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Remark on the intuitionistic fuzzy forms of two classical logic axioms. Part 2

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Abstract: It is checked which intuitionistic fuzzy implications satisfy two classical logic axioms as tautologies ad which – as intuitionistic fuzzy tautologies.
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1 Introduction

In a series of research, we discuss the intuitionistic fuzzy forms of some classical logic axioms, and check their validity in the case of intuitionistic fuzziness.

In [2] we determined the implications that satisfy the standard logical tautologies

$$(p\&q) \to r = (p \to (q \to r)),$$

$$p \to q = (p \to (p \to q)).$$

Two other well-known logical tautologies (see, e.g., [3]) are

$$(p \vee q) \to r = (p \to r)\&(q \to r), \tag{1}$$

$$(p\&q) \to r = (p \to r) \vee (q \to r). \tag{2}$$

Here, we discuss their validity for the different cases of intuitionistic fuzzy implications. In [1], 138 of them are given.

Below, we determine which of these 138 intuitionistic fuzzy implications satisfy (1) and (2).

Theorem 1. Implications $\to_2, \to_3, \to_4, \to_5, \to_{12}, \to_{13}, \to_{14}, \to_{16}, \to_{18}, \to_{19}, \to_{20}, \to_{22}, \to_{23}, \to_{25}$

, $\to_{26}$, $\to_{27}$, $\to_{28}$, $\to_{29}$, $\to_{31}$, $\to_{32}$, $\to_{33}$, $\to_{34}$, $\to_{35}$, $\to_{37}$, $\to_{40}$, $\to_{41}$, $\to_{42}$, $\to_{43}$, $\to_{44}$, $\to_{45}$, $\to_{47}$, $\to_{48}$, $\to_{49}$, $\to_{50}$, $\to_{52}$, $\to_{55}$, $\to_{56}$, $\to_{57}$, $\to_{58}$, $\to_{59}$, $\to_{60}$, $\to_{61}$, $\to_{64}$, $\to_{66}$, $\to_{67}$, $\to_{69}$, $\to_{70}$, $\to_{71}$, $\to_{72}$, $\to_{73}$, $\to_{74}$, $\to_{76}$, $\to_{77}$, $\to_{78}$, $\to_{79}$, $\to_{80}$, $\to_{81}$, $\to_{82}$, $\to_{83}$, $\to_{84}$, $\to_{85}$, $\to_{86}$, $\to_{87}$, $\to_{88}$, $\to_{89}$, $\to_{90}$, $\to_{91}$, $\to_{92}$, $\to_{93}$, $\to_{94}$, $\to_{95}$, $\to_{96}$, $\to_{97}$, $\to_{98}$, $\to_{99}$, $\to_{102}$, $\to_{105}$, $\to_{108}$, $\to_{124}$, $\to_{125}$, $\to_{127}$, $\to_{129}$, $\to_{130}$, $\to_{132}$, $\to_{134}$, $\to_{135}$, $\to_{137}$ satisfy (1).

Proof. Below, we prove that (1) is valid for implication $\to_3$. The rest assertions are proved by analogy. Let everywhere below, truth-values of p, q, r be:

$$V(p) = \langle a, b\rangle,$$

$$V(q) = \langle c, d\rangle,$$

$$V(r) = \langle e, f\rangle.$$

In [1], implication $\to_3$ is Gödel's implication, that has the form:

$$V(p \to_3 q) = \langle a, b\rangle \to_3 \langle c, d\rangle = \langle 1-(1-c).\text{sg}(a-c), d.\text{sg}(a-c)\rangle,$$

where

$$\text{sg}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}.$$

There it is defined that

$$V(p\&q) = \langle \min(a, c), \max(b, d)\rangle,$$

$$V(p \vee q) = \langle \max(a, c), \min(b, d)\rangle.$$

Therefore, the expression (1) has the form

$$(p \vee q) \to r = (p \to r)\&(q \to r)$$

or

$$(\langle a, b\rangle \vee \langle c, d\rangle) \to_3 \langle e, f\rangle = (\langle a, b\rangle \to_3 \langle e, f\rangle)\&(\langle c, d\rangle \to_3 \langle e, f\rangle).$$

The left side has the form

$$(\langle a, b\rangle \vee \langle c, d\rangle) \to_3 \langle e, f\rangle$$
$$= \langle \max(a, c), \min(b, d)\rangle \to_3 \langle e, f\rangle$$
$$= \langle 1-(1-e).\text{sg}(\max(a, c)-e), f.\text{sg}(\max(a, c)-e)\rangle.$$

