Notes on Intuitionistic Fuzzy Sets
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A study on intuitionistic L-fuzzy T_1 spaces

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Abstract: The paper yields some new conjectures of intuitionistic lattice fuzzy T_1 spaces underlying to the concepts of intuitionistic fuzzy topological spaces. These conjectures convey some appreciable and intriguing properties as "Good extension" and "Hereditary" properties. Despite these, all suppositions sustain under one-one, onto, and continuous mapping.
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1 Introduction

Intuitionistic fuzzy set (IFS) is very beneficial in providing a flexible model to elaborate uncertainty and vagueness involved in decision making. It is pretty useful in situations when the description of a problem by a linguistic variable given in terms of a membership function only seems too rough. Moreover, it uses in medical diagnosis, medical application, career determination, and real-life situations [13, 14, 22, 23].

Fuzzy sets and L-fuzzy sets were introduced by Zadeh [24] in 1965 and Goguen [15] in 1967, respectively. Later in 1983, a generalization of fuzzy sets was proposed by Atanassov [2] as intuitionistic fuzzy sets which incorporate the degree of hesitation called hesitation margin (and is defined as one minus the sum of membership and non-membership degrees respectively) as well as many types of research by the same author and his associates appeared in the literature [3–5]. Afterward, this notion was more generalized to intuitionistic L-fuzzy sets by Atanassov and Stoeva [6]. D. Coker [9–12] first defined intuitionistic fuzzy topological spaces and some of its properties which are in the sense of C. L. Chang [8]. Later, separation axioms of fuzzy topological spaces and intuitionistic fuzzy topological spaces were explored by many fuzzy topologists [7, 18–21], especially E. Ahmed *et al.* [1] defined some categories of intuitionistic fuzzy T_1 spaces, and R. Islam *et al.* [16, 17] defined some types of intuitionistic lattice fuzzy R_1 and T_0 spaces, respectively. In this paper, the authors newly define the inferences of intuitionistic lattice fuzzy T_1 spaces in four different ways by using intuitionistic fuzzy sets and query the property of its.

Throughout this paper, X and Y will be nonempty sets, $\emptyset$ be the empty set, and L will be a complete distributive lattice with the least element 0 and the greatest element 1. $A, B, \ldots$ be intuitionistic L-fuzzy sets, $G, H, \ldots$ be intuitionistic fuzzy sets, τ, s be the intuitionistic L-topologies, t be the intuitionistic topology, I = [0, 1], and the functions $\mu_A : X \to L$ and $\gamma_A : X \to L$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$), respectively.

The rest of this paper is organized as follows: Section 2 presents a brief review of some of the basic definitions of L-fuzzy sets, intuitionistic L-fuzzy sets, intuitionistic fuzzy topology, intuitionistic L-fuzzy mapping, intuitionistic L-topology, and its mapping. In Section 3, we introduce four ideas of intuitionistic L–T_1 spaces and establish a theorem of them. The concepts of "Good extension", "Hereditary" property, and its related theorems are given in Section 4. Finally, Section 5 represents a conclusion of this paper.

2 Preliminaries

We call off some basic definitions and known results of L-fuzzy sets, intuitionistic L-fuzzy sets, intuitionistic fuzzy topology, intuitionistic L-fuzzy mapping, intuitionistic L-topology and its mapping.

Definition 2.1. [21] Let X be a non-empty set and $L = (L; \leq, \wedge, \vee)$ be a complete distributive lattice with the least element 0 and the greatest element 1. An L-fuzzy set in X is a function $\alpha : X \to L$ which assigns to each element $x \in X$ a degree of membership, $\alpha(x) \in L$.

Definition 2.2. [17] Let $f: X \to Y$ be a function and α be L-fuzzy set in X. Then the image $f(\alpha)$ is an L-fuzzy set in Y which membership function is defined by

$$(f(\alpha))(y) = \{sup\,(\alpha\,(x))|f(x) = y\} \text{ if } f^{-1}(y) \neq \emptyset, x \in X$$

$$(f(\alpha))(y) = 0 \text{ if } f^{-1}\,(y) = \emptyset, x \in X.$$

Definition 2.3. [21] Let X be a non-empty set and $(L, \leq)$ be a complete distributive lattice with an involutive order reversing operation $N : L \to L$. An intuitionistic L-fuzzy set (ILFS) A in X is an object having the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$. Where the functions $\mu_A : X \to L$ and $\gamma_A : X \to L$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and every $x \in X$ satisfying $\mu_A(x) \leq N(\gamma_A(x))$.

