

$(\in, \in \vee q)$ -Intuitionistic fuzzy ideals of BCK/BCI-algebras

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Abstract: In this paper, we introduced the concept of $(\in, \in \vee q)$ -Intuitionistic fuzzy ideals of BCK-algebras. Characterizations of $(\in, \in \vee q)$ -Intuitionistic fuzzy ideals are given and condition for $(\in, \in \vee q)$ -Intuitionistic fuzzy ideals to be an (\in, \in) -Intuitionistic fuzzy ideals are established. Cartesian product and intersection of two $(\in, \in \vee q)$ -Intuitionistic fuzzy ideals are also discussed. Homomorphic image and pre-image of $(\in, \in \vee q)$ -Intuitionistic fuzzy ideals are investigated.

Keywords: BCK-algebra, Fuzzy ideal, $(\in, \in \vee q)$ -fuzzy ideal, $(\in, \in \vee q)$ -intuitionistic fuzzy ideal, Homomorphism.

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1 Introduction

In 1965, Zadeh [12] introduced the notion of a fuzzy subset of a set as method of representing uncertainty in real physical world. As a generalization of this, Intuitionistic fuzzy subset was defined by Atanassov [1] in 1986. Fuzzy sets gives the degree of membership of an element in a given set, while intuitionistic fuzzy sets gives both a degree of membership and a degree of non-membership. In 1966 Imai and Iseki [6] introduced the two classes of abstract algebras viz. BCK-algebras and BCI-algebras. It is known that the class of BCK –algebra is a proper sub-class of the class of BCI-algebras. Neggers and Kim [8] introduced a new notion, called B-algebras which are related to several classes of algebras such as BCI/BCK–algebras. Kim and Kim [7] introduced the notion of BG-algebra which is a generalisation of B-algebra. Zarandi and Saeid [13] developed intuitionistic fuzzy ideal of BG-algebra. Yun and Kim [11] studied intuitionistic fuzzy ideals of BCK-algebras in 2000. Bhakat and Das [4, 5] used the relation of “belongs to” and “quasi coincident with” between fuzzy point and fuzzy set to introduce the concept of $(\in, \in \vee q)$ -fuzzy subgroup, $(\in, \in \vee q)$ -fuzzy subring and $(\in \vee q)$ -level subset. Y. B. Yun [10] studied (α, β) -fuzzy ideals of BCK/BCI-algebra in 2004. Motivated by

this we have introduced the notion of $(\in, \in \vee q)$ -Intuitionistic fuzzy ideals of BCK-algebra and established some of their basic properties.

2 Preliminaries

Definition 2.1. ([10, 11]) A BCI-algebra is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms.

- (i) $((x * y) * (x * z)) * (z * y) = 0$
- (ii) $(x * (x * y)) * y = 0$
- (iii) $x * x = 0$
- (iv) $(x * y) = 0$ and $(y * x) = 0$ imply that $x = y$ for all $x, y, z \in X$
- (v) We can define a partial ordering " \leq " by $x \leq y$ if and only if $x * y = 0$. If a BCI-algebra X satisfies $0 * x = 0$ for all $x \in X$, then we say that X is a BCK-algebra.

In what follows, let X denote a BCK-algebra unless otherwise specified.

Definition 2.2. A non-empty subset S of a BCK-algebra X is called a subalgebra of X if $x * y \in S$, for all $x, y \in S$.

Definition 2.3. ([10]) A nonempty subset I of a BCK-algebra X is called ideal of X if

- (i) $0 \in I$
- (ii) $x * y \in I$ and $y \in I \Rightarrow x \in I$ for all $x, y \in X$

Definition 2.4. ([10]) A fuzzy set μ in X is called a fuzzy ideal of X if it satisfies the following conditions

- (i) $\mu(0) \geq \mu(x)$
- (ii) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\} \quad \forall x, y \in X$.

Definition 2.5. ([11]) An intuitionistic fuzzy set (IFS) A of a non-empty set X is an object of the form $A = \{\langle x, \mu_A(x), v_A(x) \rangle \mid x \in X\}$ where $\mu_A : X \rightarrow [0, 1]$ and $v_A : X \rightarrow [0, 1]$ with the condition $0 \leq \mu_A(x) + v_A(x) \leq 1$, $\forall x \in X$. The numbers $\mu_A(x)$ and $v_A(x)$ denote respectively the degree of membership and the degree of non-membership of the element x in the set A . For the sake of simplicity, we shall use the symbol $A = (\mu_A, v_A)$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), v_A(x) \rangle \mid x \in X\}$.

Definition 2.6. ([3]) If $A = \{\langle x, \mu_A(x), v_A(x) \rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), v_B(x) \rangle \mid x \in X\}$ be any two IFS of a set X , then $A \subseteq B$ if and only if for all $x \in X$, $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$, $A = B$ if and only if for all $x \in X$, $\mu_A(x) = \mu_B(x)$ and $v_A(x) = v_B(x)$,

$$A \cap B = \{\langle x, (\mu_A \cap \mu_B)(x), (v_A \cup v_B)(x) \rangle \mid x \in X\},$$

where $(\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\}$ and $(v_A \cup v_B)(x) = \max\{v_A(x), v_B(x)\}$,

$$A \cup B = \{\langle x, (\mu_A \cup \mu_B)(x), (v_A \cap v_B)(x) \rangle \mid x \in X\},$$

where $(\mu_A \cup \mu_B)(x) = \max\{\mu_A(x), \mu_B(x)\}$ and $(v_A \cap v_B)(x) = \min\{v_A(x), v_B(x)\}$.

Definition 2.7. ([11]) An intuitionistic fuzzy set A of a BCK-algebra X is said to be an intuitionistic fuzzy ideal of X if.

- (i) $\mu_A(0) \geq \mu_A(x)$
- (ii) $v_A(0) \leq v_A(x)$

- (iii) $\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\}$
- (iv) $v_A(x) \leq \max\{v_A(x * y), v_A(y)\} \forall x, y \in X$.

Example 2.1 Consider BCK-algebra $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	3	4	1	0

The intuitionistic fuzzy subset $A = \{\langle x, \mu_A(x), v_A(x) \rangle \mid x \in X\}$ given by $\mu_A(0) = \mu_A(2) = 0.9$, $\mu_A(1) = \mu_A(3) = \mu_A(4) = 0.6$ and $v_A(0) = v_A(2) = 0$, $v_A(1) = v_A(3) = v_A(4) = 0.3$ then A is an intuitionistic fuzzy ideal of BCK-algebra X .

3 $(\in, \in \vee q)$ -Intuitionistic fuzzy ideals of BCK-algebra

Definition 3.1. ([4]) A fuzzy set μ of the form

$$\mu(y) = \begin{cases} t & \text{if } y = x, t \in (0, 1] \\ 0 & \text{if } y \neq x \end{cases}$$

is called a fuzzy point with support x and value t and it is denoted by x_t .

Definition 3.2. ([4]) A fuzzy point x_t is said to belong to (respectively be quasi coincident with) a fuzzy set μ written as $x_t \in \mu$ (respectively $x_t q \mu$) if $\mu(x) \geq t$ (respectively $\mu(x) + t > 1$). If $x_t \in \mu$ or $x_t q \mu$ then we write $x_t \in \vee q \mu$ (Note $\in \vee q$ means $\in \vee q$ does not hold)

Definition 3.3. ([10]) A fuzzy subset μ of a BCK-algebra X is said to be $(\in, \in \vee q)$ -fuzzy ideal of X if

- (i) $x_t \in \mu \Rightarrow 0_t \in \vee q \mu$ for all $x \in X$
- (ii) $(x * y)_t, y_s \in \mu \Rightarrow x_{m(t, s)} \in \vee q \mu$ for all $x, y \in X$

Definition 3.4. ([10]) A fuzzy subset μ of a BCK-algebra X is said to be (α, β) -fuzzy ideal of X if

- (i) $x_t \alpha \mu \Rightarrow 0_t \beta \mu$ for all $x \in X$
- (ii) $(x * y)_t, y_s \alpha \mu \Rightarrow x_{m(t, s)} \beta \mu$ for all $x, y \in X$

where $m(t, s) = \min\{t, s\}$ and $\alpha, \beta \in \{\in, q, \in \vee q, \in \wedge q\}$ and $\alpha \neq \in \wedge q$

Definition 3.5. A fuzzy point x_t is said to belongs to (respectively be quasi coincident with) a IFS $A = (\mu_A, v_A)$ written as $x_t \in A$ (respt $x_t q A$) if $\mu_A(x) \geq t$ (respt $\mu_A(x) + t > 1$) and $v_A(x) < t$ (respt $v_A(x) + t \leq 1$). If $x_t \in A$ or $x_t q A$ then $x_t \in \vee q A$.

Note $x_t \in A \Rightarrow x_t \in \mu_A$ and $x_t \bar{\in} v_A$

$x_t q A \Rightarrow x_t \in \mu_A$ and $x_t \bar{q} v_A$

Definition 3.6. An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ in BCK-algebra X is said to be an $(\in, \in \vee q)$ -intuitionistic fuzzy ideals of X if it satisfies the following conditions.

