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On intuitionistic fuzzy multi-dimensional sets. Part 4

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Abstract: Three specific operations, which are extensions of operations "$\cap$", "$\cup$" and "@", are introduced over intuitionistic fuzzy multi-dimensional sets and some of their properties are studied.
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1 Introduction

In [1, 2, 3], the Temporal Intuitionistic Fuzzy Sets (TIFs) were described, and in [4, 5, 6] – the Intuitionistic Fuzzy Multi-Dimentional Sets (IFMDSs) were introduced as extensions of the TIFSs.

Here, we will introduce new operations, specific for IFMDSs, and some of their properties will be studied. Initially, short remarks on IFMDSs will be given.

2 Short remarks on intuitionistic fuzzy multi-dimensional sets

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

For every two IFSs A and B a lot of relations and operations are defined (see, e.g. [2]), the most important of which are:

$$A \subseteq B \quad \text{if and only if} \quad (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x));$$

$$A = B \quad \text{if and only if} \quad (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x));$$

$$A \cap B = \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\};$$

$$A \cup B = \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\};$$

$$A@B = \{\langle x, \frac{\mu_A(x)+\mu_B(x)}{2}, \frac{\nu_A(x)+\nu_B(x)}{2}\rangle | x \in E\}.$$

Let

$$\mathcal{P}(X) = \{Y | Y \subseteq X\},$$

where relation $\subseteq$ is the above one.

Now, following [4], we shall introduce the concept of IFMDSs.

Let sets $Z_1, Z_2, ..., Z_n$ be fixed and let for each i $(1 \leq i \leq n) : z_i \in Z_i$.

Let set E be fixed. An IFMDS A in $E \times Z_1 \times Z_2 \times ... \times Z_n$ is an object of the form

$$A(Z_1, Z_2, ..., Z_n) = \{\langle x, \mu_A(x, z_1, z_2, ..., z_n), \nu_A(x, z_1, z_2, ..., z_n)\rangle |$$

$$\langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n\},$$

where:

(a) $\mu_A(x, z_1, z_2, ..., z_n) + \nu_A(x, z_1, z_2, ..., z_n) \leq 1$ for every $\langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n$,

(b) $\mu_A(x, z_1, z_2, ..., z_n)$ and $\nu_A(x, z_1, z_2, ..., z_n)$ are the degrees of membership and non-membership, respectively, of the element $\langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n$.

3 Main result

In [4], we mentioned that for every two IFMDSs A and B in $E \times Z_1 \times Z_2 \times ... \times Z_n$ all above relations, operations and operators can be defined by analogy. Here, we will discuss with more details the definitions of the operations over IFMDSs.

Really, when $A(Z_1, Z_2, ..., Z_n)$ and $B(Z_1, Z_2, ..., Z_n)$ are two IFMDSs, operations $\cup$, $\cap$ and $@$ have the following standard forms:

$$A(Z_1, Z_2, ..., Z_n) \cap B(Z_1, Z_2, ..., Z_n)$$

$$= \{\langle x, \min(\mu_A(x, z_1, z_2, ..., z_n), \mu_B(x, z_1, z_2, ..., z_n)), \max(\nu_A(x, z_1, z_2, ..., z_n),$$

$$\nu_B(x, z_1, z_2, ..., z_n))\rangle | \langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n\},$$

$$A(Z_1, Z_2, ..., Z_n) \cup B(Z_1, Z_2, ..., Z_n)$$

$$= \{\langle x, \max(\mu_A(x, z_1, z_2, ..., z_n), \mu_B(x, z_1, z_2, ..., z_n)), \min(\nu_A(x, z_1, z_2, ..., z_n), \nu_B(x, z_1, z_2, ..., z_n))\rangle | \langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n\},$$

$$A(Z_1, Z_2, ..., Z_n) @ B(Z_1, Z_2, ..., Z_n)$$
$$= \{\langle x, \frac{\mu_A(x, z_1, z_2, ..., z_n) + \mu_B(x, z_1, z_2, ..., z_n)}{2}, \frac{\nu_A(x, z_1, z_2, ..., z_n) + \nu_B(x, z_1, z_2, ..., z_n)}{2}\rangle$$
$$| \langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n\}.$$

