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On 3-dimensional intuitionistic fuzzy index matrices

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Abstract: An extension of the concept of an Intuitionistic Fuzzy Index Matrix (IFIM) is introduced, called 3-dimensional Intuitionistic Fuzzy Index Matrix (3D-IFIM). In this paper will be introduced different operations over 3D-EIMs.
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1 Introduction

In [3, 5], 3-dimensional extended index matrix (3D-EIM) are introduced. Following this concept in this paper will be defined 3 dimensional Intuitionistic Fuzzy Index Matrix (3D-IFIM) and some operations over them will be introduced. Initially, in Section 2, will be given some elements of Intutionistic Fuzzy Logic (see [1, 4]) and some remarks over index matrices. In section 3 will be defined 3D-IFIM. In section 4 some operations over them will be introduced.

2 Basic definitions

Let us started with some remarks on Intuitionistic Fuzzy Logic from [1, 4]. The Intuitionistic Fuzzy Pair (IFP) is an object with the form $\langle a, b\rangle$, where $a, b \in [0, 1]$ and $a + b \leq 1$, that is used as an evaluation of some object or process. Its components (a and b) are interpreted as degrees of membership and non-membership, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc. Let us have two IFPs $x = \langle a, b\rangle$ and $y = \langle c, d\rangle$.

In [4] were defined following operations:

$$\neg x = \langle b, a\rangle$$
$$x \& y = \langle \min(a,c), \max(b,d)\rangle$$
$$x \vee y = \langle \max(a,c)), \min(b,d)\rangle$$
$$x + y = \langle a + c - a.c, b.d\rangle$$
$$x.y = \langle a.c, b + d - b.d\rangle$$
$$x@y = \langle \tfrac{a+c}{2}, \tfrac{b+d}{2}\rangle.$$

In [1], definitions of 138 operations "implication" and 34 operations "negation" are given – the simplest one is given above.

Let a set E be fixed. An Intuitionistic Fuzzy Set (IFS) A in E is an object of the following form (see, e.g., [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

In [3] was defined Intuituionistic Fuzzy Index Matrix (IFIM). Let $\mathcal{I}$ be a fixed set of indices. By IFIM with index sets K and L $(K, L \subset I)$, we denote the object:

$$[K, L, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j}\rangle\}]$$

$$\equiv \begin{array}{c|ccccc} & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1}\rangle & \dots & \langle \mu_{k_1,l_j}, \nu_{k_1,l_j}\rangle & \dots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n}\rangle \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_i & \langle \mu_{k_i,l_1}, \nu_{k_i,l_1}\rangle & \dots & \langle \mu_{k_i,l_j}, \nu_{k_i,l_j}\rangle & \dots & \langle \mu_{k_i,l_n}, \nu_{k_i,l_n}\rangle \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1}\rangle & \dots & \langle \mu_{k_m,l_j}, \nu_{k_m,l_j}\rangle & \dots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n}\rangle \end{array},$$

where for every $1 \le i \le m, 1 \le j \le n$: $0 \le \mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} \le 1$.

In [3, 2] the concept of a 3D-IM is introduced, as follows.

Let $\mathcal{I}$ be a fixed set of indices and $\mathcal{X}$ be a fixed set of objects. Following [2], we call "3D-IM" with index sets K, L and M $(K, L, H \subset \mathcal{I})$ the object:

$$[K, L, H, \{a_{k_i,l_j,h_g}\}]$$

$$\equiv \left\{ \begin{array}{c|ccccc} h_g & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & a_{k_1,l_1,h_g} & \vdots & a_{k_1,l_j,h_g} & \dots & a_{k_1,l_n,h_g} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & a_{k_m,l_1,h_g} & \dots & a_{k_m,l_j,h_g} & \dots & a_{k_m,l_n,h_g} \end{array} | h_g \in H \right\}$$

