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More basic operations and modal operators over 3-dimensional intuitionistic fuzzy index matrices

Velichka Traneva

"Prof. Asen Zlatarov" University
1 "Prof. Yakimov" Blvd, Bourgas 8000, Bulgaria
e-mail: veleka13@gmail.com

Abstract: An extension of the concept of an Intuitionistic Fuzzy Index Matrix (IFIM) was introduced, called 3-dimensional Intuitionistic Fuzzy Index Matrix (3D-IFIM). In this paper will be introduced more basic operations and modal operators defined over 3D-IFIMs.
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1 Introduction

Index matrix and their apparatus was introduced in 1984 in [1, 2]. In [6], 3-dimensional extended index matrix (3D-EIM) was defined. Following this concept in [7] was defined 3-Dimensional Intuitionistic Fuzzy Index Matrix (3D-IFIM) and some operations over them will be introduced. Here, will be introduced more basic operations and modal operators defined over 3D-IFIMs, which generalizes operations and modal operators over 2D-Intuitionistic fuzzy matrices from [4]

2 Basic definitions

In [3, 5] an Intuitionistic Fuzzy Pair (IFP) was defined as an object with the form $\langle a, b\rangle$, where $a, b \in [0, 1]$ and $a+b \leq 1$, that is used as an evaluation of some object or process. Its components (a and b) are interpreted as degrees of membership and non-membership, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc.

We define following operations for two IFPs $x = \langle a, b\rangle$ and $y = \langle c, d\rangle$:

$$\neg x = \langle b, a \rangle$$
$$x \& y = \langle \min(a,c), \max(b,d) \rangle$$
$$x \vee y = \langle \max(a,c)), \min(b,d) \rangle$$
$$x + y = \langle a + c - a.c, b.d \rangle$$
$$x.y = \langle a.c, b + d - b.d \rangle$$
$$x@y = \langle \tfrac{a+c}{2}, \tfrac{b+d}{2} \rangle.$$

Let a set E be fixed. An Intuitionistic Fuzzy Set (IFS) A in E is an object of the following form (see, e.g., [3]):

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let us define 3D-IFIM (see [7]). Let $\mathcal{I}$ be a fixed set of indices. A "3D-Intuitionistic Fuzzy Index Matrix" (3D-IFIM) with index sets K, L and H $(K, L, H \subset \mathcal{I})$ is called the objest:

$$[K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]$$

$$\equiv \left\{ \begin{array}{c|ccccc} h_g & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1,l_1,h_g}, \nu_{k_1,l_1,h_g} \rangle & \dots & \langle \mu_{k_1,l_j,h_g}, \nu_{k_1,l_j,h_g} \rangle & \dots & \langle \mu_{k_1,l_n,h_g}, \nu_{k_1,l_n,h_g} \rangle \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_i & \langle \mu_{k_i,l_1,h_g}, \nu_{k_i,l_1,h_g} \rangle & \dots & \langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle & \dots & \langle \mu_{k_i,l_n,h_g}, \nu_{k_i,l_n,h_g} \rangle \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & \langle \mu_{k_m,l_1,h_g}, \nu_{k_m,l_1,h_g} \rangle & \dots & \langle \mu_{k_m,l_j,h_g}, \nu_{k_m,l_j,h_g} \rangle & \dots & \langle \mu_{k_m,l_n,h_g}, \nu_{k_m,l_n,h_g} \rangle \end{array} | h_g \in H \right\},$$

where for every $1 \leq i \leq m, 1 \leq j \leq n,\ 1 \leq g \leq f$:

$$0 \leq \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g}, \mu_{k_i,l_j,h_g} + \nu_{k_i,l_j,h_g} \leq 1.$$

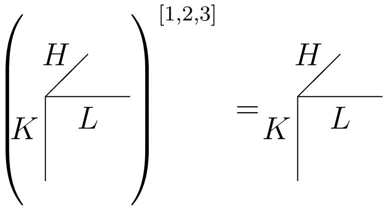
3 Operations over 3D-IFIMs

Let $\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{U}$ be fixed sets. Let operations "$*$" and "$\circ$" be defined so that: $* : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$ and $\circ : \mathcal{Z} \times \mathcal{Z} \to \mathcal{U}$. Let the index set $\mathcal{I}$ be given. We will define some operations over the 3D-IFIMs $A = [K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]$ and $B = [P, Q, R, \{\langle \rho_{p_r,q_s,r_d}, \sigma_{p_r,q_s,r_d} \rangle\}]$.