- (i) $x_t \in \mu_A \Rightarrow 0_t \in \nabla_q \mu_A$
- (ii) $x_t \bar{\in} \nu_A \Rightarrow 0_t \bar{\in} \nabla \bar{q} \nu_A$
- (iii) $(x * y)_t, y_s \in \mu_A \Rightarrow x_{m(t, s)} \in \nabla_q \mu_A$
i.e. $\mu_A(x * y) \geq t, \mu_A(y) \geq s \Rightarrow \mu_A(x) \geq m(t, s)$ or $\mu_A(x) + m(t, s) > 1, \forall x, y \in X$ where $m(t, s) = \min(t, s)$
- (iv) $(x * y)_t, y_s \bar{\in} \nu_A \Rightarrow x_{M(t, s)} \bar{\in} \nabla \bar{q} \nu_A$
i.e. $\nu_A(x * y) < t, \nu_A(y) < s \Rightarrow \nu_A(x) < M(t, s)$ or $\nu_A(x) + M(t, s) \leq 1, \forall x, y \in X$
where $M(t, s) = \max(t, s)$

Theorem 3.1. A intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a BCK-algebra X is an intuitionistic fuzzy ideal iff A is (\in, \in) -intuitionistic fuzzy ideal.

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy ideal of X , therefore we have

- (i) $\mu_A(0) \geq \mu_A(x)$
- (ii) $\nu_A(0) \leq \nu_A(x)$
- (iii) $\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\}$
- (iv) $\nu_A(x) \leq \max\{\nu_A(x * y), \nu_A(y)\} \quad \forall x, y \in X$

Let $x_t \in \mu_A \Rightarrow \mu_A(x) \geq t$ now (i) $\Rightarrow \mu_A(0) \geq \mu_A(x) \geq t \Rightarrow 0_t \in \mu_A$

And $x_t \bar{\in} \nu_A \Rightarrow \nu_A(x) < t$ now (ii) $\Rightarrow \nu_A(0) \leq \nu_A(x) < t \Rightarrow 0_t \bar{\in} \nu_A$

Let $X, y \in X$, such that $(x * y)_t, y_s \in A$ then $(x * y)_t, y_s \in \mu_A$ and $(x * y)_t, y_s \bar{\in} \nu_A$ where $t, s \in (0, 1)$, therefore $\mu_A(x * y) \geq t, \mu_A(y) \geq s$ and $\nu_A(x * y) < t, \nu_A(y) < s$
Now (iii) $\Rightarrow \mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\} \geq \min\{t, s\} = m(t, s) \Rightarrow x_{m(t, s)} \in \mu_A$
Now (iv) $\Rightarrow \nu_A(x) \leq \max\{\nu_A(x * y), \nu_A(y)\} < \max\{t, s\} = M(t, s) \Rightarrow x_{M(t, s)} \bar{\in} \nu_A$
therefore A is (\in, \in) -intuitionistic fuzzy ideal.

Conversely, let $A = (\mu_A, \nu_A)$ be an (\in, \in) -intuitionistic fuzzy ideal of X .

to prove $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy ideal of X

we know $x_{\mu_A(x)} \in \mu_A \Rightarrow 0_{\mu_A(x)} \in \mu_A \Rightarrow \mu_A(0) \geq \mu_A(x)$ [Since A is (\in, \in) -intuitionistic fuzzy ideal] (1)

Again $x_{\nu_A(x)} \in \nu_A \Rightarrow 0_{\nu_A(x)} \in \nu_A \Rightarrow \nu_A(0) \leq \nu_A(x)$ [Since A is (\in, \in) -intuitionistic fuzzy ideal] (2)

let $X, y \in X$ and $t = \mu_A(x * y), s = \mu_A(y)$

then $\mu_A(x * y) \geq t, \mu_A(y) \geq s$

i.e. $(x * y)_t \in \mu_A, y_s \in \mu_A \Rightarrow x_{m(t, s)} \in \mu_A$ [since $A = (\mu_A, \nu_A)$ be an (\in, \in) -intuitionistic fuzzy ideal of X .]

$\Rightarrow \mu_A(x) \geq m(t, s)$

$\Rightarrow \mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\}$ (3)

Again let $X, y \in X$ and $t = \nu_A(x * y), s = \nu_A(y)$ then $\nu_A(x * y) \leq t, \mu_A(y) \leq s$

Therefore $\nu_A(x * y) \leq t < t + \delta, \mu_A(y) \leq s < s + \delta$ [where δ is arbitrary small]

i.e $(x * y)_{t+\delta} \bar{\in} \nu_A, y_{s+\delta} \bar{\in} \nu_A \Rightarrow x_{M(t+\delta, s+\delta)} \bar{\in} \nu_A$ [since $A = (\mu_A, \nu_A)$ be an (\in, \in) -intuitionistic fuzzy ideal of X .]

$\Rightarrow \nu_A(x) < M(t+\delta, s+\delta)$

$\Rightarrow \nu_A(x) \leq M(t, s) < M(t+\delta, s+\delta)$

$\Rightarrow \nu_A(x) \leq \max\{\nu_A(x * y), \nu_A(y)\}$ (4)

Hence from (1), (2), (3) and (4) $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy ideal of X . □

Theorem 3.2. If $A = (\mu_A, \nu_A)$ is a (q, q) -intuitionistic fuzzy ideal of a BCK-algebra X , then it is also a (\in, \in) -intuitionistic fuzzy ideal of X .

Proof: Let $A = (\mu_A, \nu_A)$ be a (q, q) -intuitionistic fuzzy ideal of BG-algebra X .

$$\text{Let } x_t \in \mu_A \Rightarrow \mu_A(x) \geq t$$

$$\Rightarrow \mu_A(x) + \delta > t \Rightarrow \mu_A(x) + \delta - t + 1 > 1$$

$$\Rightarrow x_{(\delta-t+1)} \in \mu_A \Rightarrow 0_{(\delta-t+1)} \in \mu_A [\text{Since } (\mu_A, \nu_A) \text{ is a } (q, q)\text{-intuitionistic fuzzy ideal } X.]$$

$$\Rightarrow \mu_A(0) + \delta - t + 1 > 1 \Rightarrow \mu_A(0) + \delta > t \Rightarrow \mu_A(0) \geq t \Rightarrow 0_t \in \mu_A$$

$$\text{Therefore } x_t \in \mu_A \Rightarrow 0_t \in \mu_A \quad (5)$$

$$\text{Let } x_t \notin \nu_A \Rightarrow \nu_A(x) < t$$

$$\Rightarrow \nu_A(x) < t \Rightarrow \nu_A(x) + 1 - t < 1$$

$$\Rightarrow x_{(1-t)} \notin \nu_A \Rightarrow 0_{(1-t)} \notin \nu_A [\text{Since } (\mu_A, \nu_A) \text{ is a } (q, q)\text{-intuitionistic fuzzy ideal } X.]$$

$$\Rightarrow \nu_A(0) + 1 - t < 1 \Rightarrow \nu_A(0) < t \Rightarrow 0_t \notin \nu_A$$

$$\text{Therefore } x_t \notin \nu_A \Rightarrow 0_t \notin \nu_A \quad (6)$$

Again let $X, y \in X$ such that $(x * y)_t, y_s \in \mu_A$ then

$$\mu_A(x * y) \geq t \text{ and } \mu_A(y) \geq s$$

$$\Rightarrow \mu_A(x * y) + \delta > t \text{ and } \mu_A(y) + \delta > s \text{ where } \delta \text{ be an arbitrary small positive number}$$

$$\Rightarrow \mu_A(x * y) + \delta - t + 1 > 1 \text{ and } \mu_A(y) + \delta - s + 1 > 1$$

$$\Rightarrow (x * y)_{\delta-t+1} \in \mu_A \text{ and } (y)_{\delta-s+1} \in \mu_A$$

$$\therefore \text{we have } x_{m(\delta-t+1, \delta-s+1)} \in \mu_A [\text{Since } A = (\mu_A, \nu_A) \text{ is a } (q, q)\text{-intuitionistic fuzzy ideal } X.]$$

$$\Rightarrow \mu_A(x) + m(\delta - t + 1, \delta - s + 1) > 1$$

$$\Rightarrow \mu_A(x) + \delta + 1 - M(t, s) > 1$$

$$\Rightarrow \mu_A(x) > M(t, s) - \delta$$

$$\Rightarrow \mu_A(x) > M(t, s) \text{ since } \delta \text{ is arbitrary}$$

$$\Rightarrow \mu_A(x) > M(t, s) > m(t, s)$$

$$\Rightarrow x_{m(t, s)} \in \mu_A$$

$$\therefore (x * y)_t, y_s \in \mu_A \Rightarrow x_{m(t, s)} \in \mu_A \quad (7)$$

Again let $X, y \in X$ such that $(x * y)_t, y_s \notin \nu_A$ then

$$\Rightarrow \nu_A(x * y) < t \text{ and } \nu_A(y) < s$$

$$\Rightarrow \nu_A(x * y) + 1 - t < 1 \text{ and } \mu_A(y) + 1 - s < 1$$

$$\Rightarrow (x * y)_{1-t} \notin \nu_A \text{ and } (y)_{1-s} \notin \nu_A$$

$$\therefore \text{we have } x_{M(1-t, 1-s)} \notin \nu_A [\text{Since } (\mu_A, \nu_A) \text{ is a } (q, q)\text{-intuitionistic fuzzy ideal } X.]$$

$$\Rightarrow \nu_A(x) + M(1 - t, 1 - s) \leq 1$$

$$\Rightarrow \nu_A(x) + 1 - m(t, s) \leq 1$$

$$\Rightarrow \nu_A(x) \leq m(t, s)$$

$$\Rightarrow \nu_A(x) < m(t, s) + \delta \text{ where } \delta \text{ is arbitrary small}$$

$$\Rightarrow \nu_A(x) \leq m(t, s) < M(t, s)$$

$$\Rightarrow x_{M(t, s)} \notin \nu_A$$

$$\therefore (x * y)_t, y_s \notin \nu_A \Rightarrow x_{M(t, s)} \notin \nu_A \quad (8)$$

Hence from (5), (6), (7) and (8) $A = (\mu_A, \nu_A)$ is an (\in, \in) -intuitionistic fuzzy ideal of X . \square

Remark 3.1. Converse of above is not true i.e. every (\in, \in) -intuitionistic fuzzy ideal is not a (q, q) -intuitionistic fuzzy ideal.

