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A Study on Intuitionistic L-Fuzzy Subrings

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of an intuitionistic L-fuzzy subrings under homomorphism and anti-homomorphism.

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KEYWORDS

Intuitionistic fuzzy set, L-fuzzy set, intuitionistic L-fuzzy set, intuitionistic L-fuzzy subring, homomorphism, anti-homomorphism.

INTRODUCTION

Ever since the introduction of fuzzy sets by ZADEH [4], the fuzzy concept has invaded almost all branches of mathematics. The concept of intuitionistic fuzzy set was introduced by ATANASSOV [1], as a generalization of the notion of fuzzy set. KOG and BALKANAY [2] defined a θ-Euclidean L-fuzzy ideals of rings. PALANIAPPAN and ARJUNAN [3] defined the homomorphism, anti-homomorphism of fuzzy and anti-fuzzy ideals. In this paper, we introduce the concept of homomorphism and anti-homomorphism in intuitionistic L-fuzzy subrings and prove some results on these.

1. PRELIMINARIES

1.1 DEFINITION

An intuitionistic fuzzy set (IFS) A in X is defined as an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$, where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \le \mu_A(x)+\nu_A(x) \le 1$.

1.2 DEFINITION

Let X be a non-empty set and $L = (L, \le, \wedge, \vee)$ be a lattice with least element 0 and greatest element 1. A L-fuzzy subset A of X is a function $A : X \to L$.

1.3 DEFINITION

Let $(L, \le)$ be a complete lattice with an involutive order reversing operation $N: L \to L$. An intuitionistic L-fuzzy set (ILFS) A in X is defined as an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$, where $\mu_A : X \to L$ and $\nu_A : X \to L$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $\mu_A(x) \leq N(\nu_A(x))$.

1.4 DEFINITION

Let R be a ring. An intuitionistic L-fuzzy subset A of R is said to be an intuitionistic L-fuzzy subring of R (ILFSR) if

i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$

ii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$

iii) $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$

iv) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$ for all $x,y \in R$.

1.5 DEFINITION

Let R and R′ be any two rings, then the function $f : R \to R'$ is said to be a homomorphism if

i) $f(x+y) = f(x) + f(y)$ and

ii) $f(xy) = f(x) f(y)$, for all $x, y \in R$.

1.6 DEFINITION

Let R and R′ be any two rings, then the function $f : R \to R'$ is said to be a anti-homomorphism if

i) $f(x+y) = f(y) + f(x)$ and

ii) $f(xy) = f(y) f(x)$, for all $x, y \in R$.

1.7 DEFINITION

Let X and X′ be any two sets. Let $f:X \to X'$ be any function and let A be an IFS in X, V be an IFS in $f(X) = X'$, defined by $\mu_v(y) = \sup_{x \in f^{-1}(y)} \mu_A(x)$ and $\nu_v(y) = \inf_{x \in f^{-1}(y)} \nu_A(x)$ for all $x \in X$ and $y \in X'$. A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.1 THEOREM

Let R and R′ be rings with identity. Let $f : R \to R'$ be a homomorphism, then

(i) $f(0) = 0'$, $f(1) = 1'$ where 0, 1 and 0′, 1′ are identities of R and R′ respectively.

(ii) $f(-a) = -f(a)$ and $f(a^{-1}) = (f(a))^{-1}$, for all $a \in R$.

Proof : It is trivial.

1.2 THEOREM

Let R and R′ be rings with identity. Let $f : R \to R'$ be an anti-homomorphism, then

(i) $f(0) = 0'$, $f(1) = 1'$ where 0, 1 and 0′, 1′ are identities of R and R′ respectively.

(ii) $f(-a) = -f(a)$ and $f(a^{-1}) = (f(a))^{-1}$, for all $a \in R$.

Proof : It is trivial.

2. ILFSR of a ring R under homomorphism

2.1 THEORM

Let $f : R \to R'$ be a homomorphism. Then the homomorphic image of an intuitionistic L-fuzzy subring of a ring R is an intuitionistic L-fuzzy subring of R′.

Proof :

Since $f : R \to R'$ be a homomorphism, we have

i) $f(x+y) = f(x) + f(y)$ and

ii) $f(xy) = f(x) f(y)$, for all $x, y \in R$.

Let $V = f(A)$, where A is an ILFSR of R.

We have to prove that V is an ILFSR of R′.