The right side has the form

$$(\langle a, b\rangle \to_3 \langle e, f\rangle)\&(\langle c, d\rangle \to_3 \langle e, f\rangle)$$

$$= \langle 1-(1-e).\text{sg}(a-e), f.\text{sg}(a-e)\rangle\&\langle 1-(1-e).\text{sg}(c-e), f.\text{sg}(c-e)\rangle$$

$$= \langle \min(1-(1-e).\text{sg}(a-e), 1-(1-e).\text{sg}(c-e)), \max(f.\text{sg}(a-e), f.\text{sg}(c-e)\rangle$$

$$= \langle 1 - \max((1-e).\text{sg}(a-e), (1-e).\text{sg}(c-e)), f.\max(\text{sg}(a-e), \text{sg}(c-e)\rangle$$

$$= \langle 1 - (1-e).\max(\text{sg}(a-e), \text{sg}(c-e)), f.\max(\text{sg}(a-e), \text{sg}(c-e)\rangle.$$

Let

$$X \equiv (1 - (1-e).\text{sg}(\max(a,c)-e)) - (1 - (1-e).\max(\text{sg}(a-e), \text{sg}(c-e))).$$

Then

$$X = (1-e).\max(\text{sg}(a-e), \text{sg}(c-e)) - (1-e).\text{sg}(\max(a,c)-e)$$

$$= (1-e).(\max(\text{sg}(a-e), \text{sg}(c-e)) - \text{sg}(\max(a,c)-e)).$$

If $a \leq c$, then

$$X = (1-e).(\max(\text{sg}(a-e), \text{sg}(c-e)) - \text{sg}(c-e)) \geq 0.$$

If $a > e$, then

$$X = (1-e).(\max(1,1) - 1) = 0.$$

If $a \leq e < c$, then

$$X = (1-e).(\max(0,1) - \text{sg}(\max(c-e)) = (1-e).(1-1) = 0.$$

If $c \leq e$, then

$$X = (1-e).(\max(0,0) - \text{sg}(c-e) = (1-e).(0-0) = 0.$$

If $a > c$, then

$$X = (1-e).(\max(\text{sg}(a-e), \text{sg}(c-e)) - \text{sg}(a-e)).$$

If $a \leq e$, then

$$X = (1-e).(\max(0,0) - 0) = 0.$$

If $a > e \geq c$, then

$$X = (1-e).(\max(1,0) - 1) = 0.$$

If $a > c > e$, then

$$X = (1-e).(\max(1,1) - 1) = 0.$$

Therefore, in all cases $X = 0$.

Analogously, we check that

$$Y \equiv f.\text{sg}(\max(a,c)-e) - f.\max(\text{sg}(a-e), \text{sg}(c-e) = 0.$$

Hence, (1) is an equality for implication $\to_3$.

Theorem 2. Implications $\to_2$, $\to_3$, $\to_4$, $\to_5$, $\to_8$, $\to_{11}$, $\to_{12}$, $\to_{13}$, $\to_{16}$, $\to_{18}$, $\to_{19}$, $\to_{20}$, $\to_{22}$, $\to_{23}$, $\to_{25}$, $\to_{26}$, $\to_{27}$, $\to_{28}$, $\to_{29}$, $\to_{31}$, $\to_{32}$, $\to_{33}$, $\to_{34}$, $\to_{35}$, $\to_{37}$, $\to_{40}$, $\to_{41}$, $\to_{42}$, $\to_{43}$, $\to_{44}$, $\to_{45}$, $\to_{47}$, $\to_{48}$, $\to_{49}$, $\to_{50}$, $\to_{52}$, $\to_{55}$, $\to_{56}$, $\to_{57}$, $\to_{58}$, $\to_{59}$, $\to_{60}$, $\to_{61}$, $\to_{62}$, $\to_{63}$, $\to_{64}$, $\to_{65}$, $\to_{66}$, $\to_{67}$, $\to_{68}$, $\to_{70}$, $\to_{71}$, $\to_{72}$, $\to_{73}$, $\to_{74}$, $\to_{76}$, $\to_{77}$, $\to_{78}$, $\to_{79}$, $\to_{80}$, $\to_{81}$, $\to_{82}$, $\to_{83}$, $\to_{84}$, $\to_{85}$, $\to_{86}$, $\to_{87}$, $\to_{88}$,$\to_{89}$, $\to_{90}$, $\to_{91}$, $\to_{92}$, $\to_{93}$, $\to_{94}$, $\to_{95}$, $\to_{96}$, $\to_{97}$, $\to_{98}$, $\to_{99}$, $\to_{102}$, $\to_{105}$, $\to_{108}$, $\to_{124}$, $\to_{125}$, $\to_{127}$, $\to_{129}$, $\to_{130}$, $\to_{132}$, $\to_{134}$, $\to_{135}$, $\to_{137}$ satisfy (2).

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