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Pearson's coefficient between intuitionistic fuzzy sets

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Abstract: The correlation coefficient (Pearson's r) is one of the most frequently used tools in statistics. In this paper we discuss a correlation coefficient between Atanassov's intuitionistic fuzzy sets (A-IFSs). We have constructed the coefficient so it provides the strength of the relationship between A-IFSs and also shows if the considered sets are positively or negatively correlated. Next, the proposed correlation coefficient takes into account not only the amount of information related to the A-IFS data (expressed by the membership and non-membership values) but also the reliability of the data expressed by a so-called hesitation margin.
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1 Introduction

The correlation coefficient r (so called Pearson's coefficient) proposed by Karl Pearson in 1895 has became one of the most broadly applied indices in statistics [14]. Generally, correlation indicates how well two variables move together in an linear fashion. In other words, correlation reflects a linear relationship between two variables. It is an important measure in data analysis and classification, in particular in decision making, predicting the market behavior, medical diagnosis, pattern recognition, and other real world problems concerning environmental, political, legal, economic, financial, social, educational, artistic, etc. systems.

As in real world data are often fuzzy, the concept has been extended to fuzzy observations (cf. e.g., Chiang and Lin [6], Hong and Hwang [9], Liu and Kao [13]).

A relationship between A-IFSs (representing, e.g., preferences, attributes) seems to be of a vital importance, too, so that there are many papers discussing the correlation of A-IFSs: Gersternkorn and Mańko [7], Bustince and Burillo [3], Hong and Hwang [8], Hung [10], Hung and Wu [11], Zeng and Li [37]. In some of those papers only the strength of relationship is evaluated (cf. Gersternkorn and Mańko [7], Hong and Hwang [8], Zeng and Li [37]). In other papers (cf. Hung [10], Hung and Wu [11]), a positive and negative type of a relationship is reflected but the third term describing an A-IFS, which is important from the point of view of all similarity, distance or entropy measures (cf. Szmidt and Kacprzyk, e.g., [17], [19], [26], [21], [28]), [29]) is not accounted for.

In this paper, we discuss a concept of correlation for data represented as A-IFSs adopting the concepts from statistics. We calculate it by showing both a positive and negative relationship of the sets, and showing that it is important to take into account all three terms describing A-IFSs.

We illustrate our considerations on the examples (including benchmark data from [39]).

2 Brief introduction to A-IFSs

One of the possible generalizations of a fuzzy set in X (Zadeh [36]) given by

$$A^{'} = \{< x, \mu_{A'}(x) > |x \in X\} \tag{1}$$

where $\mu_{A'}(x) \in [0,1]$ is the membership function of the fuzzy set $A^{'}$, is an A-IFS (Atanassov [1], [2]) A is given by

$$A = \{< x, \mu_A(x), \nu_A(x) > |x \in X\} \tag{2}$$

where: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x)$, $\nu_A(x) \in [0,1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively. (Two approaches to the assigning memberships and non-memberships for A-IFSs are proposed by Szmidt and Baldwin [15]).

Obviously, each fuzzy set may be represented by the following A-IFS

$$A = \{< x, \mu_{A'}(x), 1 - \mu_{A'}(x) > |x \in X\} \tag{4}$$

An additional concept for each A-IFS in X, that is not only an obvious result of (2) and (3) but which is also relevant for applications, we will call (Atanasov [2])

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{5}$$

a *hesitation margin* of $x \in A$ which expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [2]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The hesitation margin turns out to be important while considering the distances (Szmidt and Kacprzyk [17], [19], [26], entropy (Szmidt and Kacprzyk [21], [28]), similarity (Szmidt and Kacprzyk [29]) for the A-IFSs, etc. i.e., the measures that play a crucial role in virtually all information processing tasks.

Hesitation margins turn out to be relevant for applications - in image processing (cf. Bustince et al. [5], [4]) and classification of imbalanced and overlapping classes (cf. Szmidt and Kukier [33], [34], [35]), group decision making, negotiations, voting and other situations (cf. Szmidt and Kacprzyk papers).