Let $L(X)$ denote the set of all intuitionistic L-fuzzy set in X. Obviously, every L-fuzzy set $\mu_A(x)$ in X is an intuitionistic L-fuzzy set of the form $(\mu_A, 1-\mu_A)$. Throughout this paper, we use the simpler notation $A = (\mu_A, \gamma_A)$ instead of $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$.

Definition 2.4. [6] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic L-fuzzy sets in X. Then,

(1) $A \subseteq B$ if and only if $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$

(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(3) $A^c = (\gamma_A, \mu_A)$

(4) $A \cap B = (\mu_A \cap \mu_B; \gamma_A \cup \gamma_B)$

(5) $A \cup B = (\mu_A \cup \mu_B; \gamma_A \cap \gamma_B)$

(6) $0_\sim = (0^\sim, 1^\sim)$ and $1_\sim = (1^\sim, 0^\sim)$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an ILFS of X and $B = (\mu_B, \gamma_B)$ be an ILFS of Y. Then $f^{-1}(B)$ is an ILFS of X defined by $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ and $f(A)$ is an ILFS of Y defined by $f(A) = (f(\mu_A), 1 - f(1-\gamma_A))$.

Definition 2.5. [12] An intuitionistic topology (IT for short) on a nonempty set X is a family t of IS's in X satisfies the following axioms:

(i) $\emptyset_\sim, X_\sim \in t$.

(ii) If $G_1, G_2 \in t$ then $G_1 \cap G_2 \in t$.

(iii) If $G_i \in t$ for each $i \in \Lambda$ then $\cup_{i\in\Lambda} G_i \in t$.

Then the pair (X, t) is called an intuitionistic topological space (ITS, for short) and the members of t are called intuitionistic open sets (IOS for short).

Definition 2.6. [1] An ITS (X, t) is called intuitionistic T_1 space ($I - T_1$ space) if for all $x, y \in X, x \neq y,$ there exists an IOS $G = (G_1, G_2), H = (H_1, H_2) \in t$ such that $x \in G_1, y \in G_2$ and $y \in H_1, x \in H_2$.

Definition 2.7. [17] Let $p, q \in L$ and $p + q \leq 1$. An intuitionistic L-fuzzy point (ILFP for short) $x_{(p,q)}$ of X is an ILFS of X defined by

$$x_{(p,q)}(y) = \begin{cases} (p,q)\ if\ y = x, \\ (0,1) if\ y \neq x \end{cases}$$

In this case, x is called the support of $x_{(p,q)}$ and p and q are called the value and none value of $x_{(p,q)}$, respectively. The set of all ILFP of X we denoted it by $S(X)$.

An ILFP $x_{(p,q)}$ is said to belong to an ILFS $A = (\mu_A, \gamma_A)$ of X denoted by $x_{(p,q)} \in A$, if and only if $p \leq \mu_A(x)$ and $q \geq \gamma_A(x)$ but $x_{(p,q)} \notin A$ if and only if $p \geq \mu_A(x)$ and $q \leq \gamma_A(x)$.

Definition 2.8. [17] If A is an ILFS and $x_{(p,q)}$ is an ILFP then the intersection between ILFS and ILFP is defined as $x_{(p,q)} \wedge A = \big(p \wedge \mu_A(x); q \vee \gamma_A(x)\big)$.

Definition 2.9. [17] An intuitionistic L-fuzzy topology (ILFT for short) on X is a family τ of ILFSs in X which satisfies the following conditions:

(i) $0_\sim, 1_\sim \in \tau$.
(ii) If $A_1, A_2 \in \tau$ then $A_1 \cap A_2 \in \tau$.
(iii) If $A_i \in \tau$ for each $i \in \Lambda$ then $\cup_{i\in\Lambda} A_i \in \tau$.

Then the pair (X, τ) is called an intuitionistic L-topological space (ILTS, for short) and the members of τ are called intuitionistic L-fuzzy open sets (ILFOS for short). An intuitionistic L-fuzzy set B is called an intuitionistic L-fuzzy closed set (ILFC for short) if $1 - B \in \tau$.