Now, for f(x), f(y) in R′,

$$\mu_V(f(x) - f(y)) = \mu_V(f(x - y)) \text{ as f is a homomorphism}$$
$$\geq \mu_A(x - y)$$
$$\geq \mu_A(x) \wedge \mu_A(y) \text{ as A is an ILFSR of R}$$

which implies that

$$\mu_V(f(x) - f(y)) \geq \mu_V(f(x)) \wedge \mu_V(f(y)).$$

Again,

$$\mu_V(f(x)f(y)) = \mu_V(f(xy)) \text{ as f is a homomorphism}$$
$$\geq \mu_A(xy)$$
$$\geq \mu_A(x) \wedge \mu_A(y) \text{ as A is an ILFSR of R}$$

which implies that

$$\mu_V(f(x)f(y)) \geq \mu_V(f(x)) \wedge \mu_V(f(y)).$$

Also,

$$\nu_V(f(x) - f(y)) = \nu_V(f(x - y)) \text{ as f is a homomorphism}$$
$$\leq \nu_A(x - y)$$
$$\leq \nu_A(x) \vee \nu_A(y) \text{ as A is an ILFSR of R}$$

which implies that

$$\nu_V(f(x) - f(y)) \leq \nu_V(f(x)) \vee \nu_V(f(y)).$$

Again,

$$\nu_V(f(x)f(y)) = \nu_V(f(xy)) \text{ as f is a homomorphism}$$
$$\leq \nu_A(xy)$$
$$\leq \nu_A(x) \vee \nu_A(y) \text{ as A is an ILFSR of R}$$

which implies that

$$\nu_V(f(x)f(y)) \leq \nu_V(f(x)) \vee \nu_V(f(y)).$$

Hence V is an intuitionistic L-fuzzy subring of R′.

2.2 THEORM

Let $f : R \to R'$ be a homomorphism. Then the homomorphic preimage of an intuitionistic L-fuzzy subring of a ring R′ is an intuitionistic L-fuzzy subring of R.

Proof :

Since $f : R \to R'$ be a homomorphism, we have
i) $f(x+y) = f(x) + f(y)$ and
ii) $f(xy) = f(x) f(y)$, for all $x, y \in R$.

Let V = f(A), where V is an ILFSR of R′.
We have to prove that A is an ILFSR of R.
Let $x, y \in R$, then

$$\mu_A(x - y) = \mu_V(f(x - y))$$
$$= \mu_V(f(x) - f(y)) \text{ as f is a homomorphism}$$
$$\geq \mu_V(f(x)) \wedge \mu_V(f(y)) \text{ as V is an ILFSR of R}'$$

which implies that

$$\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y).$$

Again,

$$\mu_A(xy) = \mu_V(f(xy))$$
$$= \mu_V(f(x)f(y)) \text{ as f is a homomorphism}$$
$$\geq \mu_V(f(x)) \wedge \mu_V(f(y)) \text{ as V is an ILFSR of R}'$$

which implies that

$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y).$$

Also,

$$\nu_A(x-y) = \nu_v(f(x-y))$$
$$= \nu_v(f(x)-f(y)) \text{ as f is a homomorphism}$$
$$\leq \nu_v(f(x)) \vee \nu_v(f(y)) \text{ as V is an ILFSR of R}'$$

which implies that

$$\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y).$$

Again,

$$\nu_A(xy) = \nu_v(f(xy))$$
$$= \nu_v(f(x)f(y)) \text{ as f is a homomorphism}$$
$$\leq \nu_v(f(x)) \vee \nu_v(f(y)) \text{ as V is an ILFSR of R}'$$

which implies that

$$\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y).$$

Hence A is an intuitionistic L-fuzzy subring of R.

3. ILFSR of a ring R under anti-homomorphism

3.1 THEORM

Let $f : R \to R'$ be an anti-homomorphism. Then the anti-homomorphic image of an intuitionistic L-fuzzy subring of a ring R is an intuitionistic L-fuzzy subring of R′.

Proof :

Since $f : R \to R'$ be an anti-homomorphism, we have

i) $f(x+y) = f(y) + f(x)$ and

ii) $f(xy) = f(y)\, f(x)$, for all $x, y \in R$.

Let V = f(A), where A is an ILFSR of R.

We have to prove that V is an ILFSR of R′.