Example 3.1. Consider BCK-algebra $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	3	4	1	0

The intuitionistic fuzzy subset $A = \{\langle x, \mu_A(x), v_A(x) \rangle \mid x \in X\}$ given by $\mu_A(0) = \mu_A(2) = 0.8$, $\mu_A(1) = \mu_A(3) = \mu_A(4) = 0.5$ and $v_A(0) = v_A(2) = 0$, $v_A(1) = v_A(3) = v_A(4) = 0.3$ then A is an intuitionistic fuzzy ideal of BCK-algebra X . But not a (q, q) -intuitionistic fuzzy ideal because if $x = 1$, $y = 3$, $t = 0.22$, $s = 0.55$ then $x * y = 1 * 3 = 0$ Here $\mu_A(x * y) + t = \mu_A(0) + 0.2 = 0.8 + 0.22 = 1.02 > 1$ and $\mu_A(y) + s = \mu_A(3) + 0.55 = 0.5 + .55 = 1.05 > 1$ i.e $(x * y)_t \not\in \mu_A$ and $y_s \not\in \mu_A$ but $\mu_A(x) + m(t, s) = \mu_A(1) + m(0.22, 0.55) = 0.5 + 0.22 = 0.72 < 1$

Theorem 3.3. A intuitionistic fuzzy subset $A = (\mu_A, v_A)$ of a BCK-algebra X is a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X if

- (i) $\mu_A(0) \geq m(\mu_A(x), 0.5)$
- (ii) $v_A(0) \leq M(v_A(x), 0.5)$
- (iii) $\mu_A(x) \geq m(\mu_A(x * y), \mu_A(y), 0.5)$
- (iv) $v_A(x) \leq M(v_A(x * y), v_A(y), 0.5)$

Proof. (i) First let $A = (\mu_A, v_A)$ be a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X . Assume $\mu_A(0) < m(\mu_A(x), 0.5)$

Subcase I: $\mu_A(x) < 0.5$, then

$$\mu_A(0) < m(\mu_A(x), 0.5) = \mu_A(x)$$

$$\Rightarrow \mu_A(0) < t < \mu_A(x) \text{ for some } t \in (0, 0.5)$$

$$\Rightarrow x_t \in \mu_A \text{ and } 0_t \in \mu_A \text{ Also } \mu_A(0) + t < 1 \Rightarrow 0_t \not\in \mu_A$$

$$\Rightarrow 0_t \in \overline{vq} \mu_A, \text{ which is a contradiction}$$

Subcase II: $\mu_A(x) \geq 0.5$, then

$$\mu_A(0) < m(\mu_A(x), 0.5) = 0.5$$

$$\Rightarrow 0_{0.5} \in \mu_A \text{ Also } \mu_A(0) + 0.5 < 0.5 + 0.5 = 1 \Rightarrow 0_{0.5} \not\in \mu_A$$

$$\Rightarrow 0_t \in \overline{vq} \mu_A, \text{ which is again a contradiction}$$

Therefore we must have $\mu_A(0) \geq m(\mu_A(x), 0.5)$

- (ii) Assume $v_A(0) > M(v_A(x), 0.5)$

Subcase I: $v_A(x) > 0.5$, then

$$v_A(0) > M(v_A(x), 0.5) = v_A(x)$$

$$\Rightarrow v_A(0) > t > v_A(x) \text{ for some } t \in (0.5, 1)$$

$$\Rightarrow x_t \in v_A \text{ and } 0_t \in v_A \text{ Also } v_A(0) + t > 1 \Rightarrow 0_t \not\in v_A$$

$$\Rightarrow 0_t \in \overline{vq} v_A, \text{ which is a contradiction}$$

Subcase II: $v_A(x) \leq 0.5$, then $v_A(0) > M(v_A(x), 0.5) = 0.5$

$$\Rightarrow 0_{0.5} \in v_A \text{ Also } v_A(0) + 0.5 > 0.5 + 0.5 = 1 \Rightarrow 0_{0.5} \not\in v_A$$

$\Rightarrow 0_t \in \vee q v_A$, which is again a contradiction

Therefore we must have $v_A(0) \leq M(v_A(x), 0.5)$

(iii) Sub case I: Let $m(\mu_A(x * y), \mu_A(y)) < 0.5 \forall x, y \in X$ then

$$m(\mu_A(x * y), \mu_A(y), 0.5) = m(\mu_A(x * y), \mu_A(y))$$

If possible $\mu_A(x) < m(\mu_A(x * y), \mu_A(y))$ choose a real number t such that

$$\mu_A(x) < t < m(\mu_A(x * y), \mu_A(y)) \text{ then } (x * y)_t, (y)_t \in \mu_A$$

but $\mu_A(x) < t$ i.e. $x_t \notin \mu_A$ and $\mu_A(x) + t < 2t$

i.e. $\mu_A(x) + t < 2 m(\mu_A(x * y), \mu_A(y)) < 2 \times 0.5 = 1$

$$\Rightarrow \mu_A(x) + t < 1 \text{ i.e. } x_t \bar{q} \mu_A$$

Which contradict the fact that $A = (\mu_A, v_A)$ is a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X .

$$\therefore \mu_A(x) \geq m(\mu_A(x * y), \mu_A(y)) = m(\mu_A(x * y), \mu_A(y), 0.5)$$

Subcase II: let $m(\mu_A(x * y), \mu_A(y)) \geq 0.5$

$$\text{Then } m(\mu_A(x * y), \mu_A(y)) = 0.5$$

If possible, $\mu_A(x) < m(\mu_A(x * y), \mu_A(y), 0.5) = 0.5$

Then $\mu_A(x * y) \geq 0.5$ and $\mu_A(y) \geq 0.5$

$$\Rightarrow (x * y)_{0.5}, y_{0.5} \in \mu_A \text{ but } \mu_A(x) < 0.5$$

$$\Rightarrow x_{0.5} \bar{q} \mu_A \text{ and } \mu_A(x) + 0.5 < 0.5 + 0.5 = 1 \text{ i.e. } x_{0.5} \bar{q} \mu_A$$

Which is again a contradiction that $A = (\mu_A, v_A)$ is a $(\in, \in \vee q)$ -fuzzy ideal of X

Hence we must have $\mu_A(x) \geq 0.5 = m(\mu_A(x * y), \mu_A(y), 0.5)$

(iv) Subcase I: Let $M(v_A(x * y), v_A(y)) > 0.5 \forall X, y \in X$ then

$$M(v_A(x * y), v_A(y), 0.5) = M(v_A(x * y), v_A(y))$$

If possible $v_A(x) > M(v_A(x * y), v_A(y))$ choose a real number t such that

$$v_A(x) > t > M(v_A(x * y), v_A(y))$$

$$\Rightarrow v_A(x * y) < t, v_A(y) < t \Rightarrow (x * y)_t \bar{q} v_A, (y)_t \bar{q} v_A$$

But $v_A(x) > t$ i.e. $x_t \in v_A$ and $v_A(x) + t > 2t$

i.e. $v_A(x) + t > 2M(v_A(x * y), v_A(y)) > 2 \times 0.5 = 1$

$$\Rightarrow v_A(x) + t > 1 \text{ i.e. } x_t \bar{q} v_A$$

Which contradict the fact that $A = (\mu_A, v_A)$ is a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X .

$$\therefore v_A(x) \leq M(v_A(x * y), v_A(y)) = M(v_A(x * y), v_A(y), 0.5)$$

Subcase II: let $M(v_A(x * y), v_A(y)) \leq 0.5 \forall x, y \in X$

$$\text{Then } M(v_A(x * y), v_A(y), 0.5) = 0.5$$

If possible $v_A(x) > M(v_A(x * y), v_A(y), 0.5) = 0.5$

then $v_A(x * y) \leq 0.5$ and $v_A(y) \leq 0.5$

therefore $(x * y)_{0.5}, y_{0.5} \bar{q} v_A$ but $v_A(x) > 0.5$

Therefore $x_{0.5} \in v_A$ and $v_A(x) + 0.5 > 0.5 + 0.5 = 1$ i.e. $x_t \bar{q} v_A$

Which is again a contradiction that $A = (\mu_A, v_A)$ is a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X

Hence we must have $v_A(x) \leq 0.5 = M(v_A(x * y), v_A(y), 0.5)$

Converse Part, Suppose that $A = (\mu_A, v_A)$ satisfied conditions (i), (ii), (iii) and (iv) To prove $A = (\mu_A, v_A)$ is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X'

Let $x \in X$ and $t \in (0, 1]$ be such that $x_t \in \mu_A \Rightarrow \mu_A(x) \geq t$

Therefore by (i) $\mu_A(0) \geq m(\mu_A(x), 0.5) \geq m\{t, 0.5\} = t$ or 0.5 according as if $t < 0.5$ or $t \geq 0.5$

Therefore $\mu_A(0) \geq t \Rightarrow 0_t \in \mu_A$ if $t \leq 0.5$

And $\mu_A(0) \geq 0.5 \Rightarrow \mu_A(0) + t \geq 0.5 + 0.5 = 1$ if $t \geq 0.5 \Rightarrow 0_t \in q\mu_A$

Hence $x_t \in \mu_A \Rightarrow 0_t \in v_q \mu_A$ (9)

Again let $x_t \bar{\in} v_A \Rightarrow v_A(x) < t$. Therefore by (ii) $v_A(0) \leq M(v_A(x), 0.5) \leq M\{t, 0.5\} = t$ or 0.5 according as if $t > 0.5$ or $t \leq 0.5$