Definition 2.10. [17] Let (X, τ) and (Y, s) be two ILTSs. Then a map $f: X \to Y$ is said to be

(i) Continuous if $f^{-1}(B)$ is an ILFOS of X for each ILFOS B of Y, or equivalently, $f^{-1}(B)$ is an ILFCS of X for each ILFCS B of Y,
(ii) Open if $f(A)$ is an ILFOS of Y for each ILFOS A of X,
(iii) Closed if $f(A)$ is an ILFCS of Y for each ILFCS A of X,
(iv) A homeomorphism if f is bijective, continuous and open.

3 Definitions and properties of intuitionistic lattice F–T_1 Spaces

In this section, we define four new notions of intuitionistic lattice fuzzy T_1 spaces and establish a theorem of its.

Definition 3.1. An intuitionistic L-topological space (ILTS) (X, τ) is called

(a) $IL - T_1(i)$ if for all $x, y \in X, x \neq y, \exists$ ILOS $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) = 1, \gamma_A(x) = 0; \mu_A(y) = 0, \gamma_A(y) = 1$and $\mu_B(y) = 1, \gamma_B(y) = 0, \mu_B(x) = 0, \gamma_B(x) = 1$.
(b) $IL - T_1(ii)$ if for all $x, y \in X, x \neq y, \exists$ ILOS $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) > 0, \gamma_A(x) = 0; \mu_A(y) = 0, \gamma_A(y) > 0$ and $\mu_B(y) > 0, \gamma_B(y) = 0, \mu_B(x) = 0, \gamma_B(x) > 0$.
(c) $IL - T_1(iii)$ if for all $x, y \in X, x \neq y, \exists$ ILOS $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) > \mu_A(y); \gamma_A(y) > \gamma_A(x)$ and $\mu_B(y) > \mu_B(x); \gamma_B(x) > \gamma_B(y)$.
(d) $IL - T_1(iv)$ if for all $x, y \in X, x \neq y, \exists$ ILOS $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) \neq \mu_A(y); \gamma_A(x) \neq \gamma_A(y)$ and $\mu_B(x) \neq \mu_B(y); \gamma_B(x) \neq \gamma_B(y)$.

Theorem 3.1. Let (X, τ) be an intuitionistic L-topological spaces (ILTS). Then the above four suppositions of it form in the following implications:

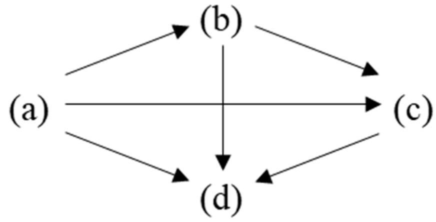

Figure 1. Implications among the lattice fuzzy T_1 concepts

Proof: Suppose that (X,τ) is an $IL-T_1(i)$. Then we have by definition, for all $x,y\in X$, $x\neq y, \exists$ ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B)\in\tau$ such that $\mu_A(x)=1,\gamma_A(x)=0;\mu_A(y)=0$, $\gamma_A(y)=1$ and $\mu_B(y)=1,\gamma_B(y)=0;\mu_B(x)=0,\gamma_B(x)=1$.

$$\Rightarrow \begin{cases}\mu_A(x)>0,\gamma_A(x)=0;\mu_A(y)=0,\gamma_A(y)>0 \text{ and}\\ \mu_B(y)>0,\gamma_B(y)=0,\mu_B(x)=0,\gamma_B(x)>0.\end{cases} \tag{1}$$

$$\Rightarrow \begin{cases}\mu_A(x)>\mu_A(y);\gamma_A(y)>\gamma_A(x) \text{ and}\\ \mu_B(y)>\mu_B(x);\gamma_B(x)>\gamma_B(y).\end{cases} \tag{2}$$

$$\Rightarrow \begin{cases}\mu_A(x)\neq\mu_A(y);\gamma_A(x)\neq\gamma_A(y) \text{ and}\\ \mu_B(x)\neq\mu_B(y);\gamma_B(x)\neq\gamma_B(y).\end{cases} \tag{3}$$