3.1 Transposition

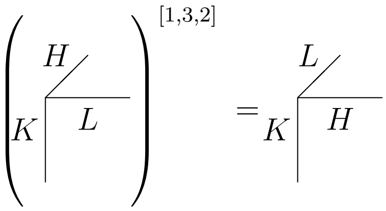
As we saw in [6], for 3D-IFIMs, similarly to the 3D-EIMs, there are 6 (=3!) cases: the standard 3D-IFIM and five different transposed 3D-IFIMs. By analogy with [6], we show that the geometrical and analytical forms of the separate transposed 3D-IFIMs are the following.

$[1,2,3]$**-transposition (identity)**



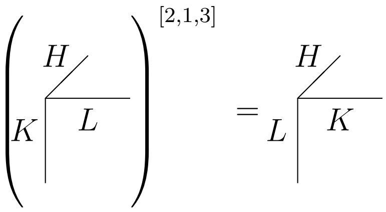
$$[K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]^{[1,2,3]} = [K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}];$$

$[1,3,2]$**-transposition**



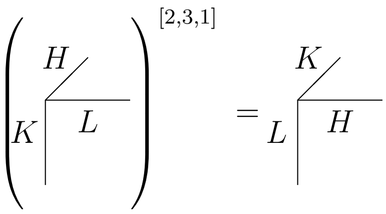
$$[K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]^{[1,3,2]} = [K, H, L, \{\langle \mu_{k_i,h_g,l_j}, \nu_{k_i,h_g,l_j} \rangle\}];$$

$[2,1,3]$**-transposition**



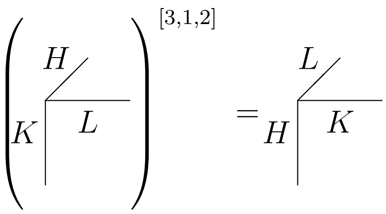
$$[K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]^{[2,1,3]} = [L, K, H, \{\langle \mu_{l_j,k_i,h_g}, \nu_{l_j,k_i,h_g} \rangle\}];$$

$[2,3,1]$**-transposition**



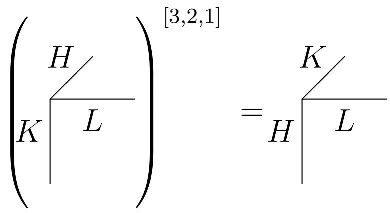
$$[K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]^{[2,3,1]} = [L, H, K, \{\langle \mu_{l_j,h_g,k_i}, \nu_{l_j,h_g,k_i} \rangle\}];$$

$[3,1,2]$**-transposition**



$$[K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]^{[3,1,2]} = [H, K, L, \{\langle \mu_{h_g,k_i,l_j}, \nu_{h_g,k_i,l_j} \rangle\}];$$

$[3, 2, 1]$**-transposition**



$$[K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]^{[3,2,1]} = [H, L, K, \{\langle \mu_{h_g,l_j,k_i}, \nu_{h_g,l_j,k_i} \rangle\}];$$

3.2 Multiplication

This operation is related to the operation "transposition". There are 36 different operations "multiplication". The first (standard) multiplication is

$$A \odot_{(\circ,*)} B = A \odot_{(\circ,*)}^{\{[1,2,3],[1,2,3]\}} B = [K \cup (P - L), Q \cup (L - P), H \cup R, \{\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle\}],$$

