Notes on Intuitionistic Fuzzy Sets
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An application of a strong and complete intuitionistic fuzzy graph by the concept of fuzzy coloring

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Abstract: The hypothesis of fuzzy coloring to a strong and complete intuitionistic fuzzy graph is conferred with a real time application. The conduction of the Academic Leadership Committee Meeting of a higher educational institution is illustrated using the concept of fuzzy coloring.
Keywords: Fuzzy coloring, Intuitionistic fuzzy graph, Panel members.
2020 Mathematics Subject Classification: 05C72, 05C90.

1 Introduction

Coloring of a Graph has been a tremendous concept originated from the year 1852 by Francis Guthrie [6]. Then in the mid 1960's, Behzad and Vizing introduced independently the concept of total coloring [4, 14]. Graph theory plays a significant role in applications of enormous fields like telecommunication, scheduling, operations research, transportation, networking, weather forecasting, medical diagnosis, etc. Graph coloring helps in the pavement of finding the path in networking, scheduling, travelling, etc., implying with a greater visibility. Fuzzy graphs were introduced by Rosenfeld in 1975 [14] and after the introduction of intuitionistic fuzzy sets (IFSs) by Krassimir T. Atanassov in 1983 [1] (see also [2, 3]), Intuitionistic Fuzzy Graphs (IFGs) were introduced by Anthony Shannon and Atanassov in 1994 [16]. R. Parvathi *et al.* conferred about properties of IFGs in 2006 [12]. Coloring to fuzzy graphs was introduced by Changiz Eslahchi and B. N. Onagh in 2006 [7]. Coloring to IFGs was introduced by S. Ismail Mohideen and M. A. Rifayathali in 2015 [8]. Fuzzy total coloring to fuzzy graphs was introduced by S. Lavanya and R. Sattanathan in 2009 [10]. Fuzzy coloring concept to fuzzy graphs was introduced by Sovan Samanta, Tarasankar Pramanik and Madhumangal Pal in the year 2015 [15]. The concepts of fuzzy coloring and total fuzzy coloring to IFGs were introduced by R. Buvaneswari and P. Revathy in the year 2023 [5].

In this paper, considering a strong and complete IFG, fuzzy coloring is incorporated to a real life application, scheduling a team meet with a graphical representation using IFGs.

2 Preliminaries

Definition 2.1. [16] An IFG of the form $\hat{G} = (V, E)$ comprises of:

1. $V = \{v_1, v_2, \ldots, v_n\} \ni \mu_i, \nu_i : V \to [0, 1]$ indicate the levels of membership and non-membership of the elements $\mu_i, \nu_i \in V$ and $0 \leq \mu_i + \nu_i \leq 1$, for all $v_i \in V$, $(i = 1, 2, \ldots, n)$.
2. $E \subseteq V \times V$ where $\mu_{ij} : V \times V \to [0, 1]$ and $v_{ij} : V \times V \to [0, 1] \ni \mu_{ij} \leq \min\{\mu_i, \mu_j\}$, $v_{ij} \leq \max\{v_i, v_j\}$ and $0 \leq \mu_{ij} + \nu_{ij} \leq 1$ for all $(\nu_i, \nu_j) \in E$.

Here, the triplets (v_i, μ_i, ν_i) and $(e_{ij}, \mu_{ij}, \nu_{ij})$ notate the levels of membership and non-membership of the vertices v_i and the edge relations $e_{ij} = (\nu_i, \nu_j) \in V \times V$ of an IFG.

Definition 2.2. [9] An IFG $\hat{G} = (V, E)$ is a strong IFG if $\mu_{ij} = \min(\mu_i, \mu_j)$ and $\nu_{ij} = \max(\nu_i, \nu_j)$ for every $\nu_i, \nu_j \in E$.

Definition 2.3. [9] An IFG $\hat{G} = (V, E)$ is a complete IFG if $\mu_{ij} = \min(\mu_i, \mu_j)$ and $\nu_{ij} = \max(\nu_i, \nu_j)$ for every $\nu_i, \nu_j \in V$.