Now, for f(x), f(y) in R′,

$$\mu_v(f(x)-f(y)) = \mu_v(f(y-x)) \text{ as f is an anti-homomorphism}$$
$$\geq \mu_A(y-x)$$
$$\geq \mu_A(y) \wedge \mu_A(x) \text{ as A is an ILFSR of R}$$
$$= \mu_A(x) \wedge \mu_A(y)$$

which implies that

$$\mu_v(f(x)-f(y)) \geq \mu_v(f(x)) \wedge \mu_v(f(y)).$$

Again,

$$\mu_v(f(x)f(y)) = \mu_v(f(yx)) \text{ as f is an anti-homomorphism}$$
$$\geq \mu_A(yx)$$
$$\geq \mu_A(y) \wedge \mu_A(x) \text{ as A is an ILFSR of R}$$
$$= \mu_A(x) \wedge \mu_A(y)$$

which implies that

$$\mu_v(f(x)f(y)) \geq \mu_v(f(x)) \wedge \mu_v(f(y)).$$

Also,

$$\nu_v(f(x)-f(y)) = \nu_v(f(y-x)) \text{ as f is an anti-homomorphism}$$
$$\leq \nu_A(y-x)$$
$$\leq \nu_A(y) \vee \nu_A(x) \text{ as A is an ILFSR of R}$$
$$= \nu_A(x) \vee \nu_A(y)$$

which implies that

$$\nu_v(f(x)-f(y)) \leq \nu_v(f(x)) \vee \nu_v(f(y)).$$

Again,

$$\nu_v(f(x)f(y)) = \nu_v(f(yx)) \text{ as f is an anti-homomorphism}$$
$$\leq \nu_A(yx)$$
$$\leq \nu_A(y) \vee \nu_A(x) \text{ as A is an ILFSR of R}$$
$$= \nu_A(x) \vee \nu_A(y)$$

which implies that

$$\nu_v(f(x)f(y)) \leq \nu_v(f(x)) \vee \nu_v(f(y)).$$

Hence V is an intuitionistic L-fuzzy subring of R′.

THEORM 3.2

Let f : R → R′ be an anti-homomorphism. Then the anti-homomorphic preimage of an intuitionistic L-fuzzy subring of a ring R′ is an intuitionistic L-fuzzy subring of R.

Proof :

Since f : R → R′ be an anti-homomorphism, we have

i) $f(x+y) = f(y) + f(x)$ and

ii) $f(xy) = f(y)\, f(x)$, for all x, y $\in$R.

Let V = f(A), where V is an ILFSR of R′.

We have to prove that A is an ILFSR of R.

Let x, y $\in$ R, then

$$\mu_A(x-y) = \mu_v(f(x-y))$$
$$= \mu_v(f(y) - f(x)) \text{ as f is an anti-homomorphism}$$
$$\geq \mu_v(f(y)) \wedge \mu_v(f(x)) \text{ as V is an ILFSR of R}'$$
$$= \mu_v(f(x)) \wedge \mu_v(f(y))$$

which implies that

$$\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y).$$

Again,

$$\mu_A(xy) = \mu_v(f(xy))$$
$$= \mu_v(f(y)f(x)) \text{ as f is an anti-homomorphism}$$
$$\geq \mu_v(f(y)) \wedge \mu_v(f(x)) \text{ as V is an ILFSR of R}'$$
$$= \mu_v(f(x)) \wedge \mu_v(f(y))$$

which implies that

$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y).$$

Also,

$$\nu_A(x-y) = \nu_v(f(x-y))$$
$$= \nu_v(f(y) - f(x)) \text{ as f is an anti-homomorphism}$$
$$\leq \nu_v(f(y)) \vee \nu_v(f(x)) \text{ as V is an ILFSR of R}'$$
$$= \nu_v(f(x)) \vee \nu_v(f(y))$$

which implies that

$$\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y).$$

Again,

$$\nu_A(xy) = \nu_v(f(xy))$$
$$= \nu_v(f(y)f(x)) \text{ as f is an anti-homomorphism}$$
$$\leq \nu_v(f(y)) \vee \nu_v(f(x)) \text{ as V is an ILFSR of R}'$$
$$= \nu_v(f(x)) \vee \nu_v(f(y))$$

which implies that

$$\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y).$$

Hence A is an intuitionistic L-fuzzy subring of R.

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