Therefore $v_A(0) \leq t \Rightarrow 0_t \bar{\in} v_A$ if $t > 0.5$

And $v_A(0) \leq 0.5 \Rightarrow v_A(0) + t \leq 0.5 + 0.5 = 1$ if $t \leq 0.5 \Rightarrow 0_t \bar{\in} v_A$

Hence $x_t \bar{\in} v_A \Rightarrow 0_t \bar{\in} v_q v_A$ (10)

Let $X, y \in X$, such that $(x * y)_t, y_s \in \mu_A$

$\therefore \mu_A(x * y) \geq t$ and $\mu_A(y) \geq s$ therefore $m(\mu_A(x * y), \mu_A(y)) \geq m(t, s)$ therefore (iii) $\Rightarrow \mu_A(x) \geq m(t, s, 0.5)$

\therefore Now if $m(t, s) \leq 0.5$ then $m(t, s, 0.5) = m(t, s)$

Therefore $\mu_A(x) \geq m(t, s)$ i.e. $x_{m(t, s)} \in \mu_A$

Again if $m(t, s) > 0.5$ then $m(t, s, 0.5) = 0.5$

$\therefore \mu_A(x) \geq m(t, s, 0.5) = 0.5$

i.e. $\mu_A(x) + m(t, s) > 0.5 + 0.5 = 1$

$\Rightarrow x_{m(t, s)} \in \mu_A$

Hence $(x * y)_t, y_s \in \mu_A \Rightarrow x_{m(t, s)} \in v_q \mu_A$ (11)

Let $X, y \in X$, such that $(x * y)_t, y_s \bar{\in} v_A$

$\therefore v_A(x * y) \leq t$ and $v_A(y) \leq s \therefore M(v_A(x * y), v_A(y)) \leq M(t, s)$ therefore (iv) $\Rightarrow v_A(x) \leq M(t, s, 0.5)$

Now if $M(t, s) \geq 0.5$ then $M(t, s, 0.5) = M(t, s)$

Therefore $v_A(x) \leq M(t, s)$

i.e. $x_{M(t, s)} \bar{\in} v_A$

Again if $M(t, s) < 0.5$ then $M(t, s, 0.5) = 0.5 \therefore v_A(x) \leq M(t, s, 0.5) = 0.5$

i.e. $v_A(x) + M(t, s) < 0.5 + 0.5 = 1 \Rightarrow x_{M(t, s)} \bar{\in} v_A$

Hence $(x * y)_t, y_s \bar{\in} v_A \Rightarrow x_{M(t, s)} \bar{\in} v_q v_A$ (12)

(9), (10), (11) and (12) $\Rightarrow A$ is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X . \square

Remark 3.2. A (\in, \in) -intuitionistic fuzzy ideal is always a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X but not conversely and can be seen from the following example.

Example 3.2. Consider a BCK-algebra $X = \{0, a, b, c\}$ with the following cayley table.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	2	4	0

Let $A = (\mu_A, v_A)$ be an Intuitionistic fuzzy set in X defined as $\mu_A(0) = 0.7, \mu_A(1) = \mu_A(3) = 0.6, \mu_A(2) = \mu_A(4) = 0.55$, and $v_A(0) = 0.2, v_A(1) = v_A(3) = 0.3, v_A(2) = v_A(4) = 0.4$ then $A = (\mu_A, v_A)$ is an $(\in, \in \vee q)$ -Intuitionistic fuzzy ideal by above theorem but it is not a (\in, \in) -intuitionistic fuzzy ideal since $3_{0.6} = (2 * 1)_{0.6}, 1_{0.6} \in \mu_A$ but $2_{0.6} \notin \mu_A$.

Theorem 3.4. An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a BCK-algebra X is a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X and $\mu_A(x) < 0.5, \nu_A(x) > 0.5 \forall X, y \in X$, then $A = (\mu_A, \nu_A)$ is also a (\in, \in) -intuitionistic fuzzy ideal of X .

Proof. Let $A = (\mu_A, \nu_A)$ be an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X and $\mu_A(x) < 0.5$ and $\nu_A(x) > 0.5 \forall X, y \in X$.

Let $x_t \in \mu_A \Rightarrow \mu_A(x) \geq t \Rightarrow t \leq \mu_A(x) < 0.5$ and also $\mu_A(0) < 0.5$

$\Rightarrow \mu_A(0) + t < 0.5 + 0.5 = 1 \Rightarrow 0_t \in \mu_A$ Since μ_A is $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X

Therefore $0_t \in \mu_A$ Hence $x_t \in \mu_A \Rightarrow 0_t \in \mu_A$ (13)

Let $(x * y)_t \in \mu_A, y_s \in \mu_A$

$t \leq \mu_A(x * y) < 0.5$ and $s \leq \mu_A(y) < 0.5$

$\therefore m(t, s) < 0.5$ also $\mu_A(x) < 0.5$ thus $\mu_A(x) + m(t, s) < 0.5 + 0.5 = 1$.

Since μ_A is $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X therefore either $\mu_A(x) \geq m(t, s)$ or $\mu_A(x) + m(t, s) > 1$. so we must have $\mu_A(x) \geq m(t, s) \Rightarrow x_{m(t, s)} \in \mu_A$

Therefore $(x * y)_t \in \mu_A, y_s \in \mu_A \Rightarrow x_{m(t, s)} \in \mu_A$ (14)

Again let $x_t \in \nu_A \Rightarrow \nu_A(x) < t \Rightarrow 0.5 < \nu_A(x) \leq t$ and $0.5 < \nu_A(y)$

Therefore $\nu_A(x) + t > 0.5 + 0.5 = 1$

$\Rightarrow 0_t \in \nu_A$ Since μ_A is $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X

Therefore $0_t \in \nu_A$ Hence $x_t \in \nu_A \Rightarrow 0_t \in \nu_A$ (15)

Again let $(x * y)_t \in \nu_A, y_s \in \nu_A$

Therefore $0.5 < \nu_A(x * y) \leq t$ and $0.5 < \nu_A(y) \leq s$

$\therefore M(t, s) > 0.5$ also $\nu_A(x) > 0.5$ thus $\nu_A(x) + M(t, s) > 0.5 + 0.5 = 1$.

Since ν_A is $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X therefore either $\nu_A(x) \leq m(t, s)$

or $\nu_A(x) + M(t, s) < 1$ so we must have $\nu_A(x) \leq m(t, s) \Rightarrow x_{M(t, s)} \in \nu_A$

therefore $(x * y)_t \in \nu_A, y_s \in \nu_A \Rightarrow x_{M(t, s)} \in \nu_A$ (16)

Hence (13), (14), (15) and (16) $\Rightarrow A = (\mu_A, \nu_A)$ is an (\in, \in) -intuitionistic fuzzy ideal of X . \square

Remark 3.3. Every (\in, q) -intuitionistic fuzzy ideal of BCK-algebra X is always a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X .

Theorem 3.5. An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a BCK-algebra X is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X iff the sets $(\mu_A) t = \{x \in X \mid \mu_A(x) \geq t$, where $t \in (0, 0.5)$, $\mu_A(0) \geq t\}$ and $(\nu_A) s = \{x \in X \mid \nu_A(x) < s$, where $s \in (0.5, 1]$, $\nu_A(0) < s\}$ are ideal of X .

Proof: Assume $A = (\mu_A, \nu_A)$ is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X .

Let $X, \in X$ such that $x \in (\mu_A) t$ where $t \in (0, 0.5]$.

Therefore $\mu_A(x) \geq t$, now by theorem 3.3

$\mu_A(0) \geq m(\mu_A(x), 0.5) \geq m(t, 0.5) = t$

$\Rightarrow \mu_A(0) \geq t \Rightarrow 0 \in (\mu_A)_t$

Again let $X, y \in X$ such that $x * y, y \in (\mu_A) t$ where $t \in (0, 0.5]$.

Therefore $\mu_A(x * y) \geq t, \mu_A(y) \geq t$, now by theorem 3.3

$\mu_A(x) \geq m(\mu_A(x * y), \mu_A(y), 0.5) \geq m(t, t, 0.5) = t$

$\Rightarrow \mu_A(x) \geq t \Rightarrow x \in (\mu_A)_t$

Therefore, $x * y, y \in (\mu_A) t \Rightarrow x \in (\mu_A)_t$

Hence $(\mu_A)_t$ is an ideal of X .

Let $x \in X$ such that $x \in (v_A)_s$ where $s \in (0.5, 1]$

Therefore $v_A(x) < s$, Now by theorem 3.3

$$v_A(0) \leq M(v_A(x), 0.5) < M(s, 0.5) = s$$

$$\Rightarrow v_A(0) < s \Rightarrow 0 \notin (v_A)_s$$

Again let $X, y \in X$ such that $x * y, y \in (v_A)_s$ where $s \in (0.5, 1]$

Therefore $v_A(x * y) < s, v_A(y) < s,$

Now by theorem 3.3

$$v_A(x) \leq M(v_A(x * y), v_A(y), 0.5) < M(s, s, 0.5) = s$$

$$\Rightarrow v_A(x) < s \Rightarrow x \notin (v_A)_s \text{ Therefore } x * y, y \in (v_A)_s \Rightarrow x \notin (v_A)_s$$

Hence $(v_A)_s$ is an ideal of X .

Conversely, let $A = (\mu_A, v_A)$ be intuitionistic fuzzy subset of X and the sets $(\mu_A)_t = \{x \in X \mid \mu_A(x) \geq t\}$, where $t \in (0, 0.5]\}$ and $(v_A)_s = \{x \in X \mid v_A(x) < s\}$, where $s \in (0.5, 1]\}$ are ideal of X , to prove $A = (\mu_A, v_A)$ is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X .