Definition 2.4. [10] The collection $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k\}$ are fuzzy sets in V is known as the k-fuzzy coloring of $\hat{G} = (V, E)$ when

1. $\vee\Gamma = V$
2. $\gamma_i \wedge \gamma_j = \varnothing$
3. for all xy, strong edges of $\hat{G}$, $\wedge\{\gamma_i(x), \gamma_j(y)\} = \varnothing, (1 \leq i \leq k)$.

Definition 2.5. [13] A collection $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k\}$ of IFS on a set V is known as the k-vertex coloring of $\hat{G} = (V, E)$ when

1. $\vee\gamma_i(x) = V$, for all $x \in V$
2. $\gamma_i \wedge \gamma_j = \varnothing$
3. for all xy strong edges of $\hat{G}$, $\min\{\gamma_i(\mu(x)), \gamma_i(\mu(y))\} = 0; \max\{\gamma_i(v(x)), \gamma_i(v(y))\} = 1$, $(1 \leq i \leq k)$.

The nominal utility of k of $\hat{G}$ possessing k-vertex coloring is notated as $\chi(\hat{G})$ gives the vertex chromatic number of an IFG $\hat{G}$.

Definition 2.6. [13] A collection $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k\}$ of IFS on E is known as the k-edge coloring of $\hat{G} = (V, E)$ whereas

1. $\vee\gamma_i(xy) = E$, for all $xy \in E$
2. $\gamma_i \wedge \gamma_j = \varnothing$
3. for all xy, incident edges of E, $\min\{\gamma_i(\mu(xy))\} = 0;\ \max\{\gamma_i(v(xy))\} = 1,\ (1 \le i \le k)$.

The nominal utility of k of $\hat{G}$ possessing k-edge coloring is indicated as $\chi'(\hat{G})$, gives the edge chromatic number of an IFG $\hat{G}$.

Definition 2.7. [13] A collection $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k\}$ of IFS on V and E is known as the k-total coloring of $\hat{G} = (V, E)$ whereas

1. $\vee\gamma_i(x) \vee \gamma_i(xy) = V \vee E$ for all $x \in V,\ xy \in E$
2. $\gamma_i \wedge \gamma_j = \varnothing$
3. for all xy, strong and incident vertices and edges of $\hat{G}$, $\min\{\gamma_i(\mu(x)), \gamma_i(\mu(y)), \gamma_i(\mu(xy))\}$ $= 0;\ \max\{\gamma_i(v(x)), \gamma_i(v(y)), \gamma_i(v(xy))\} = 1, (1 \le i \le k)$.

In $\hat{G}$, the nominal utility of k possessing a k-total coloring is indicated as $\chi^T(\hat{G})$ gives the total chromatic number of an IFG $\hat{G}$.

Definition 2.8. [5] An ordered pair $I\left[\chi_F(\hat{G})\right] = \{x, (C_k, 1)\}$ is said to be the fuzzy vertex chromatic index, where x is the fuzzy vertex chromatic number denoting the least number of distinct fuzzy colors C_k required in coloring the vertices of an IFG $\hat{G} = (V, E)$.

Definition 2.9. [5] An ordered pair $I\left[\chi'_F(\hat{G})\right] = \{x', (C_k, 1)\}$ is said to be the fuzzy edge chromatic index, whereas x' denotes fuzzy edge chromatic number implying the least number of distinct fuzzy colors C_k required in coloring the edges of an IFG $\hat{G} = (V, E)$.

Definition 2.10. [5] An ordered pair $I\left[\chi^T_F(\hat{G})\right] = \left\{x^T, (C_k, 1)\right\}$ is said to be the total fuzzy chromatic index, whereas x^T denotes total fuzzy chromatic number implying the least number of distinct fuzzy colors C_k required in coloring the vertices and edges of an IFG $\hat{G} = (V, E)$.