2.1 A geometrical representation

One of possible geometrical representations of an intuitionistic fuzzy sets is given in Figure 1 (cf. Atanassov [2]). It is worth noticing that although we use a two-dimensional figure (which is more convenient to draw in our further considerations), we still adopt our approach (e.g., Szmidt

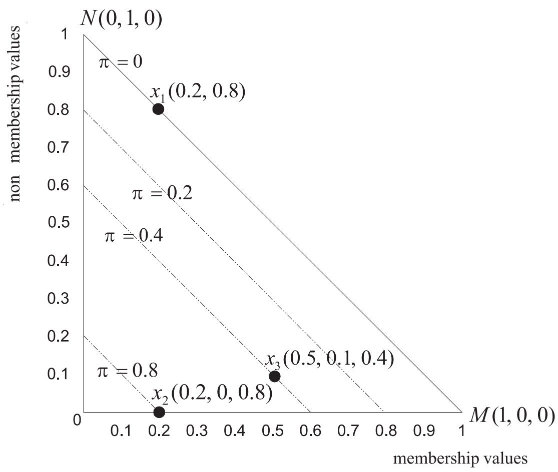

Figure 1: Geometrical representation

and Kacprzyk [19], [26], [21], [28]), [29]) taking into account all three terms (membership, non-membership and hesitation margin values) describing an intuitionistic fuzzy set. Any element belonging to an intuitionistic fuzzy set may be represented inside an MNO triangle. In other words, the MNO triangle represents the surface where the coordinates of any element belonging to an A-IFS can be represented. Each point belonging to the MNO triangle is described by the three coordinates: (μ, ν, π). Points M and N represent the crisp elements. Point $M(1, 0, 0)$ represents elements fully belonging to an A-IFS as $\mu = 1$, and may be seen as the representation of the ideal positive element. Point $N(0, 1, 0)$ represents elements fully not belonging to an A-IFS as $\nu = 1$, i.e. can be viewed as the ideal negative element. Point $O(0, 0, 1)$ represents elements about which we are not able to say if they belong or not belong to an A-IFS (the intuitionistic fuzzy index $\pi = 1$). Such an interpretation is intuitively appealing and provides means for the representation of many aspects of imperfect information. Segment MN (where $\pi = 0$) represents elements belonging to the classic fuzzy sets ($\mu+\nu = 1$). For example, point $x_1(0.2, 0.8, 0)$ (Figure 1), like any element from segment MN represents an element of a fuzzy set. A line parallel to MN describes the elements with the same values of the hesitation margin. In Figure 1 we can see point $x_3(0.5, 0.1, 0.4)$ representing an element with the hesitation margin equal 0.4, and point $x_2(0.2, 0, 0.8)$ representing an element with the hesitation margin equal 0.8. The closer a line that is parallel to MN is to O, the higher the hesitation margin.

3 Correlation

The correlation coefficient (Pearson's r) between two variables is a measure of the linear relationship between them.

The correlation coefficient is 1 in the case of a positive (increasing) linear relationship, -1 in the case of a negative (decreasing) linear relationship, and some value between -1 and 1 in all other cases. The closer the coefficient is to either -1 or 1, the stronger the correlation between the variables.

3.1 Correlation between crisp sets

Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a random sample of size n from a joint probability density function $f_{X,Y}(x, y)$, let $\overline{X}$ and $\overline{Y}$ be the sample means of variables X and Y, respectively, then the sample correlation coefficient $r(X, Y)$ is given as (e.g., [14]):

$$r(A, B) = \frac{\sum\limits_{i=1}^{n}(x_i - \overline{X})(y_i - \overline{Y})}{(\sum\limits_{i=1}^{n}(x_i - \overline{X})^2 \sum\limits_{i=1}^{n}(y_i - \overline{Y})^2)^{0.5}} \tag{6}$$

where: $\overline{X} = \frac{1}{n}\sum\limits_{i=1}^{n} x_i, \quad \overline{Y} = \frac{1}{n}\sum\limits_{i=1}^{n} y_i.$