Let the IFMDS A be defined over $E \times Z'_1 \times Z'_2 \times ... \times Z'_n$ and IFMDS B be defined over $E \times Z''_1 \times Z''_2 \times ... \times Z''_n$. Then, these three operations have the forms:

$$A(Z'_1, Z'_2, ..., Z'_n) \cap B(Z''_1, Z''_2, ..., Z''_n)$$
$$= \{\langle x, \mu_{A(Z'_1, Z'_2, ..., Z'_n) \cap B(Z''_1, Z''_2, ..., Z''_n)}(x, z_1, z_2, ..., z_n),$$
$$\nu_{A(Z'_1, Z'_2, ..., Z'_n) \cap B(Z''_1, Z''_2, ..., Z''_n)}(x, z_1, z_2, ..., z_n)\rangle$$
$$| \langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n\},$$

where

$$\langle x, \mu_{A(Z'_1, Z'_2, ..., Z'_n) \cap B(Z''_1, Z''_2, ..., Z''_n)}(x, z_1, z_2, ..., z_n), \nu_{A(Z'_1, Z'_2, ..., Z'_n) \cap B(Z''_1, Z''_2, ..., Z''_n)}(x, z_1, z_2, ..., z_n)\rangle$$

$$= \begin{cases} \langle x, \min(\mu_{A(Z'_1, Z'_2, ..., Z'_n)}(x, z_1, z_2, ..., z_n), \mu_{B(Z''_1, Z''_2, ..., Z''_n)}(x, z_1, z_2, ..., z_n)), \\ \quad \max(\nu_{A(Z'_1, Z'_2, ..., Z'_n)}(x, z_1, z_2, ..., z_n), \nu_{B(Z''_1, Z''_2, ..., Z''_n)}(x, z_1, z_2, ..., z_n))\rangle, \\ \quad \text{if } (\forall i : 1 \leq i \leq n)(z_i = z'_i = z''_i \in Z_i = Z'_i \cap Z''_i) \\ \langle x, 0, 1\rangle, \quad \text{otherwise} \end{cases}$$

$$A(Z'_1, Z'_2, ..., Z'_n) \cup B(Z''_1, Z''_2, ..., Z''_n)$$
$$= \{\langle x, \mu_{A(Z'_1, Z'_2, ..., Z'_n) \cup B(Z''_1, Z''_2, ..., Z''_n)}(x, z_1, z_2, ..., z_n),$$
$$\nu_{A(Z'_1, Z'_2, ..., Z'_n) \cup B(Z''_1, Z''_2, ..., Z''_n)}(x, z_1, z_2, ..., z_n)\rangle$$
$$| \langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n\},$$

where

$$\langle x, \mu_{A(Z'_1, Z'_2, ..., Z'_n) \cup B(Z''_1, Z''_2, ..., Z''_n)}(x, z_1, z_2, ..., z_n), \nu_{A(Z'_1, Z'_2, ..., Z'_n) \cup B(Z''_1, Z''_2, ..., Z''_n)}(x, z_1, z_2, ..., z_n)\rangle$$

$$
= \begin{cases}
\langle x, \mu_{A(Z'_1,Z'_2,...,Z'_n)}(x,z_1,z_2,...,z_n), \nu_{A(Z'_1,Z'_2,...,Z'_n)}(x,z_1,z_2,...,z_n)\rangle, \\
\qquad \text{if } (\forall i: 1 \leq i \leq n)(z_i = z'_i \in Z_i = Z'_i)\&(\exists i: 1 \leq i \leq n)(z_i = z'_i \in Z'_i - Z''_i); \\
\langle x, \mu_{B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n), \nu_{B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n)\rangle, \\
\qquad \text{if } (\forall i: 1 \leq i \leq n)(z_i = z''_i \in Z_i = Z''_i)\&(\exists i: 1 \leq i \leq n)(z_i = z''_i \in Z''_i - Z'_i); \\
\langle x, \max(\mu_{A(Z'_1,Z'_2,...,Z'_n)}(x,z_1,z_2,...,z_n), \mu_{B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n)), \\
\quad \min(\nu_{A(Z'_1,Z'_2,...,Z'_n)}(x,z_1,z_2,...,z_n), \nu_{B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n))\rangle, \\
\qquad \text{if } (\forall i: 1 \leq i \leq n)(z_i = z'_i = z''_i \in Z_i = Z'_i \cap Z''_i); \\
\langle x, 0, 1\rangle, \quad \text{otherwise}.
\end{cases}
$$