3 Fuzzy coloring of an intuitionistic fuzzy graph

Combining any two colors gives a new color. White color, when combined with a distinct color, a new color is not obtained, but rather the solidity (reliability) of the distinct color reduces to some extent. Thus, the strength of darkness of the color decreases. The term solidity implies the fuzzy term. Generally, when it is a graph, a fuzzy graph or an IFG, no adjacent vertices and its edges can be colored using the same color. In accordance to the fuzzy coloring concept, by reducing the solidity of the color, the adjoining vertices are colored utilizing the same color if the edges are weak or there exist no edges between the two vertices. The fuzzy color (F_C) is used in coloring the vertices and edges of an IFG. Considering a distinct color, namely C_K and $(\omega \leq 1)$ components of color C_K is blended together with an $(1 - \omega)$ components of the white color, then the obtained blend is called the fusion of the color C_K, and its consequential shade is called F_C of color C_K accompanied with the membership value ω.

Definition 3.1. [15] The collection of the distinct colors is $C = \sum_{i=1}^{n}(c_i), n = 1$. The collection of F_C is the fuzzy set (F_C, f) where $f : C \to [0, 1]$, along $f(c_i)$, which is the membership value of a color c_i, which denotes the quantity for every component of the fusion obtained. A color $\tilde{c}_i = (c_i, f(c_i))$ gives F_C correlating with distinct color c_i. In the blend of F_C, the membership value of the distinct color is equal to 1.

F_C's can be formed more in number with any one distinct color. Different measures of white color yields different fuzzy colors of the distinct color considered. For example, if blue is the color accounted, a "fuzzy blue" color is easily formed by blending 0.9 components of blue color and 0.1 component of the color white. The "fuzzy blue" color shall be notated as (B,0.9) where 'B' denotes the "Blue" color. Also, another fuzzy blue color (B,0.8) is formed by blending 0.8 components of blue color and 0.2 components of white color and so on. The fuzzy blue color represented by (B,1) denotes the fuzzy blue color's maximum solidity. By reducing the solidity, a larger number of fuzzy colors can be formed.

4 Procedure of fuzzy coloring to a strong and complete intuitionistic fuzzy graphs

In fuzzy coloring concept, a crisp graph, fuzzy graph or an IFG consisting of strong and complete edges, fuzzy coloring is accomplished with the full solidity of the distinct fuzzy colors, namely, (G,1) and (B,1).

Strong and complete intuitionistic fuzzy graphs

In a strong and complete IFG, all the vertices are connected to all the other vertices [5], as depicted in Figure 1.

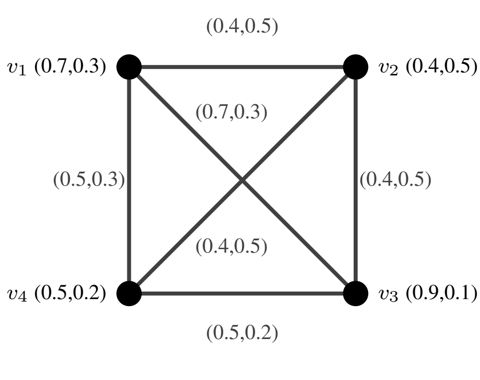

Figure 1. Strong and complete IFG

In accordance with the fuzzy coloring and total fuzzy coloring concept, Figures 2 (a), (b) and (c) imply the fuzzy vertex chromatic number as $\chi_F(\hat{G}) = 2$ along with the fuzzy vertex chromatic index $I\left[\chi_F(\hat{G})\right] = \{2, (B, 1), (R, 1)\}$, the fuzzy edge chromatic number to be $\chi'_F(\hat{G}) = 4$ along with the fuzzy edge chromatic index $I\left[\chi_F{}'(\hat{G})\right] = \{4, (R, 1), (G, 1), (Y, 1), (B, 1)\}$, and then the total fuzzy chromatic number to be $\chi_F{}^T(\hat{G}) = 4$ along with the total fuzzy chromatic index $I\left[\chi_F{}^T(\hat{G})\right] = \{4, (R, 1), (G, 1), (Y, 1), (B, 1)\}$.