$$\equiv \left\{ \begin{array}{c|ccccc} h_1 & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & a_{k_1,l_1,h_1} & \vdots & a_{k_1,l_j,h_1} & \dots & a_{k_1,l_n,h_1} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & a_{k_m,l_1,h_1} & \dots & a_{k_m,l_j,h_1} & \dots & a_{k_m,l_n,h_1} \end{array}, \right.$$

$$\begin{array}{c|ccccc} h_2 & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & a_{k_1,l_1,h_2} & \vdots & a_{k_1,l_j,h_2} & \dots & a_{k_1,l_n,h_2} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & a_{k_m,l_1,h_2} & \dots & a_{k_m,l_j,h_2} & \dots & a_{k_m,l_n,h_2} \end{array},$$

$$\left. \dots, \quad \begin{array}{c|ccccc} h_f & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & a_{k_1,l_1,h_f} & \vdots & a_{k_1,l_j,h_f} & \dots & a_{k_1,l_n,h_f} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & a_{k_m,l_1,h_f} & \dots & a_{k_m,l_j,h_f} & \dots & a_{k_m,l_n,h_f} \end{array} \right\},$$

where $K = \{k_1, k_2, ..., k_m\}$, $L = \{l_1, l_2, ..., l_n\}$, $H = \{h_1, h_2, ..., h_f\}$, and for $1 \leq i \leq m$, $1 \leq j \leq n$, $1 \leq g \leq f$: $a_{k_i,l_j,h_g} \in \mathcal{X}$.

3 Definition of a 3-dimensional intuitionistic fuzzy index matrix

Now, we define a "3D-Intuitionistic Fuzzy Index Matrix" (3D-IFIM) with index sets K, L and H ($K, L, H \subset \mathcal{I}$):

$$[K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]$$

$$\equiv \left\{ \begin{array}{c|ccccc} h_g & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1,l_1,h_g}, \nu_{k_1,l_1,h_g} \rangle & \dots & \langle \mu_{k_1,l_j,h_g}, \nu_{k_1,l_j,h_g} \rangle & \dots & \langle \mu_{k_1,l_n,h_g}, \nu_{k_1,l_n,h_g} \rangle \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_i & \langle \mu_{k_i,l_1,h_g}, \nu_{k_i,l_1,h_g} \rangle & \dots & \langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle & \dots & \langle \mu_{k_i,l_n,h_g}, \nu_{k_i,l_n,h_g} \rangle \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & \langle \mu_{k_m,l_1,h_g}, \nu_{k_m,l_1,h_g} \rangle & \dots & \langle \mu_{k_m,l_j,h_g}, \nu_{k_m,l_j,h_g} \rangle & \dots & \langle \mu_{k_m,l_n,h_g}, \nu_{k_m,l_n,h_g} \rangle \end{array} | h_g \in H \right\}$$

,

where for every $1 \leq i \leq m, 1 \leq j \leq n,\ 1 \leq g \leq f$:

$$0 \leq \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g}, \mu_{k_i,l_j,h_g} + \nu_{k_i,l_j,h_g} \leq 1.$$

4 Operations over 3D-IFIMs

Let $\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{U}$ be fixed sets. Let operations "$*$" and "$\circ$" be defined so that: $* : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$ and $\circ : \mathcal{Z} \times \mathcal{Z} \to \mathcal{U}$. Let the index set $\mathcal{I}$ be given. We will define some operations over the 3D-IFIMs $A = [K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]$ and $B = [P, Q, R, \{\langle \rho_{p_r,q_s,r_d}, \sigma_{p_r,q_s,r_d} \rangle\}]$.

