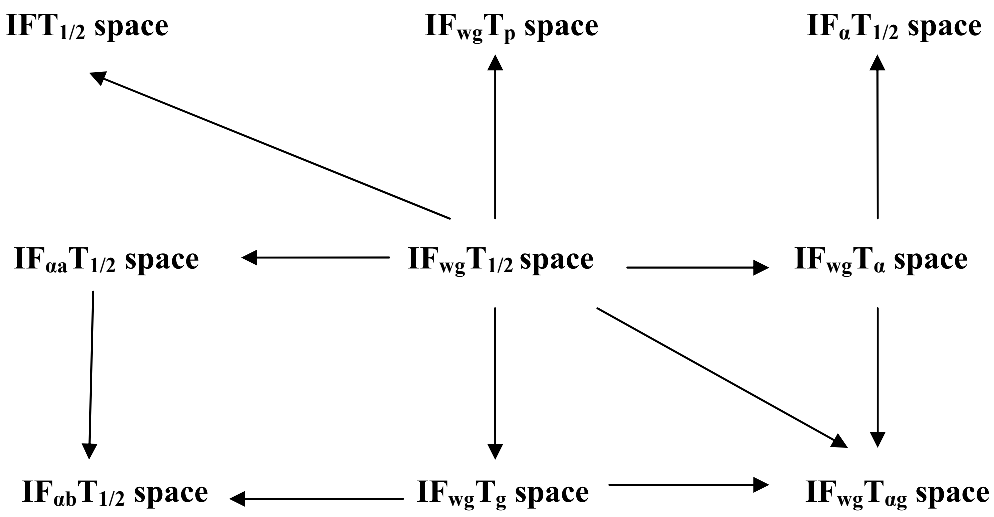
Proof: Let (X, τ) be an $IF_{wg}T_g$ space and let A be an IFαGCS in X. Since every IFαGCS is an IFWGCS, A is an IFWGCS in X. By hypothesis, A is an IFGCS in X. Hence (X, τ) is an $IF_{\alpha b}T_{1/2}$ space. □

Theorem 3.20: Every $IF_{wg}T_{\alpha}$ space in an IFTS (X, τ) is an $IF_{\alpha}T_{1/2}$ space.

Proof: Let (X, τ) be an $IF_{wg}T_{\alpha}$ space and let A be an IFαGCS in X. Since every IFαGCS is an IFWGCS, A is an IFWGCS in X. By hypothesis, A is an IFαCS in X. Hence (X, τ) is an $IF_{\alpha}T_{1/2}$ space.

□

Remark 3.21: We have the following implication diagram.



However, none of the above implications is reversible.

Theorem 3.22: If (X, τ) is an $IF_{wg}T_g$ space and an $IFT_{1/2}$ space then (X, τ) is an $IF_{wg}T_{1/2}$ space.

Proof: Let A be an IFWGCS in (X, τ). Since (X, τ) is an $IF_{wg}T_g$ space, A is an IFGCS in (X, τ). Further, since (X, τ) is an $IFT_{1/2}$ space, A is an IFCS in (X, τ). Hence (X, τ) is an $IF_{wg}T_{1/2}$ space. □

Theorem 3.23: If (X, τ) is an $IF_{wg}T_{\alpha g}$ space and an $IF_{\alpha a}T_{1/2}$ space then (X, τ) is an $IF_{wg}T_{1/2}$ space.

Proof: Let A be an IFWGCS in (X, τ). Since (X, τ) is an $IF_{wg}T_{\alpha g}$ space, A is an IFαGCS in (X, τ). Further, since (X, τ) is an $IF_{\alpha a}T_{1/2}$ space, A is an IFCS in (X, τ). Hence (X, τ) is an $IF_{wg}T_{1/2}$ space. □

Theorem 3.24: If (X, τ) is an $IF_{wg}T_{\alpha g}$ space and an $IF_{\alpha b}T_{1/2}$ space then (X, τ) is an $IF_{wg}T_g$ space.

Proof: Let A be an IFWGCS in (X, τ). Since (X, τ) is an $IF_{wg}T_{\alpha g}$ space, A is an IFαGCS in (X, τ). Further, since (X, τ) is an $IF_{\alpha b}T_{1/2}$ space, A is an IFGCS in (X, τ). Hence (X, τ) is an $IF_{wg}T_g$ space. □

Theorem 3.25: If (X, τ) is an $IF_{wg}T_{\alpha g}$ space and an $IF_{\alpha}T_{1/2}$ space then (X, τ) is an $IF_{wg}T_{\alpha}$ space.

Proof: Let A be an IFWGCS in (X, τ). Since (X, τ) is an $IF_{wg}T_{\alpha g}$ space, A is an IFαGCS in (X, τ). Further, since (X, τ) is an $IF_{\alpha}T_{1/2}$ space, A is an IFαCS in (X, τ). Hence (X, τ) is an $IF_{wg}T_{\alpha}$ space. □

4 Conclusion

The quantum of research work done in all generalizations is a pointer of their enormous application potentials. These generalizations have also affected the different core branches of mathematics. One such branch is the topology which has generalizations as fuzzy topology, intuitionistic fuzzy topology and soft topology etc. In this present work we have introduced intuitionistic fuzzy $_{wg}T_g$ spaces, intuitionistic fuzzy $_{wg}T_{\alpha}$ spaces and intuitionistic fuzzy $_{wg}T_{\alpha g}$ spaces in intuitionistic fuzzy topological spaces and have established several interesting theorems with necessary counter examples.

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