3.2 Correlation between fuzzy sets

Suppose that we have a random sample $x_1, x_2, \ldots, x_n \in X$ with a sequence of paired data $(\mu_A(x_1), \mu_B(x_1))$, $(\mu_A(x_2), \mu_B(x_2))$, …, $(\mu_A(x_n), \mu_B(x_n))$ which correspond to the membership values of fuzzy sets A and B defined on X, then the correlation coefficient $r_f(A, B)$ is given as ([6]):

$$r_f(A, B) = \frac{\sum\limits_{i=1}^{n}(\mu_A(x_i) - \overline{\mu_A})(\mu_B(x_i) - \overline{\mu_B})}{(\sum\limits_{i=1}^{n}(\mu_A(x_i) - \overline{\mu_A})^2)^{0.5}(\sum\limits_{i=1}^{n}(\mu_B(x_i) - \overline{\mu_B})^2)^{0.5}} \tag{7}$$

where: $\overline{\mu_A} = \frac{1}{n}\sum\limits_{i=1}^{n} \mu_A(x_i), \quad \overline{\mu_B} = \frac{1}{n}\sum\limits_{i=1}^{n} \mu_B(x_i).$

3.3 Correlation between A-IFSs

We propose a correlation coefficient for two A-IFSs, A and B, so that we could express not only a relative strength but also a positive or negative relationship between A and B. Next, we take into account all three terms describing an A-IFSs (membership, non-membership values and the hesitation margins) because each of them influences the results.

Suppose that we have a random sample $x_1, x_2, \ldots, x_n \in X$ with a sequence of paired data $[(\mu_A(x_1), \nu_A(x_1), \pi_A(x_1)), (\mu_B(x_1), \nu_B(x_1), \pi_B(x_1))]$, $[(\mu_A(x_2), \nu_A(x_2), \pi_A(x_2)), (\mu_B(x_2), \nu_B(x_2), \pi_B(x_2))]$, …, $[(\mu_A(x_n), \nu_A(x_n), \pi_A(x_n)), (\mu_B(x_n), \nu_B(x_n), \pi_B(x_n))]$ which correspond to the membership values, non-memberships values and hesitation margins of A-IFSs A and B defined on X, then the correlation coefficient $r_{A-IFS}(A, B)$ is given by Definition 1.

Definition 1 The correlation coefficient $r_{A-IFS}(A, B)$ between two A-IFSs, A and B in X, is:

$$r_{A-IFS}(A, B) = \frac{1}{3}(r_1(A, B) + r_2(A, B) + r_3(A, B)) \tag{8}$$

where

$$r_1(A, B) = \frac{\sum\limits_{i=1}^{n}(\mu_A(x_i) - \overline{\mu_A})(\mu_B(x_i) - \overline{\mu_B})}{(\sum\limits_{i=1}^{n}(\mu_A(x_i) - \overline{\mu_A})^2)^{0.5}(\sum\limits_{i=1}^{n}(\mu_B(x_i) - \overline{\mu_B})^2)^{0.5}} \tag{9}$$

$$r_2(A,B) = \frac{\sum_{i=1}^{n}(\nu_A(x_i) - \overline{\nu_A})(\nu_B(x_i) - \overline{\nu_B})}{(\sum_{i=1}^{n}(\nu_A(x_i) - \overline{\nu_A})^2)^{0.5}(\sum_{i=1}^{n}(\nu_B(x_i) - \overline{\nu_B})^2)^{0.5}} \tag{10}$$

$$r_3(A,B) = \frac{\sum_{i=1}^{n}(\pi_A(x_i) - \overline{\pi_A})(\pi_B(x_i) - \overline{\pi_B})}{(\sum_{i=1}^{n}(\pi_A(x_i) - \overline{\pi_A})^2)^{0.5}(\sum_{i=1}^{n}(\pi_B(x_i) - \overline{\pi_B})^2)^{0.5}} \tag{11}$$

where: $\overline{\mu_A} = \frac{1}{n}\sum_{i=1}^{n}\mu_A(x_i)$, $\overline{\mu_B} = \frac{1}{n}\sum_{i=1}^{n}\mu_B(x_i)$, $\overline{\nu_A} = \frac{1}{n}\sum_{i=1}^{n}\nu_A(x_i)$,
$\overline{\nu_B} = \frac{1}{n}\sum_{i=1}^{n}\nu_B(x_i)$, $\overline{\pi_A} = \frac{1}{n}\sum_{i=1}^{n}\pi_A(x_i)$, $\overline{\pi_B} = \frac{1}{n}\sum_{i=1}^{n}\pi_B(x_i)$,