4.1 Addition-$(\circ, *)$

$$A \oplus_{(\circ,*)} B = [K \cup P, L \cup Q, H \cup R, \{\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle\}],$$

where

$$\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle =$$

$$= \begin{cases} \langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle, & \text{if } t_u = k_i \in K, v_w = l_j \text{ and } x_y = h_g \in H - R \\ & \quad \text{or } t_u = k_i \in K, v_w = lj \in L - Q \text{ and } x_y = h_g \in H \\ & \quad \text{or } t_u = k_i \in K - P, v_w = lj \in L \text{ and } x_y = h_g \in H; \\ \\ \langle \rho_{p_r,q_s,r_d}, \sigma_{p_r,q_s,r_d} \rangle, & \text{if } t_u = p_r \in P, v_w = q_s \in Q \text{ and } x_y = rd \in R - H \\ & \quad \text{or } t_u = p_r \in P, v_w = q_s \in Q - L \text{ and } x_y = r_d \in R \\ & \quad \text{or } t_u = p_r \in P - K, v_w = q_s \in Q \text{ and } x_y = r_d \in R; \\ \\ \langle \circ(\mu_{k_i,l_j,h_g}, \rho_{p_r,q_s,r_d}), & \text{if } t_u = k_i = p_r \in K \cap P, v_w = l_j = q_s \in L \cap Q \\ \quad *(\nu_{k_i,l_j,h_g}, \sigma_{p_r,q_s,r_d}) \rangle, & \quad \text{and } x_y = h_g = r_d \in H \cap R \\ \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

where

$$\langle \circ, * \rangle \in \{\langle max, min \rangle, \langle min, max \rangle\}.$$

4.2 Termwise multiplication-$(\circ, *)$

$$A \otimes_{(\circ,*)} B = [K \cap P, L \cap Q, H \cap R, \{\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle\}],$$

where

$$\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle = \langle \circ(\mu_{k_i,l_j,h_g}, \rho_{p_r,q_s,r_d}), *(\nu_{k_i,l_j,h_g}, \sigma_{p_r,q_s,r_d}) \rangle$$

Here, $\langle \circ, * \rangle$ is as above.

4.3 Structural subtraction

$$A \ominus B = [K - P, L - Q, H - R, \{\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle\}],$$

where "$-$" is the set-theoretic difference operation and $\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle = \langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle$, for $t_u = k_i \in K - P, v_w = l_j \in L - Q$ and $x_y = h_g \in H - R$.

4.4 Negation

$$\neg A = [K, L, H, \{\neg\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g}\rangle\}],$$

where $\neg$ is one of the itutionistic fuzzy negations introduced in [3].

4.5 Termwise subtraction

$$A -_{\circ,*} B = A \oplus_{\circ,*} \neg B,$$

where $\langle \circ, * \rangle$ is as above.

5 Some examples

5.1 Example 1

Let $A = $

h	c	d
a	$\langle 0.5, 0.3\rangle$	$\langle 0.4, 0.2\rangle$
b	$\langle 0.1, 0.8\rangle$	$\langle 0.7, 0.1\rangle$

, $B = $

h	c	g
a	$\langle 0.3, 0.1\rangle$	$\langle 0.6, 0.2\rangle$
c	$\langle 0.3, 0.6\rangle$	$\langle 0.3, 0.6\rangle$
f	$\langle 0.5, 0.2\rangle$	$\langle 0.6, 0.1\rangle$

. Then

$A \oplus_{(\max,\min)} B = $

h	c	d	g
a	$\langle 0.5, 0.1\rangle$	$\langle 0.4, 0.2\rangle$	$\langle 0.6, 0.2\rangle$
b	$\langle 0.1, 0.8\rangle$	$\langle 0.7, 0.1\rangle$	$\langle 0.0, 1.0\rangle$
c	$\langle 0.3, 0.6\rangle$	$\langle 0.0, 1.0\rangle$	$\langle 0.3, 0.6\rangle$
f	$\langle 0.5, 0.2\rangle$	$\langle 0.0, 1.0\rangle$	$\langle 0.6, 0.1\rangle$

; $A \otimes_{(\max,\min)} B = $

h	c
a	$\langle 0.5, 0.1\rangle$

;

$A -_{(\max,\min)} B = $

h	c	d	g
a	$\langle 0.5, 0.3\rangle$	$\langle 0.4, 0.2\rangle$	$\langle 0.2, 0.6\rangle$
b	$\langle 0.1, 0.8\rangle$	$\langle 0.7, 0.1\rangle$	$\langle 0.0, 1.0\rangle$
c	$\langle 0.6, 0.3\rangle$	$\langle 0.0, 1.0\rangle$	$\langle 0.6, 0.3\rangle$
f	$\langle 0.2, 0.5\rangle$	$\langle 0.0, 1.0\rangle$	$\langle 0.1, 0.6\rangle$

.