Suppose $A = (\mu_A, v_A)$ is not an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X , then there exist $a, b \in X$ such that at least one of $\mu_A(0) < m(\mu_A(a), 0.5)$, $v_A(0) > M(v_A(a), 0.5)$, $\mu_A(a) < m(\mu_A(a * b), \mu_A(b), 0.5)$ and $v_A(a) > M(v_A(a * b), v_A(b), 0.5)$ hold.

Suppose $\mu_A(0) < m(\mu_A(a), 0.5)$ holds. Let $t = [\mu_A(0) + m(\mu_A(a), 0.5)]/2$, then $t \in (0, 0.5)$ and $\mu_A(0) < t < m(\mu_A(a), 0.5)$ (17)

i.e. $\Rightarrow \mu_A(0) < t \Rightarrow 0 \notin (\mu_A)_t$, which is a contradiction [since $(\mu_A)_t$ is ideal]

Hence we must have $\mu_A(0) \geq m(\mu_A(a), 0.5)$ (18)

Again let $v_A(0) > M(v_A(a), 0.5)$ holds. Let $t = [v_A(0) + M(v_A(a), 0.5)]/2$, then $t \in (0, 0.5)$ and $v_A(0) > s > M(v_A(a), 0.5)$

$\Rightarrow v_A(0) > s \Rightarrow 0 \in (v_A)_s$ which is a contradiction [since $(v_A)_s$ is ideal]

Therefore we must have $v_A(0) \leq M(v_A(a), 0.5)$ (19) Again suppose $\mu_A(a) < m(\mu_A(a * b), \mu_A(b), 0.5)$ holds. Let $t = [\mu_A(a) + m(\mu_A(a * b), \mu_A(b), 0.5)]/2$, then $t \in (0, 0.5)$ and $\mu_A(a) < t < m(\mu_A(a * b), \mu_A(b), 0.5)$ (20)

i.e. $\mu_A(a * b) > t, \mu_A(b) > t$

$\Rightarrow a * b \in (\mu_A)_t, b \in (\mu_A)_t$

$\Rightarrow a \in (\mu_A)_t$ [since $(\mu_A)_t$ is ideal]

Therefore $\mu_A(a) > t$, which contradict (20)

Hence we must have $\mu_A(x) \geq m(\mu_A(x * y), \mu_A(y), 0.5)$ (21)

Next let $v_A(a) > M(v_A(a * b), v_A(b), 0.5)$ holds. Let $s = [v_A(a) + M(v_A(a * b), v_A(b), 0.5)]/2$, then $s \in (0.5, 1]$

and $v_A(a) > s > M(v_A(a * b), v_A(b), 0.5)$ (22)

i.e. $v_A(a * b) < s, v_A(b) < s$

$\Rightarrow a * b \notin (v_A)_s, b \notin (v_A)_s \Rightarrow a \notin (v_A)_s$ [since $(v_A)_s$ is ideal]

Therefore $v_A(a) < s$, which contradict (22)

Therefore we must have $v_A(x) \leq M(v_A(x * y), v_A(y), 0.5)$ (23)

Hence (18), (19), (21) and (23) $\Rightarrow A = (\mu_A, v_A)$ is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X . \square

Theorem 3.6. Let S be a subset of a BCK-algebra X . Consider the intuitionistic fuzzy set $A_s =$

(μ_S, v_S) in X defined by

$$\mu_S(x) = \begin{cases} 1 & \text{if } x \in S, \\ 0 & \text{otherwise} \end{cases} \quad v_S(x) = \begin{cases} 0 & \text{if } x \in S, \\ 1 & \text{otherwise} \end{cases}$$

Then S is an ideal of X iff $A_S = (\mu_S, v_S)$ is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal X .

Proof. Let S be an ideal of X .

Now $(\mu_S)_t = \{x \in X \mid \mu_S(x) \geq t\} = S$, And $(v_S)_t = \{x \in X \mid v_S(x) < t\} = S$ which is an ideal. Hence by theorem 3.5, $A_S = (\mu_S, v_S)$ is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal X .

Conversely, assume that $A_S = (\mu_S, v_S)$ is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal X , to prove S is an ideal of X .

Let $x \in S$. Then $\mu_S(0) \geq m(\mu_S(x), 0.5) = m(1, 0.5) = 0.5 \Rightarrow \mu_S(0) \geq 0.5 \Rightarrow \mu_S(0) = 1 \Rightarrow 0 \in S$.

And $v_S(0) \leq M(v_S(x), 0.5) = M(0, 0.5) = 0.5 \Rightarrow v_S(0) \leq 0.5 \Rightarrow v_S(0) = 0 \Rightarrow 0 \in S$.

Again let $x * y, y \in S$. Then $\mu_S(x) \geq m(\mu_S(x * y), \mu_S(y), 0.5) = m(1, 1, 0.5) = 0.5 \Rightarrow \mu_S(x) \geq 0.5 \Rightarrow \mu_S(x) = 1 \Rightarrow x \in S$.

And $v_S(x) \leq M(v_S(x * y), v_S(y), 0.5) = M(0, 0, 0.5) = 0.5 \Rightarrow v_S(x) \leq 0.5 \Rightarrow v_S(x) = 0 \Rightarrow x \in S$.

Hence S is an ideal of X . \square

Theorem 3.7. Let S be an ideal of X , then there exists $(\in, \in \vee q)$ -intuitionistic fuzzy ideal $A = (\mu_A, v_A)$ of X such that $(\mu_A)_t = (v_A)_s = S$ for every $t \in (0, 0.5)$ and $s \in (0.5, 1]$.

Proof. Let $A = (\mu_A, v_A)$ be an intuitionistic fuzzy set in X defined by

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in S, \\ u & \text{otherwise} \end{cases} \quad v_A(x) = \begin{cases} 0 & \text{if } x \in S, \\ s & \text{otherwise} \end{cases}$$

where $u < t \in (0, 0.5]$. Therefore $(\mu_A)_t = \{x \in X \mid \mu_A(x) \geq t\} = S$, $(v_A)_s = \{x \in X \mid v_A(x) < s\} = S$ and hence $(\mu_A)_t = (v_A)_s = S$ is an ideal.

Now if $A = (\mu_A, v_A)$ is not an $(\in, \in \vee q)$ -fuzzy ideal of X then there exist $a, b \in X$ such that at least one of $\mu_A(0) < m(\mu_A(a), 0.5)$, $v_A(0) > M(v_A(a), 0.5)$, $\mu_A(a) < m(\mu_A(a * b), \mu_A(b), 0.5)$ and $v_A(a) > M(v_A(a * b), v_A(b), 0.5)$ hold.

Suppose $\mu_A(0) < m(\mu_A(a), 0.5)$ holds. Then choose a real number $t \in (0, 1)$ such that $\mu_A(0) < t < m(\mu_A(a), 0.5)$

$\Rightarrow \mu_A(0) < t \Rightarrow 0 \in (\mu_A)_t$, which is a contradiction [since $(\mu_A)_t$ is ideal]

Hence we must have $\mu_A(0) \geq m(\mu_A(a), 0.5)$

Suppose $v_A(0) > M(v_A(a), 0.5)$ holds. Then choose a real number $s \in (0, 1)$ such that $v_A(0) > s > M(v_A(a), 0.5)$

$\Rightarrow v_A(0) > s \Rightarrow 0 \in (v_A)_s$, which is a contradiction [since $(v_A)_s$ is ideal]

Hence we must have $v_A(0) \leq M(v_A(a), 0.5)$

Suppose $\mu_A(a) < m(\mu_A(a * b), \mu_A(b), 0.5)$ holds. Then choose a real number $t \in (0, 1)$ such that $\mu_A(a) < t < m(\mu_A(a * b), \mu_A(b), 0.5)$ (24)

i.e. $\mu_A(a * b) > t$, $\mu_A(b) > t$

$\Rightarrow a * b \in (\mu_A)_t$, $b \in (\mu_A)_t$

$\Rightarrow a \in (\mu_A)_t = S$ [since $(\mu_A)_t$ is ideal]

Therefore $\mu_A(a) = 1 > t$, which contradicts (24)

Hence we must have $\mu_A(x) \geq m(\mu_A(x * y), \mu_A(y), 0.5)$

Again if $v_A(a) > M(v_A(a * b), v_A(b), 0.5)$ holds then choose a real number $s \in (0, 1)$

And $v_A(a) > s > M(v_A(a * b), v_A(b), 0.5)$ hold (25)

i.e. $v_A(a * b) < s, v_A(b) < s,$

$$\Rightarrow a * b \in (v_A)_s, b \in (v_A)_s \Rightarrow a \in (v_A)_s = S [\text{since } (v_A)_s \text{ is ideal}]$$

Therefore $v_A(a) = 0 < s$, which contradicts (25)

Hence we must have $v_A(x) \leq M(v_A(x * y), v_A(y), 0.5)$

Thus, $A = (\mu_A, v_A)$ is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X . \square

Definition 3.7. Let $A = (\mu_A, v_A)$ be intuitionistic fuzzy subset of BCK-algebra X and $t \in (0, 1]$, then let

$$\begin{aligned} (\mu_A)_t &= \{x \in X \mid x_t \in \mu_A\} = \{x \in X \mid \mu_A(x) \geq t\} \\ &\subset \mu_A >_t = \{x \in X \mid x_t \in \mu_A\} = \{x \in X \mid \mu_A(x) + t > 1\} \\ [\mu_A]_t &= \{x \in X \mid x_t \in \in \vee q \mu_A\} = \{x \in X \mid \mu_A(x) \geq t \text{ or } \mu_A(x) + t > 1\} \end{aligned}$$

Where $(\mu_A)_t$ is called t level set of μ_A , $\mu_A >_t$ is called q level set of μ_A and $[\mu_A]_t$ is called $\in \vee q$ level set of μ_A clearly $[\mu_A]_t = \mu_A >_t \cup (\mu_A)_t$

$$\begin{aligned} (v_A)_t &= \{x \in X \mid x_t \in v_A\} = \{x \in X \mid v_A(x) \leq t\} \\ &\subset v_A >_t = \{x \in X \mid x_t \in \overline{v_A}\} = \{x \in X \mid v_A(x) + t \leq 1\} \\ [v_A]_t &= \{x \in X \mid x_t \in \overline{\in \vee q v_A}\} = \{x \in X \mid v_A(x) \leq t \text{ or } v_A(x) + t \leq 1\} \end{aligned}$$

Where $(v_A)_t$ is called t level set of v_A , $v_A >_t$ is called q level set of v_A and $[v_A]_t$ is called $\in \vee q$ level set of v_A clearly $[v_A]_t = v_A >_t \cup (v_A)_t$

Theorem 3.8. Let $A = (\mu_A, v_A)$ be intuitionistic fuzzy subset of BCK-algebra X , then A is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X iff $[\mu_A]_t$ and $[v_A]_t$ is an ideal of X for all $t \in (0, 1]$. We call $[\mu_A]_t$ and $[v_A]_t$ as $\in \vee q$ level ideals of μ .