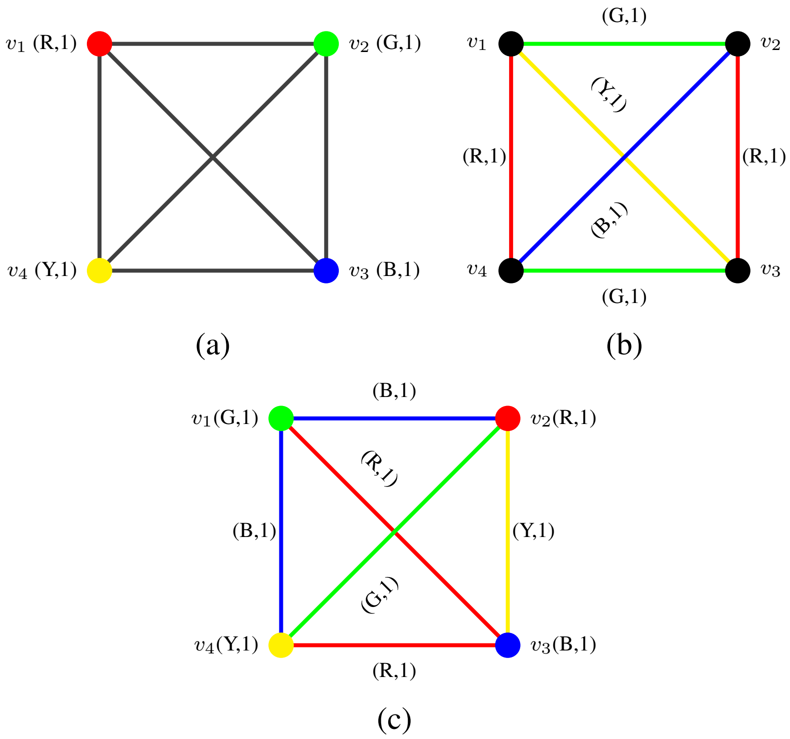

Figure 2. Fuzzy coloring of a strong and complete IFG
(a) Fuzzy vertex coloring (b) Fuzzy edge coloring (c) Total fuzzy coloring

The concept of a strong and complete IFG with the fuzzy coloring approach plays a significant role in real time applications like scheduling the team meetings. One of the practical applications is the conduction of the "Academic Leadership Committee Meeting" of a higher educational institution, discussed in this paper.

5 Example: Academic Leadership Committee Meeting

In the example below, the Academic Leadership Committee Meeting of a higher educational institution, namely, the XYZ College of Engineering and Technology has seven panel members as follows:

1. Dr. K. Jim – Principal, Head of the Institution (KJ)
2. Dr. S. Harry – Dean, Research and Development (SH)
3. Dr. B. John – Dean, Placement (BJ)

4. Dr. V. Rose – Dean, Student Affairs (VR)
5. Dr. J. Jack – Dean, Internal Quality Assurance Cell (JJ)
6. Dr. A. Danny – Head of Department of Science and Humanities (AD)
7. Dr. V. Nick – Dean, Innovation and Head of Department of Mechanical Engineering (VN)

It has six panels with the aforementioned panel members:

a) Academic Affairs (AA): KJ, SH, VR, JJ, AD
b) Research and Development (RD): KJ, SH, JJ, VN
c) Placement Coordination (PC): BJ, VR, JJ
d) Student Affairs (SA): KJ, VR, AD, VN
e) Internal Quality Assurance Cell (IQAC): KJ, SH, JJ
f) Hostel Affairs (HA): KJ, VR, AD, VN

When the Academic Leadership Committee plans for a team meeting, first and foremost, every individual panel meets with their panel members and, if required, two or more panels connect with their applicable other panels, followed by the "Academic Leadership Committee" where all panels meet together to proceed over the suggestions of each panel and take resolutions. Two panels cannot meet at a stretch at the same time, when they have the members in common and the panels without common panel members can meet at a stretch [11]. This is a usual process of execution during the conduction of a meeting. The situation of a conduction of the Academic Leadership Committee meeting can be considered as an intuitionistic fuzzy case as it is influenced by turbulent factors such as complexity in executions and hesitations.