Hence from (1), (2) and (3) we see that $(a)\Longrightarrow(b)\Longrightarrow(c)\Longrightarrow(d)$.
Again suppose that (X,τ) is an $IL-T_1(i)$. Then for all $x,y\in X, x\neq y, \exists$ ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B)\in\tau$ such that $\mu_A(x)=1,\gamma_A(x)=0;\mu_A(y)=0,\gamma_A(y)=1$ and $\mu_B(y)=1,\gamma_B(y)=0;\mu_B(x)=0,\gamma_B(x)=1$.

$$\Rightarrow \begin{cases}\mu_A(x)>\mu_A(y);\gamma_A(y)>\gamma_A(x) \text{ and}\\ \mu_B(y)>\mu_B(x);\gamma_B(x)>\gamma_B(y).\end{cases} \tag{4}$$

$$\Rightarrow \begin{cases}\mu_A(x)\neq\mu_A(y);\gamma_A(x)\neq\gamma_A(y) \text{ and}\\ \mu_B(x)\neq\mu_B(y);\gamma_B(x)\neq\gamma_B(y).\end{cases} \tag{5}$$

From (4) and (5) we see that $(a)\Longrightarrow(c)$ and $(a)\Longrightarrow(d)$.
And finally let (X,τ) is an $IL-T_1(ii)$. Then from (1) for all $x,y\in X, x\neq y,\exists$ ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B)\in\tau$ such that $\mu_A(x)>0,\gamma_A(x)=0;\mu_A(y)=0,\gamma_A(y)>0$ and $\mu_B(y)>0,\gamma_B(y)=0;\mu_B(x)=0,\gamma_B(x)>0$.

$$\Rightarrow \begin{cases}\mu_A(x)\neq\mu_A(y);\gamma_A(x)\neq\gamma_A(y) \text{ and}\\ \mu_B(x)\neq\mu_B(y);\gamma_B(x)\neq\gamma_B(y).\end{cases} \tag{6}$$

From (6) we see that $(b)\Longrightarrow(d)$. □

None of the reverse implications is true in general which can be seen in the following counter examples:

Example 3.1.1. Let $X=\{x,y\}$, and τ be an ILT on X generated by $\{0_\sim,1_\sim,A,B\}$ where $A=\left\{\langle\frac{x}{0.6},\frac{x}{0.4}\rangle,\langle\frac{y}{0.7},\frac{y}{0.3}\rangle\right\}$, and $B=\left\{\langle\frac{x}{0.4},\frac{x}{0.6}\rangle,\langle\frac{y}{0.3},\frac{y}{0.7}\rangle\right\}$. Hence we see that (X,τ) is an $IL-T_1(iv)$ but not $IL-T_1(i)$, $IL-T_1(ii)$, and $IL-T_1(iii)$. Therefore $(d)\nRightarrow(a)$, $(d)\nRightarrow(b)$, and $(d)\nRightarrow(c)$.

Example 3.1.2. Let $X = \{x, y\}$, and τ be an ILT on X generated by $\{0_\sim, 1_\sim, A, B\}$ where $A = \left\{\langle\frac{x}{0.7}, \frac{x}{0.3}\rangle, \langle\frac{y}{0.5}, \frac{y}{0.4}\rangle\right\}$ and $B = \left\{\langle\frac{x}{0.4}, \frac{x}{0.6}\rangle, \langle\frac{y}{0.5}, \frac{y}{0.5}\rangle\right\}$. Hence we see that (X, τ) is an $IL - T_1(iii)$ but not $IL - T_1(i)$, and $IL - T_1(ii)$. Here $(c) \nRightarrow (a)$, $(c) \nRightarrow (b)$. Finally if we consider $A = \left\{\langle\frac{x}{0.1}, \frac{x}{0}\rangle, \langle\frac{y}{0.4}, \frac{y}{0}\rangle\right\}$ and $B = \left\{\langle\frac{x}{0}, \frac{x}{0.3}\rangle, \langle\frac{y}{0.1}, \frac{y}{0}\rangle\right\}$. Hence we see that (X, τ) is an (b) but not (a).