where

$$\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle$$

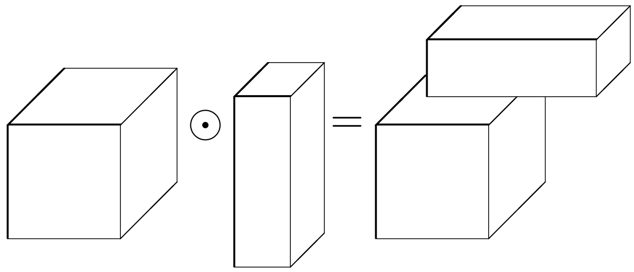
$$= \begin{cases} \langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle, & \text{if } t_u = k_i \in K \\ & \& \ v_w = l_j \in L - P - Q \ \& \ x_y = h_g \in H \\ & \text{or } t_u = k_i \in K - P - Q \\ & \& \ v_w = l_j \in L \ \& \ x_y = h_g \in H \\ \langle \rho_{p_r,q_s,r_d}, \sigma_{p_r,q_s,r_d} \rangle, & \text{if } t_u = p_r \in P \\ & \& \ v_w = q_s \in Q - K - L \ \& \ x_y = r_d \in R \\ & \text{or } t_u = p_r \in P - L - K \\ & \& \ v_w = q_s \in Q \ \& \ x_y = r_d \in R \\ \langle \underset{l_j = p_r \in L \cap P}{\circ} (*(\mu_{k_i,l_j,h_g}, \rho_{p_r,q_s,r_d})), & \text{if } t_u = k_i \in K \ \& \ v_w = q_s \in Q \\ \underset{l_j = p_r \in L \cap P}{*} (\circ(\nu_{k_i,l_j,h_g}, \sigma_{p_r,q_s,r_d})) \rangle, & \& \ x_y = h_g = r_d \in H \cap R \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases},$$

where

$$\langle \circ, * \rangle \in \{\langle \max, \min \rangle, \langle \min, \max \rangle\}$$

.

The geometrical interpretation of this operation is (see [4])



Let $[x, y, z]$ and $[u, v, w]$ be permutations of a triple $[1, 2, 3]$.

The operation $\{[x, y, z], [u, v, w]\}$-multiplication is defined by:

$$A \odot_{(\circ,*)}^{\{[x,y,z],[u,v,w]\}} B = A^{[x,y,z]} \odot_{(\circ,*)} B^{[u,v,w]}.$$

3.3 Reduction

By analogy with [6], we introduce operations "reduction", "projection" and "substitution" over an 3D-IFIM.

Let us introduce operations $(k, \bot, \bot)$-, $(\bot, l, \bot)$- and $(\bot, \bot, h)$-reduction of a given 3D-IFIM $A = [K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]$:

$$A_{(k,\bot,\bot)} = [K - \{k\}, L, H, \langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle]$$

where

$\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle = \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}$ for $t_u = k_i \in K - \{k\}$, $v_w = l_j \in L$ and $x_y = h_g \in H$,

$$A_{(\bot,l,\bot)} = [K, L - \{l\}, H, \langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle],$$

where

$\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle = \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}$ for $t_u = k_i \in K$, $v_w = l_j \in L - \{l\}$ and $x_y = h_g \in H$

and

$$A_{(\bot,\bot,h)} = [K, L, H - \{h\}, \langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle],$$

where

$\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle = \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}$ for $t_u = k_i \in K$, $v_w = l_j \in L$ and $x_y = h_g \in H - \{h\}$.

Second, we define

$$A_{(k,l,h)} = ((A_{(k,\bot,\bot)})_{(\bot,l,\bot)})_{(\bot,\bot,h)},$$

i.e.,

$$A_{(k,l,h)} = [K - \{k\}, L - \{l\}, H - \{h\}, \langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle],$$

where

$$\langle \varphi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y} \rangle = \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}$$

for $t_u = k_i \in K - \{k\}$, $v_w = l_j \in L - \{l\}$ and $x_y = h_g \in H - \{h\}$.

For every 3D-IFIM A and for every $k_1, k_2 \in K$, $l_1, l_2 \in L$, $h_1, h_2 \in H$,

$$(A_{(k_1,l_1,h_1)})_{(k_2,l_2,h_2)} = (A_{(k_2,l_2,h_2)})_{(k_1,l_1,h_1)}.$$

Third, let $P = \{p_1, p_2, ..., p_s\} \subseteq K$, $Q = \{q_1, q_2, ..., q_t\} \subseteq L$ and $R = \{r_1, r_2, ..., r_u\} \subseteq H, p \in K, l \in L, h \in H$.