This scenario can be graphically represented as the IFG depicted in Figure 3, pertaining the vertices to be the panels' names and the panel members in common are connected by the adjacent edges.

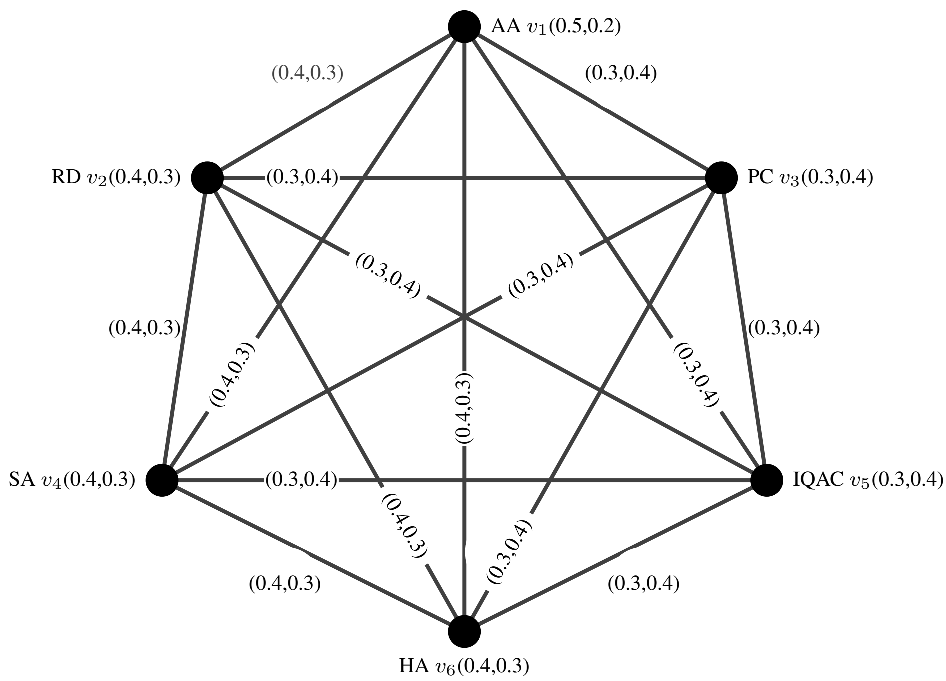

Figure 3. IFG representation of Academic Leadership Committee

Since all the panels are commonly connected under the "Academic Leadership Committee", each panel is strong and the "Academic Leadership Committee" is completely connected to all the panels. This is graphically represented by the IFG in Figure 4.

In accordance with the fuzzy coloring perception, Figure 4 imply the representation of the panels showing the fuzzy vertex chromatic number as $\chi_F(\hat{G}) = 6$ along with the fuzzy vertex chromatic index $I\left[\chi_F(\hat{G})\right] = \{6,\ (R,1)\,, (Y,1)\,, (P,1)\,, (G,1)\,, (B,1)\,, (BL,1)\}$.