4 "Good extension" and "Hereditary" properties of intuitionistic lattice *F–T*1 (*j*) concepts, where *j* = *i*, *ii*, *iii*, *iv*

In this section, we discuss the "Good extension" and "Hereditary" properties and set up its associated theorems. Furthermore, we observe that all concepts preserve under one-one, onto, and continuous mapping.

Definition 4.1. [1] Let (X, t) be an intuitionistic topological space and let $\tau = \{1_G : G \in t\}$, $1_{(G_1,\ G_2)} = (1_{G_1}, 1_{G_2})$, then (X, τ) is the corresponding intuitionistic *L*-topological space of $(X,\ t)$. Let *P* be a property of intuitionistic topological spaces and *LP* be its intuitionistic *L*-fuzzy topology analogue. Then *LP* is called a 'Good extension' of *P* iff the statement (X, t) has *P* iff (X, τ) has *LP* holds good for every intuitionistic topological space (X, t).

Theorem 4.1. Let (X, t) be an intuitionistic T_1 space and let (X, τ) be an intuitionistic $L - T_1(j)$ spaces, where $j = i, ii, iii, iv$. Then the "Good extension" property of intuitionistic lattice fuzzy $T_1(j)$ spaces is shown in such a way that

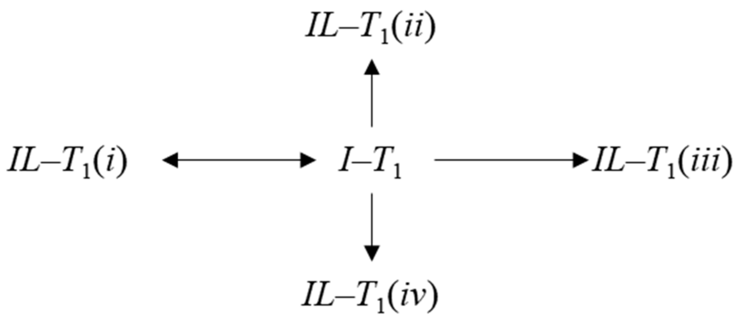

Figure 2. "Good extension" property among the lattice fuzzy T_1 concepts

Proof: Suppose that (X, t) is an $I - T_1$ space. We prove that (X, τ) is $IL - T_1(i)$. Since (X, t) is $I - T_1$ space, then if for all $x, y \in X, x \neq y, \exists$ an IFOS $G = (G_1, G_2),\ H = (H_1, H_2) \in t$ such that $x \in G_1, y \in G_2$ and $y \in H_1, x \in H_2$.

$$\Longrightarrow 1_{G_1}(x) = 1, 1_{G_2}(y) = 1 \text{ and } 1_{H_1}(y) = 1, 1_{H_2}(x) = 1$$

Let $1_{G_1} = \mu_A, 1_{G_2} = \gamma_A$ and $1_{H_1} = \mu_B, 1_{H_2} = \gamma_B$. Then $\mu_A(x) = 1, \gamma_A(x) = 0, \mu_A(y) = 0,$ $\gamma_A(y) = 1$ and $\mu_B(y) = 1, \gamma_B(y) = 0, \mu_B(x) = 0, \gamma_B(x) = 1$. Since $\{(\mu_A, \gamma_A), (\mu_B, \gamma_B)\} \in \tau$, thence (X, τ) is $IL - T_1(i)$. Hence $I - T_1 \Longrightarrow IL - T_1(i)$.

Conversely, suppose that (X,τ) is $IL-T_1(i)$. We prove that (X,t) is $I-T_1$. Since (X,τ) is $IL-T_1(i)$, we have by definition, for all $x,y \in X, x \neq y, \exists$ an ILOS $A=(\mu_A,\gamma_A)$, $B=(\mu_B,\gamma_B) \in \tau$ such that $\mu_A(x)=1, \gamma_A(x)=0, \mu_A(y)=0, \gamma_A(y)=1$ and $\mu_B(y)=1$, $\gamma_B(y)=0,\ \mu_B(x)=0, \gamma_B(x)=1$.

Let ${\mu_A}^{-1}\{1\}=\{x\}=G_1, {\gamma_A}^{-1}\{1\}=\{y\}=G_2$ and ${\mu_B}^{-1}\{1\}=\{y\}=H_1, {\gamma_B}^{-1}\{1\}=\{x\}=H_2 \Rightarrow x \in G_1, y \in G_2$ and $x \in H_2, y \in H_1$.