5.2 Example 2

Let us consider the following problem: An international company offers for sale products $P_1, P_2, ..., P_p$ in different countries $C_1, C_2, ..., C_c$. Let $M_1, M_2, ..., M_m$ be equal time periods.

An 3D-IFIM with elements corresponding to this problem has the form:

$$F = [P, C, M\{\langle \mu_{P_i,C_j,M_g}, \nu_{P_i,C_j,M_g}\rangle\}]$$

$$\equiv \left\{ \begin{array}{c|ccccc} M_g & C_1 & \dots & C_j & \dots & C_n \\ \hline P_1 & \langle \mu_{P_1,C_1,M_g}, \nu_{P_1,C_1,M_g}\rangle & \dots & \langle \mu_{P_1,C_j,M_g}, \nu_{P_1,C_j,M_g}\rangle & \dots & \langle \mu_{P_1,C_n,M_g}, \nu_{P_1,C_n,M_g}\rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ P_i & \langle \mu_{P_i,C_1,M_g}, \nu_{P_i,C_1,M_g}\rangle & \dots & \langle \mu_{P_i,C_j,M_g}, \nu_{P_i,C_j,M_g}\rangle & \dots & \langle \mu_{P_i,C_n,M_g}, \nu_{P_i,C_n,M_g}\rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ P_m & \langle \mu_{P_m,C_1,M_g}, \nu_{P_m,C_1,M_g}\rangle & \dots & \langle \mu_{P_m,C_j,M_g}, \nu_{P_m,C_j,M_g}\rangle & \dots & \langle \mu_{P_m,C_n,M_g}, \nu_{P_m,C_n,M_g}\rangle \end{array} \right\},$$

where $M_g \in M$ and $P = \{P_1, P_2, ..., P_p\}$, $M = \{M_1, M_2, ..., M_m\}$, $C = \{C_1, C_2, ..., C_c\}$, and for $1 \leq i \leq p$, $1 \leq j \leq c$, $1 \leq g \leq m$ and $F_{P_i,C_j,M_g} = \langle m_{P_i,C_j,M_g}, n_{P_i,C_j,M_g} \rangle$, where m_{P_i,C_j,M_g} is the quantity of implemented planned deliveries of the product P_i, divided by the total quantity planned deliveries of this product for the country C_j for time period M_g, and n_{P_i,C_j,M_g} is the quantity of the outstanding planned deliveries of the product P_i, divided by the total quantity planned deliveries of this product for the country C_j for time period M_g, as above. Therefore, $m_{P_i,C_j,M_g} + n_{P_i,C_j,M_g} \leq 1$ and number $1 - m_{P_i,C_j,M_g} - n_{P_i,C_j,M_g}$ corresponds to the quantity of planned but unconfirmed deliveries of product P_i for country C_j.

The information on deliveries of this company in this mathematical form by matrices is the basis for multidimensional analysis of business data and provides the capability for complex calculations, trend analysis, and sophisticated data modeling. It is the foundation for desicion making Planning, Budgeting, Forecasting, Financial Reporting and Data Warehouse Reporting. As notes in [6], "management is seen as an information process, i.e the process of formation, perception, transmission, processing and storage of information". Stages in the process of decision-making are: identification of the problem, development of options for solutions, evaluation of options, choosing the best alternative, implementation of the decision and analysis of results.

6 Conclusion

In future, other operations over 3-dimensional intuitionistic fuzzy index matrices will be discussed and their applications will be researched for describing of databases, data warehouses and management processes.

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