Now, we define the following four operations:

$$A_{(P,l,h)} = (...((A_{(p_1,l,h)})_{(p_2,l,h)})...)_{(p_s,l,h)},$$

$$A_{(k,Q,h)} = (...((A_{(k,l_1,h)})_{(k,l_2,h)})...)_{(k,l_t,h)},$$

$$A_{(k,q,H)} = (...((A_{(k,l,r_1)})_{(k,l,r_2)})...)_{(k,l,r_u)},$$

$$A_{(P,Q,H)} = (...((A_{(p_1,Q,H)})_{(p_2,Q,H)})...)_{(p_s,Q,H)}$$

$$= (...((A_{(P,q_1,H)})_{(P,q_2,H)})...)_{(P,q_t,H)} = (...((A_{(P,Q,r_1)})_{(P,Q,r_2)})...)_{(P,Q,r_u)}.$$

Obviously,

$$A_{(K,L,H)} = I_\emptyset, A_{(\emptyset,\emptyset,\emptyset)} = A.$$

3.4 Projection

Let $P \subseteq K$, $Q \subseteq L$, $R \subseteq H$. Then,

$$pr_{P,Q,R}A = [P, Q, R, \langle \rho_{p_r,q_s,r_d}, \sigma_{p_r,q_s,r_d} \rangle],$$

where

$$(\forall k_i \in P)(\forall l_j \in Q)(\forall h_g \in R)(\langle \rho_{p_r,q_s,r_d}, \sigma_{p_r,q_s,r_d} \rangle = \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}.$$

3.5 Substitution

Let the 3D-IFIM $A = [K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}]$ be given.

Let us define local substitution over the IFIM is defined for the pairs of indices (p, k) and/or (q, l) and/or (r, h), respectively, by

$$\left[\frac{p}{k}; \bot; \bot\right] A = \left[(K - \{k\}) \cup \{p\}, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}\right],$$

$$\left[\bot; \frac{q}{l}; \bot\right] A = \left[K, (L - \{l\}) \cup \{q\}, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}\right],$$

$$\left[\bot; \bot; \frac{r}{h}\right] A = \left[K, L, (H - \{h\}) \cup \{r\}, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}\right].$$

Second,

$$\left[\frac{p}{k}; \frac{q}{l}; \frac{r}{h}\right] A = \left[\frac{p}{k}; \bot; \bot\right] \left[\bot; \frac{q}{l}; \bot\right] \left[\bot; \bot; \frac{r}{h}\right] A$$

$$= \left[(K - \{k\}) \cup \{p\}, (L - \{l\}) \cup \{q\}, (H - \{h\}) \cup \{r\}, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g} \rangle\}\right].$$

Let the sets of indices $P = \{p_1, p_2, ..., p_m\}$, $Q = \{q_1, q_2, ..., q_n\}$, $R = \{r_1, r_2, ..., r_s\}$ be given, where $m = card(K), n = card(L), s = card(H)$.

Third, for them we define sequentially:

$$\left[\frac{P}{K};\perp;\perp\right]A = \left[\frac{p_1}{k_1}\frac{p_2}{k_2}...\frac{p_n}{k_n};\perp;\perp\right]A,$$

$$\left[\perp;\frac{Q}{L};\perp\right]A = \left[\perp;\frac{q_1}{l_1}\frac{q_2}{l_2}...\frac{q_n}{l_n};\perp\right]A,$$

$$\left[\perp;\perp;\frac{R}{H}\right]A = \left[\perp;\frac{r_1}{h_1}\frac{r_2}{h_2}...\frac{r_s}{h_s};\perp\right]A,$$

$$\left[\frac{P}{K};\frac{Q}{L};\frac{R}{H}\right]A = \left[\frac{p_1}{k_1}\frac{p_2}{k_2}...\frac{p_m}{k_m};\frac{q_1}{l_1}\frac{q_2}{l_2}...\frac{q_n}{l_n};\frac{r_1}{h_1}\frac{r_2}{h_2}...\frac{r_n}{h_s}\right]A = \left[P, Q, R, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g}\rangle\}\right].$$