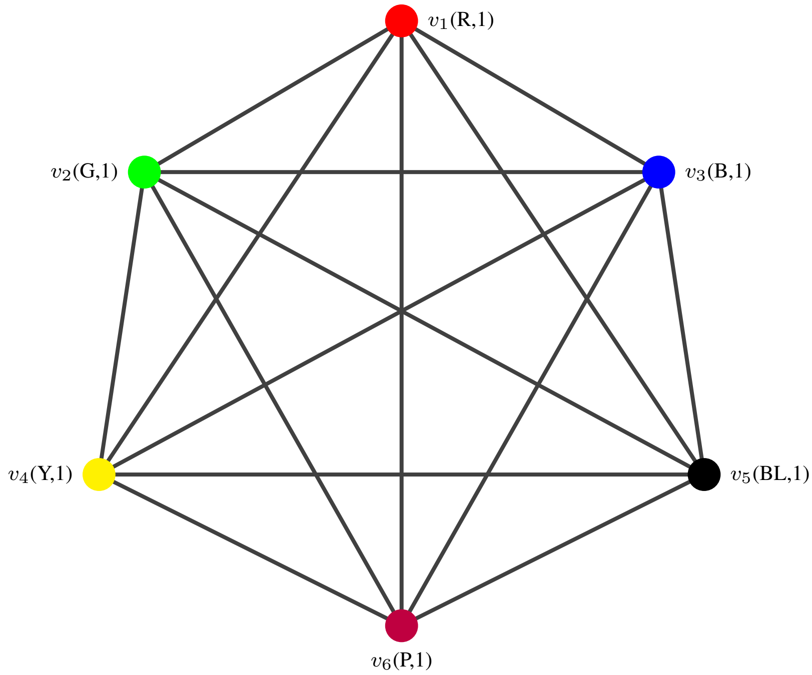

Figure 4. Panel connectivity of Academic Leadership Committee

Now, to show the possibility of the panels who are having panel members in common, it is depicted graphically as an IFG and also by shading the adjacent edges with the fuzzy colors by reducing its solidity, for easy identification for the conduct of the meet smartly, which results in time saving. Here, the panels, namely, Student Affairs and Hostel Affairs have the same four panel members where they can meet at the same time and complete the discussions regarding both the panels and arrive decisions and save time, too. To portray this, an adjacent edge connecting the panels (vertices) v_4 and v_6 is to be shaded with strong fuzzy color pertaining to the maximum solidity, for example (R,1). Then with the count of the common panel members in the remaining panels, the edges connecting the panels can be shaded with the next levels of solidity reduced fuzzy colors, namely, (R,$0.a$), (R,$0.b$), (R,$0.c$) and so on. The decision to call upon the two panels at a same time is arrived quick by an IFG representation as depicted in Figure 5 with the utilization of fuzzy colors.

In accordance with the fuzzy coloring concept, Figure 5 implies the fuzzy edge chromatic number to be ${\chi_F}'(\hat{G}) = 6$ along with the fuzzy edge chromatic index,

$$I\left[{\chi_F}'(\hat{G})\right] = \{6, (R,1)\,, (G,0.b)\,, (B,0.b)\,, (P,0.b)\,, (BL,0.a)\,, (Y,0.a)\}.$$

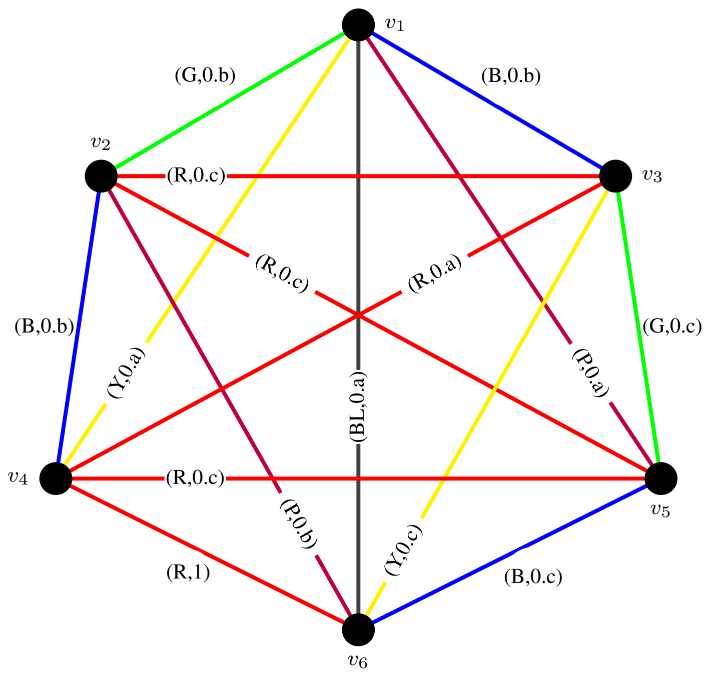

Figure 5. Scheduling of Academic Leadership Committee meeting

This graphical representation of fuzzy colored IFG implying the fuzzy vertex and the fuzzy edge chromatic number along with their indices portrays the easy accessibility with which panels can all meet together at the same time and how many panel members will be in common and come to a quick decision to schedule the meetings. However, each panel conducting a meeting individually at different schedules does not require this. This paper depicts the problem of analyzing the possibility of two or more panels to meet at a same time stretch for an enhancement of a quicker task completion.