Proof. Assume that A is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X , to prove $[\mu_A]_t$ and $[v_A]_t$ is an ideal of X .

Let $x \in [\mu_A]_t$ for $t \in (0, 1]$.

Then $x_t \in \in \vee q \mu_A \Rightarrow x_t \in \mu_A$ and $x_t \in \mu_A$, i.e. $\mu_A(x) \geq t$ or $\mu_A(x) + t > 1$. Since A is an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X , $\mu_A(0) \geq m(\mu_A(x), 0.5) \forall x \in X$.

Now we have the following cases.

Case I : $\mu_A(x) \geq t$, let $t > 0.5$

$$\text{Then } \mu_A(0) \geq m(\mu_A(x), 0.5) \geq m(t, 0.5) = 0.5$$

$$\Rightarrow \mu_A(0) \geq 0.5 \Rightarrow \mu_A(x) + t > 0.5 + 0.5 = 1 \Rightarrow x_t \in \mu_A$$

$$\text{Again if } t \leq 0.5, \text{ then } \mu_A(0) \geq m(\mu_A(x), 0.5) \geq m(t, 0.5) = t$$

$$\Rightarrow \mu_A(0) \geq t \Rightarrow 0_t \in \mu_A$$

$$\text{Hence } 0_t \in \in \vee q \mu_A \Rightarrow 0 \in [\mu_A]_t$$

Case II : $\mu_A(x) + t > 1$, let $t > 0.5$

$$\text{Then } \mu_A(0) \geq m(\mu_A(x), 0.5) \geq m(1-t, 0.5) = 1-t$$

$$\Rightarrow \mu_A(0) > 1-t \Rightarrow \mu_A(0) + t > 1 \Rightarrow 0_t \in \mu_A$$

$$\text{Again if } t \leq 0.5, \text{ then } \mu_A(0) \geq m(\mu_A(x), 0.5) \geq m(1-t, 0.5) = 0.5$$

$$\Rightarrow \mu_A(0) \geq 0.5 \geq t \Rightarrow 0_t \in \mu_A$$

$$\text{Hence } 0_t \in \in \vee q \mu_A \Rightarrow 0 \in [\mu_A]_t$$

Again let $x * y, y \in [\mu_A]_t$ for $t \in (0, 1]$,

Then $(x * y)_t \in \in \vee q \mu_A$ and $y_t \in \in \vee q \mu_A$

i.e $\mu_A(x * y) \geq t$ or $\mu_A(x * y) + t > 1$ and $\mu_A(y) \geq t$ or $\mu_A(y) + t > 1$

Since, A is an $(\in, \in \vee q)$ - intuitionistic fuzzy ideal X

$$\mu_A(x) \geq m(\mu_A(x), \mu_A(y), 0.5) \quad \forall x, y \in X$$

Now we have the following cases.

Case I : $\mu_A(x * y) \geq t, \mu_A(y) \geq t$, let $t > 0.5$

$$\text{Then } \mu_A(x) \geq m(\mu_A(x * y), \mu_A(y), 0.5) \geq m(t, t, 0.5) = 0.5$$

$$\Rightarrow \mu_A(x) \geq 0.5 \Rightarrow \mu_A(x) + t > 0.5 + 0.5 = 1 \Rightarrow x \in \mu_A$$

$$\text{Again if } t \leq 0.5, \text{ then } \mu_A(x) \geq m(\mu_A(x * y), \mu_A(y), 0.5) \geq m(t, t, 0.5) = t$$

$$\Rightarrow \mu_A(x) \geq t \Rightarrow x \in \mu_A$$

$$\text{Hence } x \in \vee q \mu_A \Rightarrow x \in [\mu_A]_t$$

Case II : $\mu_A(x * y) \geq t, \mu_A(y) + t > 1$, let $t > 0.5$

$$\text{Then } \mu_A(x) \geq m(\mu_A(x * y), \mu_A(y), 0.5) > m(t, 1-t, 0.5) = 1-t$$

$$\Rightarrow \mu_A(x) > 1-t \Rightarrow \mu_A(x) + t > 1 \Rightarrow x \in \mu_A$$

$$\text{Again if } t \leq 0.5, \text{ then } \mu_A(x) \geq m(\mu_A(x * y), \mu_A(y), 0.5) \geq m(t, 1-t, 0.5) = t$$

$$\Rightarrow \mu_A(x) \geq t \Rightarrow x \in \mu_A$$

$$\text{Hence } x \in \vee q \mu_A \Rightarrow x \in [\mu_A]_t$$

Case III : $\mu_A(x) + t > 1, \mu_A(y) \geq t$

This is similar to case II

Case IV : $\mu_A(x * y) + t > 1, \mu_A(y) + t > 1$, let $t > 0.5$

$$\text{Then } \mu_A(x) \geq m(\mu_A(x * y), \mu_A(y), 0.5) > m(1-t, 1-t, 0.5) = 1-t$$

$$\Rightarrow \mu_A(x) > 1-t \Rightarrow \mu_A(x) + t > 1 \Rightarrow x \in \mu_A$$

$$\text{Again if } t \leq 0.5, \text{ then } \mu_A(x) \geq m(\mu_A(x * y), \mu_A(y), 0.5) \geq m(1-t, 1-t, 0.5) = 0.5 \geq t$$

$$\Rightarrow \mu_A(x) \geq t \Rightarrow x \in \mu_A$$

$$\text{Hence } x \in \vee q \mu_A \Rightarrow x \in [\mu_A]_t$$

$$\text{Hence from above four cases } x * y, y \in [\mu_A]_t \Rightarrow x \in [\mu_A]_t$$

Hence $[\mu_A]_t$ is an ideal of X. Similarly we can prove $[\nu_A]_t$ is an ideal of X

Conversely, let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set in X, such that $[\mu_A]_t$ and $[\nu_A]_t$ is an ideal of X for all $t \in (0, 1]$, to prove $A = (\mu_A, \nu_A)$ is an $(\in, \in \vee q)$ - intuitionistic fuzzy ideal X.

Suppose A is not an $(\in, \in \vee q)$ - intuitionistic fuzzy ideal X, then there exist a, b $\in X$ such that at least one of $\mu_A(a) < m(\mu_A(a), 0.5)$, $\nu_A(a) > M(\nu_A(a), 0.5)$, $\mu_A(a) < m(\mu_A(a * b), \mu_A(b), 0.5)$ and $\nu_A(a) > M(\nu_A(a * b), \nu_A(b), 0.5)$ hold.

Suppose $\mu_A(a) < m(\mu_A(a), 0.5)$ is true, then choose $t \in (0, 1]$ such that $\mu_A(a) < t < m(\mu_A(a), 0.5)$.

Then $\mu_A(a) < t \Rightarrow 0 \notin (\mu_A)_t \subset [\mu_A]_t$ which is a contradiction, as $[\mu_A]_t$ is an ideal

Again if $\nu_A(a) > M(\nu_A(a), 0.5)$ is true, then choose $s \in (0, 1]$ such that $\nu_A(a) > s > M(\nu_A(a), 0.5)$.

Then $\nu_A(a) > s \Rightarrow 0 \notin (\nu_A)_s \subset [\nu_A]_s$ which is a contradiction, as $[\nu_A]_s$ is an ideal.

Suppose $\mu_A(a) < m(\mu_A(a * b), \mu_A(b), 0.5)$ is true, then choose $t \in (0, 1]$ such that

$$\mu_A(a) < t < m(\mu_A(a * b), \mu_A(b), 0.5) \quad (26)$$

Then $\mu_A(a * b) \geq t, \mu_A(b) \geq t \Rightarrow a * b, b \in (\mu_A)_t \subset [\mu_A]_t$ which is an ideal.

Therefore $a \in [\mu_A]_t \Rightarrow \mu_A(a) \geq t$ or $\mu_A(a) + t > 1$ which contradict (26).

Again if $\nu_A(a) > M(\nu_A(a * b), \nu_A(b), 0.5)$ is true, then choose $t \in (0, 1]$ such that

$$\nu_A(a) > t > M(\nu_A(a * b), \nu_A(b), 0.5) \quad (27)$$

Then $\nu_A(a * b) < t, \nu_A(b) < t \Rightarrow a * b, b \in (\nu_A)_t \subset [\nu_A]_t$ which is an ideal. Therefore

$a \in [v_A]_t \Rightarrow v_A(a) < t$ or $v_A(a) + t < 1$ which contradicts (27).