$$
\begin{aligned}
& A(Z'_1,Z'_2,...,Z'_n)@B(Z''_1,Z''_2,...,Z''_n) \\
&= \{\langle x, \mu_{A(Z'_1,Z'_2,...,Z'_n)@B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n), \\
&\quad \nu_{A(Z'_1,Z'_2,...,Z'_n)@B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n)\rangle \\
&\quad |\langle x,z_1,z_2,...,z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n\},
\end{aligned}
$$

where

$$
\langle x, \mu_{A(Z'_1,Z'_2,...,Z'_n)@B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n), \nu_{A(Z'_1,Z'_2,...,Z'_n)@B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n)\rangle
$$

$$
= \begin{cases}
\langle x, \frac{\mu_{A(Z'_1,Z'_2,...,Z'_n)}(x,z_1,z_2,...,z_n)}{2}, \frac{\nu_{A(Z'_1,Z'_2,...,Z'_n)}(x,z_1,z_2,...,z_n)+1}{2}\rangle, \\
\qquad \text{if } (\forall i: 1 \leq i \leq n)(z_i = z'_i \in Z_i = Z'_i)\&(\exists i: 1 \leq i \leq n)(z_i = z'_i \in Z'_i - Z''_i); \\
\langle x, \frac{\mu_{B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n)}{2}, \frac{\nu_{B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n)+1}{2}\rangle, \\
\qquad \text{if } (\forall i: 1 \leq i \leq n)(z_i = z''_i \in Z_i = Z''_i)\&(\exists i: 1 \leq i \leq n)(z_i = z''_i \in Z''_i - Z'_i); \\
\langle x, \frac{\mu_{A(Z'_1,Z'_2,...,Z'_n)}(x,z_1,z_2,...,z_n)+\mu_{B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n)}{2}, \\
\quad \frac{\nu_{A(Z'_1,Z'_2,...,Z'_n)}(x,z_1,z_2,...,z_n)+\nu_{B(Z''_1,Z''_2,...,Z''_n)}(x,z_1,z_2,...,z_n)}{2}\rangle, \\
\qquad \text{if } (\forall i: 1 \leq i \leq n)(z_i = z'_i = z''_i \in Z_i = Z'_i \cap Z''_i); \\
\langle x, 0, 1\rangle, \quad \text{otherwise}.
\end{cases}
$$

Let us define

$$
E^* = \{\langle x, 1, 0\rangle | x \in E\}.
$$

For the case of IFMDSs over $E \times Z_1 \times Z_2 \times ... \times Z_n$:

$$\mathcal{P}(E^*) = \{A(Z_1, Z_2, ..., Z_n) \mid A(Z_1, Z_2, ..., Z_n)$$

$$= \{\langle x, \mu_{A(Z_1,Z_2,...,Z_n)}(x, z_1, z_2, ..., z_n), \nu_{A(Z_1,Z_2,...,Z_n)}(x, z_1, z_2, ..., z_n)\rangle$$

$$\mid \langle x, z_1, z_2, ..., z_n\rangle \in E \times Z_1 \times Z_2 \times ... \times Z_n\}\}.$$

Theorem. For a fixed universe E,

1. $\langle \mathcal{P}(E^*), \cap, E^*\rangle$ is a commutative monoid;
2. $\langle \mathcal{P}(E^*), \cup, O^*\rangle$ is a commutative monoid;
3. $\langle \mathcal{P}(E^*), @\rangle$ is a groupoid;
4. None of these objects is a (commutative) group.