Since $G=(G_1,G_2)$ and $H=(H_1,H_2) \in t$, henceforth (X,t) is an $I-T_1(i)$. Therefore, we have $I-T_1 \Leftrightarrow IL-T_1(i)$. Consequently, it is shown that $I-T_1 \Rightarrow IL-T_1(ii), I-T_1 \Rightarrow IL-T_1(iii)$ and $I-T_1 \Rightarrow IL-T_1(iv)$. □

None of the reverse implications is true in general which can be seen from the following counter examples:

Example 4.1.1. Let $X=\{x,y\}$, and τ be an ILT on X generated by $\{0_\sim,1_\sim,A,B\}$ where $A=\left\{\langle\frac{x}{0.4},\frac{x}{0}\rangle,\langle\frac{y}{0},\frac{y}{0.7}\rangle\right\}$ and $B=\left\{\langle\frac{x}{0},\frac{x}{0.5}\rangle,\langle\frac{y}{0.4},\frac{y}{0}\rangle\right\}$. Hence we see that (X,τ) is an $IL-T_1(ii)$, $IL-T_1(iii)$, and $IL-T_1(iv)$ but not $I-T_1$. Hence proved. □

Definition 4.2. [17] Let (X,τ) be an ILTS and $U \subseteq X$. We define $\tau_U=\{A|U: A \in \tau\}$ the subspace ILTS on U induced by τ. Then (U,τ_U) is called the subspace of (X,τ) with the underlying set U. An *IL*-topological property '*P*' is called hereditary if each subspace of an *IL*-topological space with property '*P*' also has property '*P*'.

Theorem 4.2. Let (X,τ) be an ILTS and $(U,\ \tau_U)$ be a subspace of its. Then this space (X,τ) holds the hereditary property in the following way that:

(a) (X,τ) is $IL-T_1(i) \Rightarrow (U,\tau_U)$ is $IL-T_1(i)$.
(b) (X,τ) is $IL-T_1(ii) \Rightarrow (U,\tau_U)$ is $IL-T_1(ii)$.
(c) (X,τ) is $IL-T_1(iii) \Rightarrow (U,\tau_U)$ is $IL-T_1(iii)$.
(d) (X,τ) is $IL-T_1(iv) \Rightarrow (U,\tau_U)$ is $IL-T_1(iv)$.

Proof: We prove only (a). Suppose (X,τ) is $IL-T_1(i)$, we prove that (U,τ_U) is $IL-T_1(i)$. Let $x,y \in U, x \neq y$. Then $x,y \in X, x \neq y$ as $U \subseteq X$. Since (X,τ) is $IL-T_1(i)$, we have for all $x,y \in X, x \neq y, \exists$ ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B) \in \tau$ such that $\mu_A(x)=1, \gamma_A(x)=0$; $\mu_A(y)=0, \gamma_A(y)=1$and $\mu_B(y)=1, \gamma_B(y)=0, \mu_B(x)=0, \gamma_B(x)=1$. For $U \subseteq X$, we find ILOS $A\big|U=\left(\mu_{A|U},\gamma_{A|U}\right), B\big|U=\left(\mu_{B|U},\gamma_{B|U}\right) \in \tau_U$ such that $\mu_{A|U}(x)=1, \gamma_{A|U}(x)=0$, $\mu_{A|U}(y)=0,\ \gamma_{A|U}(y)=1$ and $\mu_{B|U}(y)=1, \gamma_{B|U}(y)=0, \mu_{B|U}(x)=0, \gamma_{B|U}(x)=1$ as $U \subseteq X$. Hence (U,τ_U) is $IL-T_1(i)$. (b), (c), and (d) can be proved in similar way. □

Theorem 4.3. Let (X,τ) and (Y,s) be two ILTS, $f:(X,\tau) \to (Y,s)$ be one-one, onto and continuous map. Then these spaces maintain in the succeeding feature.