4 Extended intuitionistic fuzzy modal operators defined over 3D-IFIMs

Let $x = \langle a, b\rangle$ be an IFP and let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. The main modal operators defined over x are:

$$\Box x = \{\langle x, a, 1 - a\rangle | x \in E\}$$

$$\Diamond x = \{\langle x, 1 - b, b\rangle | x \in E\}$$

Some of the extended modal operators defined over x have the following forms (see [3, 5]):

$$F_{\alpha,\beta}(x) = \langle a + \alpha.(1 - a - b), b + \beta.(1 - a - b)\rangle, \text{ where } \alpha + \beta \leq 1$$
$$G_{\alpha,\beta}(x) = \langle \alpha.a, \beta.b\rangle$$
$$H_{\alpha,\beta}(x) = \langle \alpha.a, b + \beta.(1 - a - b)\rangle$$
$$H^*_{\alpha,\beta}(x) = \langle \alpha.a, b + \beta.(1 - \alpha.a - b)\rangle$$
$$J_{\alpha,\beta}(x) = \langle a + \alpha.(1 - a - b), \beta.b\rangle$$
$$J^*_{\alpha,\beta}(x) = \langle a + \alpha.(1 - a - \beta.b), \beta.b\rangle$$

The level operators have the forms:

$$P_{\alpha,\beta}x = \langle \max(\alpha, a), \min(\beta, b)\rangle$$

$$Q_{\alpha,\beta}x = \langle \min(\alpha, a), \max(\beta, b)\rangle,$$

for $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$.

Let us define operators over 3D-IFIMs. Let $O_{\alpha,\beta}$ be one of the above modal operators, let α, β satisfy the respective conditions, given above.

Then

$$(O_{\alpha,\beta})(A) = \left\{ \begin{array}{c|ccc} h_g & l_1, & \dots & l_n \\ \hline k_1 & O_{\alpha,\beta}(\langle \mu_{k_1,l_1,h_g}, \nu_{k_1,l_1,h_g} \rangle) & \dots & O_{\alpha,\beta}(\langle \mu_{k_1,l_n,h_g}, \nu_{k_1,l_n,h_g} \rangle) \\ \vdots & \vdots & \dots & \vdots \\ k_m & O_{\alpha,\beta}(\langle \mu_{k_m,l_1,h_g}, \nu_{k_m,l_1,h_g} \rangle) & \dots & O_{\alpha,\beta}(\langle \mu_{k_m,l_n,h_g}, \nu_{k_m,l_n,h_g} \rangle) \end{array} |h_g \in H \right\}.$$

5 Example

Let $A = \begin{array}{c|cc} h & c & d \\ \hline a & \langle 0.5, 0.3 \rangle & \langle 0.4, 0.2 \rangle \\ b & \langle 0.1, 0.8 \rangle & \langle 0.7, 0.1 \rangle \end{array}$, $B = \begin{array}{c|c} h & g \\ \hline d & \langle 0.5, 0.4 \rangle \\ s & \langle 0.1, 0.8 \rangle \\ t & \langle 0.3, 0.6 \rangle \end{array}$.

Then

$$A \otimes_{(\max,\min)} B = \begin{array}{c|cc} h & g & c \\ \hline a & \langle \max(\min(0.4, 0.5)), \min(\max(0.2, 0.1)) \rangle & \langle 0.5, 0.3 \rangle \\ b & \langle \max(\min(0.7, 0.5)), \min(\max(0.1, 0.1)) & \langle 0.1, 0.8 \rangle \\ s & \langle 0.1, 0.8 \rangle & \langle 0.0, 1.0 \rangle \\ t & \langle 0.3, 0.8 \rangle & \langle 0.0, 1.0 \rangle \end{array}$$

$$= \begin{array}{c|cc} h & g & c \\ \hline a & \langle 0.4, 0.2 \rangle & \langle 0.5, 0.3 \rangle \\ b & \langle 0.5, 0.1 & \langle 0.1, 0.8 \rangle \\ s & \langle 0.1, 0.8 \rangle & \langle 0.0, 1.0 \rangle \\ t & \langle 0.3, 0.8 \rangle & \langle 0.0, 1.0 \rangle \end{array}.$$

6 Conclusion

In future, other applications of the apparatus of the 3D-intuitionistic fuzzy index matrices will be studied, for instance, in the areas of optimal management, data bases, data warehousing, etc.

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