When $n = 1$ and $Z_1 = T$, where T is a fixed set with finite number of distinct time-elements or it is a time-interval, the IFMDS is reduced to a Temporal IFS (TIFS). It has the form

$$A(T) = \{\langle x, \mu_A(x, t), \nu_A(x, t)\rangle | \langle x, t\rangle \in E \times T\}.$$

Suppose that we have two TIFSs:

$$A(T') = \{\langle x, \mu_A(x, t), \nu_A(x, t)\rangle | \langle x, t\rangle \in E \times T'\},$$

and

$$B(T'') = \{\langle x, \mu_B(x, t), \nu_B(x, t)\rangle | \langle x, t\rangle \in E \times T''\},$$

where T' and T'' have the above sense. Then, the above operations obtain the following forms

$$A(T') \cap B(T'') = \{\langle x, \mu_{A(T')\cap B(T'')}(x, t), \nu_{A(T')\cap B(T'')}(x, t)\rangle | \langle x, t\rangle \in E \times T\},$$

where

$$\langle x, \mu_{A(T')\cap B(T'')}(x, t), \nu_{A(T')\cap B(T'')}(x, t)\rangle$$

$$= \begin{cases} \langle x, \min(\mu_A(x, t'), \mu_B(x, t'')), \max(\nu_A(x, t'), \nu_A(x, t''))\rangle, \\ \qquad\qquad\qquad\qquad \text{if } t = t' = t'' \in T = T' \cap T''; \\ \\ \langle x, 0, 1\rangle, \qquad\qquad\quad \text{otherwise.} \end{cases}$$

$$A(T') \cup B(T'') = \{\langle x, \mu_{A(T')\cup B(T'')}(x, t), \nu_{A(T')\cup B(T'')}(x, t)\rangle | \langle x, t\rangle \in E \times T\},$$

where

$$\langle x, \mu_{A(T')\cup B(T'')}(x, t), \nu_{A(T')\cup B(T'')}(x, t)\rangle$$

$$= \begin{cases} \langle x, \mu_A(x, t'), \nu_A(x, t')\rangle, & \text{if } t = t' \in T = T' - T'' \\ \\ \langle x, \mu_B(x, t''), \nu_A(x, t'')\rangle, & \text{if } t = t'' \in T = T'' - T' \\ \\ \langle x, \max(\mu_A(x, t'), \mu_B(x, t'')), \min(\nu_A(x, t'), \nu_A(x, t''))\rangle, \\ & \text{if } t = t' = t'' \in T = T' \cap T'' \\ \\ \langle x, 0, 1\rangle, & \text{otherwise.} \end{cases}$$

$$A(T')@B(T'') = \{\langle x, \mu_{A(T')@B(T'')}(x,t), \nu_{A(T')@B(T'')}(x,t)\rangle | \langle x,t\rangle \in E \times T\},$$

where

$$\langle x, \mu_{A(T')\cup B(T'')}(x,t), \nu_{A(T')\cup B(T'')}(x,t)\rangle$$

$$= \begin{cases} \langle x, \frac{\mu_A(x,t')}{2}, \frac{\nu_A(x,t')+1}{2}\rangle, & \text{if } t = t' \in T = T' - T''; \\ \langle x, \frac{\mu_B(x,t'')}{2}, \frac{\nu_A(x,t'')}{2}\rangle, & \text{if } t = t'' \in T = T'' - T'; \\ \langle x, \frac{\mu_A(x,t')+\mu_B(x,t'')}{2}, \frac{\nu_A(x,t')+\nu_A(x,t'')}{2}\rangle, & \text{if } t = t' = t'' \in T = T' \cap T''; \\ \langle x, 0, 1\rangle, & \text{otherwise.} \end{cases}$$

Operations $\cup$ and $\cap$ over $A(T')$ and $B(T'')$ coincide with these from [7].

In a next research of the authors, different operations "negation" will be introduced over IFMDSs.

4 Conclusion

The so constructed IFMDSs can have different applications. For example, they can be used in procedures for decision making. Now, we can have a universe of objects for estimation E and sets of values of the estimations - $Z_1, Z_2, ..., Z_n$ given by different experts. They will generate a space of values that can be interpreted by an IFMDS in which, using, e.g. the new topological operators, we can find the extremal values.

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