The proposed correlation coefficient (8) depends on two factors: the amount of information expressed by the membership and non-membership degrees (9)–(10), and the reliability of information expressed by the hesitation margins (11).

Remark: analogously as for the crisp and fuzzy data, $r_{A-IFS}(A,B)$ makes sense for A-IFS variables whose values vary. If, for instance, the temperature is constant and the amount of ice cream sold is the same, then it is impossible to conclude about their relationship (as, from the mathematical point of view, we avoid zero in the denominator).

The correlation coefficient $r_{A-IFS}(A,B)$ (8) fulfills the following properties:

1. $r_{A-IFS}(A,B) = r_{A-IFS}(B,A)$
2. If $A = B$ then $r_{A-IFS}(A,B) = 1$
3. $|r_{A-IFS}(A,B)| =\leq 1$

The above properties are not only fulfilled by the correlation coefficient $r_{A-IFS}(A,B)$ (8) but also by its every component (9)–(11).

Remark: It is should be emphasized that $r_{A-IFS}(A,B) = 1$ occurs not only for $A = B$ but also in the cases of a perfect linear correlation of the data (the same concerns each component (9)–(11)).

We will show now a simplified example. The size of the data set is too small to look at them as for significant samples, but the purpose is just for illustration.

Example 1 Let A and B be A-IFSs in $X = \{x_1, x_2, x_3\}$:

$$A = \{(x_1, 0.1, 0.2, 0.7), (x_2, 0.2, 0.09, 0.71), (x_3, 0.3, 0.01, 0.69)\}$$
$$B = \{(x_1, 0.3, 0, 0.7), (x_2, 0.2, 0.2, 0.6), (x_3, 0.1, 0.6, 0.3)\}$$

It is easy to notice that

- the membership values of the elements in A (i.e.: $0.1, 0.2, 0.3$) increase whereas the membership values of the elements in B (i.e.: $0.3, 0.2, 0.1$) decrease. In the result (9) we have $r_1(A,B) = -1$.
- the non-membership values of the elements in A (i.e.: $0.2, 0.09, 0.01$) decrease whereas the non-membership values of the elements in B (i.e.: $0.0, 0.2, 0.6$) increase. In the result (10) we have $r_2(A,B) = -0.96$.

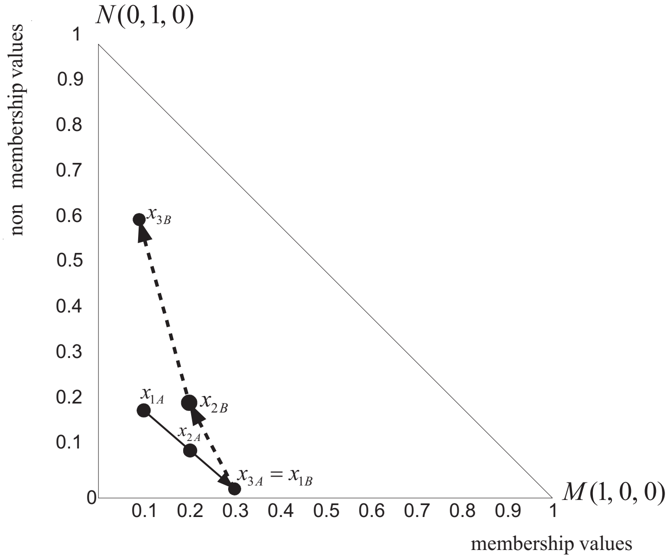

Figure 2: Visualization of the data from Example 1: it is easy to notice that there is no perfect linear relationship among elements from A and B

- the hesitation margins of the elements in A (i.e.: $(0.7, 0.71, 0.69)$ decrease and the hesitation margins of the elements in B (i.e.: $0.7, 0.6, 0.2$) decrease. In the result (11) we have $r_3(A, B) = 0.73$.