6 Conclusion

In this paper, the real time application of conduction of the "Academic Leadership Committee Meeting" of a higher educational institution is discussed with the concept of fuzzy coloring correlating with the case of a strong and complete IFG. The authors propose to work further on more real time applications using the fuzzy coloring approach.

References

[1] Atanassov, K. T. (1983). Intuitionistic fuzzy sets. *VII ITKR's Session*, Sofia, June 1983 (Deposed in Central Sci. - Techn. Library of Bulg. Acad. of Sci., 1697/84) (in Bulgarian). Reprinted: *Int. J. Bioautomation*, 2016, 20(S1), S1–S6.

[2] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96.

[3] Atanassov, K. T. (1999). *Intuitionistic Fuzzy Sets: Theory and Applications*, Springer Physica-Verlag.

[4] Behzad. M. (1965). *Graphs and Their Chromatic Numbers.* Ph.D Thesis, Michigan State University, East Lansing.

[5] Buvaneswari, R., & Revathy, P. (2023). Fuzzy coloring and total fuzzy coloring of various types of intuitionistic fuzzy graphs. *Notes on Intuitionistic Fuzzy Sets*, 29(4), 383–400.

[6] Chartrand, G., & Zhang, P. (2009). *Chromatic Graph Theory*. CRC Press, A Chapman and Hall Book.

[7] Eslahchi, C., & Onagh, B. N. (2006). Vertex-strength of fuzzy graphs. *International Journal of Mathematics and Mathematical Sciences*, 1, 1–9.

[8] Ismail Mohideen, S., & Rifayathali, M. A. (2015). Coloring of intuitionistic fuzzy graph using (α, β) -cuts. *International Research Journal of Mathematics, Engineering and IT*, 2(12), 14–26.

[9] Karunambigai, M. G., Parvathi, R., & Buvaneswari, R. (2011). Constant intuitionistic fuzzy graphs. *Notes on Intuitionistic Fuzzy Sets,* 17(1), 37–47.

[10] Lavanya, S., & Sattanathan, R. (2009). Fuzzy total coloring of fuzzy graphs. *International Journal of Information Technology and Knowledge Management*, 2(1), 37–39.

[11] Nivethana, V., & Parvathi, A. (2013). Fuzzy total coloring and chromatic number of a complete fuzzy graph. *International Journal of Emerging Trends in Engineering and Development*, 6(3), 377–384.

[12] Parvathi, R., & Karunambigai, M. G. (2006). Intuitionistic fuzzy graphs. In: Reusch, B. (ed.). *Computational Intelligence, Theory and Applications*, Vol 38., 139–150. Springer, Berlin, Heidelberg.

[13] Prasanna, A., Rifayathali, M. A., & Ismail Mohideen, S. (2017). Strong intuitionistic fuzzy graph coloring. *International Journal of Latest Engineering Research and Applications (IJLERA)*, 2(8), 163–169.

[14] Rosenfeld, A. (1975). Fuzzy graphs. *Academic Press*, 77–95.

[15] Samanta, S., Pramanik, T., & Pal, M. (2015). Fuzzy coloring of fuzzy graphs. *Afrika Matematika*, 27, 37–50.

[16] Shannon, A., & Atanassov, K. T. (1994). A first step to a theory of the intuitionistic fuzzy graphs. *Proceedings of the First Workshop on Fuzzy Based Expert Systems (D. Lakov, Ed.)*, Sofia, 28–30 September 1994, 59–61.

[17] Vizing, V. G. (1963). The cartesian product of graphs.*Vychislitel'nye Sistemy*, 9, 30–43.