Hence we must have

$$\begin{aligned}\mu_A(0) &\geq m(\mu_A(a), 0.5) \\ v_A(0) &\leq M(v_A(a), 0.5) \\ \mu_A(x) &\geq m(\mu_A(x * y), \mu_A(y), 0.5) \\ v_A(x) &\leq M(v_A(x * y), v_A(y), 0.5) \quad \forall x, y \in X\end{aligned}$$

Hence $A = (\mu_A, v_A)$ is an $(\in, \in \vee q)$ - intuitionistic fuzzy ideal of X . \square

Theorem 3.9. Every $(\in \vee q, \in \vee q)$ - intuitionistic fuzzy ideal is an $(\in, \in \vee q)$ - intuitionistic fuzzy ideal.

Proof. It follows from definition. \square

Theorem 3.10. Let $A = (\mu_A, v_A)$ and $B = (\mu_B, v_B)$ be two $(\in, \in \vee q)$ - intuitionistic fuzzy ideals of a BCK-algebra X . Then $A \cap B$ is also a $(\in, \in \vee q)$ - intuitionistic fuzzy ideal of X .

Proof. Let $x, y \in X$. Now we have $(A \cap B)(x) = \{<x, (\mu_A \cap \mu_B)(x), (v_A \cup v_B)(x)> \mid x \in X\}$

$$= m\{\mu_A(0), \mu_B(0)\}$$

$$\geq m\{m\{\mu_A(x), 0.5\}, m\{\mu_B(x), 0.5\}\}$$

$$= m\{m\{\mu_A(x), \mu_B(x)\}, 0.5\} [A \text{ is } (\in, \in \vee q)\text{-intuitionistic fuzzy ideal}]$$

$$= m\{m\{\mu_A(x), \mu_B(x)\}, 0.5\}$$

$$= m\{(\mu_A \cap \mu_B)(x), 0.5\}$$

$$(\mu_A \cap \mu_B)(0) \geq m\{(\mu_A \cap \mu_B)(x), 0.5\}$$

$$\text{And } (v_A \cup v_B)(0) = M\{v_A(0), v_B(0)\}$$

$$\leq M\{M\{v_A(x), 0.5\}, M\{v_B(x), 0.5\}\}$$

$$= M\{M\{v_A(x), \mu_B(x)\}, 0.5\} [A \text{ is } (\in, \in \vee q)\text{-intuitionistic fuzzy ideal}]$$

$$= M\{(v_A \cup v_B)(x), 0.5\}$$

$$(v_A \cup v_B)(0) \leq M\{(v_A \cup v_B)(x), 0.5\}$$

$$\text{Again } (\mu_A \cap \mu_B)(x) = m\{\mu_A(x), \mu_B(x)\}$$

$$\geq m\{m\{\mu_A(x * y), \mu_A(y), 0.5\}, m\{\mu_B(x * y), \mu_B(y), 0.5\}\}$$

$$= m\{m\{\mu_A(x * y), \mu_B(x * y)\}, m\{\mu_A(y), \mu_B(y)\}, 0.5\} [A \text{ is } (\in, \in \vee q)\text{-intuitionistic fuzzy ideal}]$$

$= m\{(\mu_A \cap \mu_B)(x * y), (\mu_A \cap \mu_B)(y), 0.5\}$

$$(\mu_A \cap \mu_B)(x) \geq m\{(\mu_A \cap \mu_B)(x * y), (\mu_A \cap \mu_B)(y), 0.5\}$$

$$\text{And } (v_A \cup v_B)(x) = M\{v_A(x), v_B(x)\}$$

$$\leq M\{M\{v_A(x * y), v_A(y), 0.5\}, M\{v_B(x * y), v_B(y), 0.5\}\}$$

$$= M\{M\{v_A(x * y), \mu_B(x * y)\}, M\{v_A(y), v_B(y)\}, 0.5\} [A \text{ is } (\in, \in \vee q)\text{-intuitionistic fuzzy ideal}]$$

$$= M\{(v_A \cup v_B)(x * y), (v_A \cup v_B)(y), 0.5\}$$

$$(v_A \cup v_B)(x) \leq M\{(v_A \cup v_B)(x * y), (v_A \cup v_B)(y), 0.5\}$$

Hence from above $(A \cap B)$ $(\in, \in \vee q)$ - intuitionistic fuzzy ideal of X . \square

The above theorem can be generalised as

Theorem 3.11. Let $\{A_i = (\mu_{A_i}, \nu_{A_i}) \mid i = 1, 2, 3, \dots\}$ be a family of $(\in, \in \vee q)$ -intuitionistic fuzzy ideals of a BCK-algebra X , then $\cap_{i=1}^n A_i$ is also a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X , where $\cap A_i(x) = \{\langle x, m\{\mu_{A_i}(x)\} : i = 1, 2, 3, \dots \rangle, M\{\nu_{A_i}(x)\} : i = 1, 2, 3, \dots \} > | x \in X \}$.

4 Cartesian product of BCK-algebras and their $(\in, \in \vee q)$ -intuitionistic fuzzy ideals

Theorem 4.1. Let X, Y be two BCK-algebras, then their Cartesian product $X \times Y = \{(x, y) \mid x \in X, y \in Y\}$ is also a BCK-algebra under the binary operation $*$ defined in $X \times Y$ by $(x, y)*(p, q) = (x*p, y*q)$ for all $(x, y), (p, q) \in X \times Y$

Proof. Straightforward. \square

Definition 4.1. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two $(\in, \in \vee q)$ -intuitionistic fuzzy ideals of a BCK-algebra X . Then their Cartesian product $A \times B$ is defined by $(A \times B)(x, y) = \{\langle (x, y), m\{\mu_A(x), \mu_B(y)\}, M\{\nu_A(x), \nu_B(y)\} \rangle > | X, y \in X\}$ where $\mu_A, \mu_B : X \rightarrow [0, 1]$ and $\nu_A, \nu_B : X \rightarrow [0, 1] \forall X, y \in X$.

Theorem 4.2. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two $(\in, \in \vee q)$ -intuitionistic fuzzy ideals of a BCK-algebra X . Then $A \times B$ is also a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X .

Proof. Similar to Theorem 3.10. \square

5 Homomorphism of BCK-algebras and $(\in, \in \vee q)$ -intuitionistic fuzzy ideals

Definition 5.1. Let X and X' be two BCK-algebras, then a mapping $f : X \rightarrow X'$ is said to be homomorphism, if $f(x * y) = f(x) * f(y) \forall x, y \in X$.

Theorem 5.1. Let X and X' be two BCK-algebras and $f : X \rightarrow X'$ be homomorphism. If $A = (\mu_A, \nu_A)$ be a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X' , then $f^{-1}(A)$ is $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X .

Proof. $f^{-1}(A) = f^{-1}(\mu_A, \nu_A)(x)$ is defined as $f^{-1}(\mu_A, \nu_A)(x) = (\mu_A, \nu_A)(f(x)) \forall x \in X$.

Let $A = (\mu_A, \nu_A)$ be an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X'

Let $x, y \in X$ such that $(x * y)_t, y_s \in f^{-1}(A) = f^{-1}(\mu_A, \nu_A) = (f^{-1}(\mu_A), f^{-1}(\nu_A))$.

then $(x * y)_t, y_s \in f^{-1}(\mu_A)$ and $(x * y)_t, y_s \in f^{-1}(\nu_A)$

Case I. Let $x_t \in f^{-1}(\mu_A)$

$\Rightarrow f^{-1}(\mu_A)(x) \geq t \Rightarrow f^{-1}(\mu_A)(x) \geq t \Rightarrow \mu_A f(x) \geq t$

$\Rightarrow (f(x))_t \in \mu_A \Rightarrow (f(0))_t \in \vee q \mu_A$ [since $A = (\mu_A, \nu_A)$ be an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal]

$\Rightarrow (f(0))_t \in \mu_A$ and $(f(0))_t \leq \mu_A$

$\Rightarrow \mu_A(f(0)) \geq t$ or $\mu_A(f(0)) + t > 1$

$$\Rightarrow f^{-1}(\mu_A)(0)) \geq t \text{ or } f^{-1}(\mu_A)(0)) + t > 1$$

$$\Rightarrow 0_t \in f^{-1}(\mu_A) \text{ or } 0_t \in qf^{-1}(\mu_A)$$

$$\Rightarrow 0_t \in Vqf^{-1}(\mu_A)$$

Case II. Let $x_t \in f^{-1}(v_A)$

$$\Rightarrow f^{-1}(v_A)(x) < t \Rightarrow v_A f(x) < t$$

$$\Rightarrow (f(x))_t \in v_A$$

$\Rightarrow (f(0))_t \in \overline{\vee q} v_A$ [since $A = (\mu_A, v_A)$ be an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal]

$$\Rightarrow (f(0))_t \in v_A \text{ and } (f(0))_t \in \overline{q} v_A$$

$$\Rightarrow v_A(f(0)) < t \text{ or } v_A(f(0)) + t \leq 1$$

$$\Rightarrow f^{-1}(v_A)(0)) < t \text{ or } f^{-1}(v_A)(0)) + t \leq 1$$

$$\Rightarrow 0_t \in f^{-1}(v_A) \text{ or } 0_t \in \overline{q} f^{-1}(v_A)$$

$$\Rightarrow 0_t \in \overline{\vee q} f^{-1}(v_A)$$

Case III. Let $(x * y)_t, y_s \in f^{-1}(\mu_A)$

$$\Rightarrow f^{-1}(\mu_A)(x * y) \geq t \text{ and } f^{-1}(\mu_A)(y) \geq s$$

$$\Rightarrow f^{-1}(\mu_A)(x * y) \geq t \text{ and } f^{-1}(\mu_A)(y) \geq s$$

$$\Rightarrow f^{-1}(\mu_A)(x * y) \geq t \text{ and } f^{-1}(\mu_A)(y) \geq s$$

$$\Rightarrow \mu_A f(x * y) \geq t \text{ and } \mu_A f(y) \geq s$$

$$\Rightarrow (f(x * y))_t \in \mu_A \text{ and } (f(y))_s \in \mu_A$$

$\Rightarrow (f(x) * f(y))_t \in \mu_A \text{ and } (f(y))_s \in \mu_A$ [since f is homomorphism]