Therefore, finally, from (8) we obtain $r_{A-IFS}(A, B) = \frac{1}{3}(-1 - 0.96 + 0.73) = -0.41$.

If we exclude from considerations the hesitation margins, and take into account two components (9) and (10) only, we obtain $r_{A-IFS}(A, B) = \frac{1}{2}(-1 - 0.96) = -0.98$ which means that there is a substantial negative linear relationship between A and B (which is difficult to agree).

In Figure 2 there is a geometrical interpretation (cf. Section 2.1) of the data from Example 1.

It is worth emphasizing that for practical purposes (e.g., in decision making) it seems rather useful to know correlation (11) concerning lack of knowledge represented by the variables considered. If, for example, the data represent reactions of patients to a new medicine, it seems unavoidable to carefully examine just the part (11) of the correlation coefficient (8) as it may happen that a new treatment/medicine increases unforeseen reactions. In such situations it may be important not only to assess all components separately but even to give them different weights in (8).

Now we will verify if the situation is similar (if all the parts of (8) count) for a well known benchmark example - Iris Plants Database [38]. The data set Iris contains 3 classes (150 data examples in total; each class of 50 examples) and 4 continuous attributes: sepal length (SL) , sepal width (SW), petal length (PL), and petal width (PW).

To describe the data via the A-IFSs, we use the algorithm – based on the mass assignment theory – proposed by Szmidt and Baldwin [15] to assign the parameters of an A-IFS model which describes each attribute in terms of membership values, non-membership values, and hesitation margin values. Having description of the attributes in terms of A-IFSs, we have calculated the three components of (8) for each pair of the attributes. The results are in Table 1–Table 3.

Table 1: The values of the correlation component (9) between each pair of the attributes for Iris data

Attribute	SL	SW	PL	PW
SL	-	0.3	0.86	0.84
SW		-	0.6	0.6
PL			-	0.99
PW				-

Table 2: The values of the correlation component (10) between each pair of the attributes for Iris data

Attribute	SL	SW	PL	PW
SL	-	0.24	0.85	0.83
SW		-	0.51	0.5
PL			-	0.99
PW				-

Table 3: The values of the correlation component (11) between each pair of the attributes for Iris data

Attribute	SL	SW	PL	PW
SL	-	-0.14	0.67	0.61
SW		-	0.19	-0.14
PL			-	0.6
PW				-

It is easy to notice that the attributes PL and PW are strongly correlated [0.99 for both (9) and (10), and 0.6 for (11)]. Both attributes are also strongly correlated with attribute SL (PL and SL: 0.86, 0.85, and 0.67 for (9)–(11), respectively, and PW and SL: 0.84, 0.83, 0,61 for (9)–(11), respectively). We may notice again, that the values (11) are significant. The correlation (8) among the attributes is given in Table 4. It is well known, e.g. [39] that the same attributes, i.e. PL, PW, and next: SL, are the best one while discriminating the three classes.

Certainly, we may find an example when $r_3(A, B)$ (11) does not influence the correlation coefficient $r_{A-IFS}(A, B)$ (8) in a sense of the final result (an obtained number). But such situations are the exceptions, not a rule.

4 Conclusions

We have discussed a new correlation coefficient between A-IFSs. The coefficient discussed, like Pearson's coefficient between crisp sets, measures how strong is relationship between A-IFSs, and indicates if the sets are positively or negatively correlated. It is worth stressing that we have taken into account all three terms describing A-IFS (the membership, non-membership values

Table 4: The values of the correlation (8) between each pair of the attributes for Iris data

Attribute	SL	SW	PL	PW
SL	-	0.13	0.79	0.76
SW		-	0.43	0.32
PL			-	0.86
PW				-

and hesitation margins). Each term plays an important role in data analysis and decision making, so that each of them should be reflected while assessing the correlation between A-IFSs.

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