$\Rightarrow (f(x))_{m(t, s)} \in Vq \mu_A$ [since $A = (\mu_A, v_A)$ be an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal]

$$\Rightarrow \mu_A(f(x)) \geq m(t, s) \text{ or } \mu_A(f(x)) + m(t, s) > 1$$

$$\Rightarrow f^{-1}(\mu_A)(x) \geq m(t, s) \text{ or } f^{-1}(\mu_A)(x)) + m(t, s) > 1$$

$$\Rightarrow x_{m(t, s)} \in f^{-1}(\mu_A) \text{ or } x_{m(t, s)} \in qf^{-1}(\mu_A)$$

$$\Rightarrow x_{m(t, s)} \in \overline{\vee q} f^{-1}(\mu_A)$$

Case IV. Let $(x * y)_t, y_s \in f^{-1}(v_A)$

$$\Rightarrow f^{-1}(v_A)(x * y) < t \text{ and } f^{-1}(v_A)(y) < s$$

$$\Rightarrow v_A f(x * y) < t \text{ and } v_A f(y) < s$$

$$\Rightarrow (f(x * y))_t \in v_A \text{ and } (f(y))_s \in v_A$$

$\Rightarrow (f(x) * f(y))_t \in v_A \text{ and } (f(y))_s \in v_A$ [since f is homomorphism]

$\Rightarrow (f(x))_{M(t, s)} \in \overline{\vee q} v_A$ [since $A = (\mu_A, v_A)$ be an $(\in, \in \vee q)$ -intuitionistic fuzzy ideal]

$$\Rightarrow v_A(f(x)) \leq M(t, s) \text{ or } v_A(f(x)) + M(t, s) \leq 1$$

$$\Rightarrow f^{-1}(v_A)(x) \leq M(t, s) \text{ or } f^{-1}(v_A)(x)) + M(t, s) \leq 1$$

$$\Rightarrow x_{M(t, s)} \in f^{-1}(v_A) \text{ or } x_{M(t, s)} \in \overline{q} f^{-1}(v_A)$$

$$\Rightarrow x_{M(t, s)} \in \overline{\vee q} f^{-1}(v_A)$$

Hence from above $f^{-1}(A) = f^{-1}(\mu_A, v_A) = (f^{-1}(\mu_A), f^{-1}(v_A))$ is a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X . \square

Theorem 5.2. Let X and X' be two BCK-algebras and $f : X \rightarrow X'$ be an onto homomorphism. If $A = (\mu_A, v_A)$ be an intuitionistic fuzzy subset of X' such that $f^{-1}(A)$ is a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X , then A is also a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X' .

Proof. Let $x', y' \in X'$ such that $(x' * y')_t, y'_s \in A = (\mu_A, v_A)$ where $t, s \in [0, 1]$.

Then $\mu_A(x' * y') \geq t$ and $\mu_A(y') \geq s$ and $v_A(x' * y') \leq t$ and $v_A(y') \leq s$

since f is onto so there exists $X, y \in X$ such that $f(x) = x'$, $f(y) = y'$ also f is homomorphism so $f(x * y) = f(x) * f(y) = x' * y'$

$$x'_t \in A = (\mu_A, v_A)$$

then $x'_t \in \mu_A$, $x'_t \bar{\in} v_A$

If $x'_t \in \mu_A$

so $\mu_A(f(x)) \geq t$

$$\Rightarrow f^{-1}(\mu_A)(x) \geq t \Rightarrow (x)_t \in f^{-1}(\mu_A)$$

$\Rightarrow \exists t \in V_q f^{-1}(\mu_A)$ [Since $f^{-1}(\mu_A)$ is a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X]

$\Rightarrow \exists t \in f^{-1}(\mu_A)$ and $0_t \in f^{-1}(\mu_A)$

$$\Rightarrow f^{-1}(\mu_A)(0) \geq t \text{ or } f^{-1}(\mu_A)(0) + t > 1$$

$$\Rightarrow \mu_A(f(0)) \geq t \text{ or } \mu_A(f(0)) + t > 1$$

$$\Rightarrow \mu_A(0') \geq t \text{ or } \mu_A(0') + t > 1$$

$$\Rightarrow 0'_t \in V_q \mu_A$$

Therefore $x'_t \in \mu_A \Rightarrow 0'_t \in V_q \mu_A$

Again consider $x'_t \bar{\in} v_A$

Again $v_A(x') < t$

$$\Rightarrow v_A(f(x)) < t \Rightarrow f^{-1}(v_A)(x) < t \Rightarrow x_t \bar{\in} f^{-1}(v_A)$$

$\Rightarrow 0_t \bar{\in} V_q f^{-1}(v_A)$ [Since $f^{-1}(v_A)$ is $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X]

$\Rightarrow 0_t \bar{\in} f^{-1}(v_A)$ and $0_t \bar{\in} f^{-1}(v_A)$

$$\Rightarrow f^{-1}(v_A)(0) < t \text{ or } f^{-1}(v_A)(0) + t \leq 1$$

$$\Rightarrow (v_A)f(0) < t \text{ or } (v_A)f(0) + t \leq 1$$

$$\Rightarrow (v_A)(0') < t \text{ or } (v_A)(0') + t \leq 1$$

$$\Rightarrow (0')_t \bar{\in} v_A \text{ or } (0')_t \bar{\in} v_A$$

$$\Rightarrow (0')_t \bar{\in} V_q v_A$$

Again let $(x' * y')_t, y'_s \in A = (\mu_A, v_A)$ where $t, s \in [01]$

Therefore $(x' * y')_t, y'_s \in \mu_A$

$\mu_A(f(x * y)) \geq t$ and $\mu_A(f(y)) \geq s$

$$\Rightarrow f^{-1}(\mu_A)(x * y) \geq t \text{ and } f^{-1}(\mu_A)(y) \geq s$$

$$\Rightarrow (x * y)_t \in f^{-1}(\mu_A) \text{ and } (y)_s \in f^{-1}(\mu_A)$$

$\Rightarrow (x)_{m(t, s)} \in V_q f^{-1}(\mu_A)$ [Since $f^{-1}(\mu_A)$ is a $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X]

$$\Rightarrow f^{-1}(\mu_A)(x) \geq m(t, s) \text{ or } f^{-1}(\mu_A)(x) + m(t, s) > 1$$

$$\Rightarrow \mu_A(f(x)) \geq m(t, s) \text{ or } \mu_A(f(x)) + m(t, s) > 1$$

$$\Rightarrow \mu_A(x') \geq m(t, s) \text{ or } \mu_A(x') + m(t, s) > 1$$

$$\Rightarrow x'_{m(t, s)} \in V_q \mu_A$$

also $(x' * y')_t, y'_s \in v_A$

$$\Rightarrow v_A(x' * y') < t \text{ and } v_A(y') < s$$

$$\Rightarrow v_A(f(x * y)) < t \text{ and } v_A(f(y)) < s$$

$$\Rightarrow f^{-1}(v_A)(x * y) < t \text{ and } f^{-1}(v_A)(y) < s$$

$$\Rightarrow (x * y)_t \bar{\in} f^{-1}(v_A) \text{ and } (y)_s \bar{\in} f^{-1}(v_A)$$

$\Rightarrow x_{m(t, s)} \bar{\in} V_q f^{-1}(v_A)$ [Since $f^{-1}(v_A)$ is $(\in, \in \vee q)$ -intuitionistic fuzzy ideal of X]

$$\Rightarrow f^{-1}(v_A)(x) < m(t, s) \text{ or } f^{-1}(v_A)(x) + m(t, s) \leq 1$$

$$\Rightarrow (v_A)f(x) < m(t, s) \text{ or } (v_A)f(x) + m(t, s) \leq 1$$

$$\Rightarrow (v_A)(f(x)) < M(t, s) \text{ or } (v_A)(f(x)) + M(t, s) \leq 1$$

$$\Rightarrow (v_A)(x') < M(t, s) \text{ or } (v_A)(x') + M(t, s) \leq 1$$

$$\Rightarrow (x')_{M(t, s)} \bar{\in} v_A \text{ or } (x')_{M(t, s)} \bar{q} v_A$$

$$\Rightarrow (x')_{M(t, s)} \bar{\in} \overline{V q} v_A$$

Hence from above v_A is a $(\in, \in \vee q)$ -intuitionistic fuzzy fuzzy ideal of X . \square

6 Conclusion

In this paper, we have introduced the concept of $(\in, \in \vee q)$ -intuitionistic fuzzy fuzzy ideals of BG-algebra and investigated some of their useful properties. In my opinion, these definitions and results can be extended to other algebraic systems also. In the notions of (α, β) -fuzzy ideals we can define twelve different types of ideals by three choices of α and four choices of β . In the present paper, we have mainly discussed $(\in, \in \vee q)$ -type fuzzy ideal.

In future, the following studies may be carried out:

- (1) $(\in, \in \vee q)$ -intuitionistic fuzzy BCI-positive implicative ideals of BCI-algebra
- (2) $(\in, \in \vee q)$ -intuitionistic doubt fuzzy ideals of BCK-algebra.
- (3) $(\in, \in \vee q)$ -intuitionistic fuzzy maximal ideals of BCK-algebras.

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