(a) (X,τ) is $IL-T_1(i) \Leftrightarrow (Y,s)$ is $IL-T_1(i)$
(b) (X,τ) is $IL-T_1(ii) \Leftrightarrow (Y,s)$ is $IL-T_1(ii)$
(c) (X,τ) is $IL-T_1(iii) \Leftrightarrow (Y,s)$ is $IL-T_1(iii)$
(d) (X,τ) is $IL-T_1(iv) \Leftrightarrow (Y,s)$ is $IL-T_1(iv)$

Proof: We prove only (a). Suppose (X,τ) is $IL-T_1(i)$, we prove that (Y,s) is $IL-T_1(i)$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, whereas $\exists\, x_1, x_2 \in X$, such that $f(x_1)=y_1$, $f(x_2)=y_2$ and $x_1 \neq x_2$ as f is one-one. Again since (X,τ) is $IL-T_1(i)$, we have for all $x_1, x_2 \in X$, $x_1 \neq x_2$, there exists an ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B)\in\tau$ such that $\mu_A(x_1)=1, \gamma_A(x_1)=0, \mu_A(x_2)=0, \gamma_A(x_2)=1$ and $\mu_B(x_2)=1, \gamma_B(x_2)=0, \mu_B(x_1)=0, \gamma_B(x_1)=1$. Now there exists an ILOS $f(A)=\big(f(\mu_A), 1-f(1-\gamma_A)\big), f(B)=\big(f(\mu_B), 1-f(1-\gamma_B)\big)\in s$ such that $f(\mu_A)(y_1)=\{\sup \mu_A(x_1): f(x_1)=y_1\}=1$

$\{1-f(1-\gamma_A)\}(y_1)=1-f(1-\gamma_A)(y_1)=1-\{\sup(1-\gamma_A)(x_1): f(x_1)=y_1\}$
$=1-\{\sup\big(1-\gamma_A(x_1)\big): f(x_1)=y_1\}=1-\{\sup(1-0)\}=1-1=0$ and

$$f(\mu_A)(y_2)=\{\sup \mu_A(x_2): f(x_2)=y_2\}=0$$

$\{1-f(1-\gamma_A)\}(y_2)=1-f(1-\gamma_A)(y_2)=1-\{\sup(1-\gamma_A)(x_2): f(x_2)=y_2\}$
$=1-\{\sup\big(1-\gamma_A(x_2)\big): f(x_2)=y_2\}=1-\{\sup(1-1)\}=1-0=1.$ And similarly,

$$f(\mu_B)(y_2)=1; \{1-f(1-\gamma_B)\}(y_2)=0\,; f(\mu_B)(y_1)=0; \{1-f(1-\gamma_B)\}(y_1)=1.$$

Hence (Y,s) is $IL-T_1(i)$.

Conversely suppose that (Y,s) is $IL-T_1(i)$. We prove that (X,τ) is $IL-T_1(i)$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2 \Longrightarrow f(x_1)\neq f(x_2)$ as f is one-one. Put $f(x_1)=y_1$, and $f(x_2)=y_2$, then $y_1 \neq y_2$. Since (Y,s) is $IL-T_1(i)$, $\exists$ ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B)\in s$ such that $\mu_A(y_1)=1, \gamma_A(y_1)=0$; $\mu_A(y_2)=0, \gamma_A(y_2)=1$and $\mu_B(y_1)=0, \gamma_B(y_1)=1$; $\mu_B(y_2)=1, \gamma_B(y_2)=0$

$$\Longrightarrow \begin{cases} \mu_A f(x_1)=1, \gamma_A f(x_1)=0; \mu_A f(x_2)=0, \gamma_A f(x_2)=1 \text{ and} \\ \mu_B f(x_1)=0, \gamma_B f(x_1)=1; \mu_B f(x_2)=1, \gamma_B f(x_2)=0. \end{cases}$$

$$\Longrightarrow \begin{cases} f^{-1}\mu_A(x_1)=1, f^{-1}\gamma_A(x_1)=0; f^{-1}\mu_A(x_2)=0, f^{-1}\gamma_A(x_2)=1 \text{ and} \\ f^{-1}\mu_B(x_1)=0, f^{-1}\gamma_B(x_1)=1; f^{-1}\mu_B(x_2)=1, f^{-1}\gamma_B(x_2)=0. \end{cases}$$

Since $A=(\mu_A,\gamma_A)$, and $B=(\mu_B,\gamma_B)\in s$, therefore $f^{-1}(A)=(f^{-1}(\mu_A), f^{-1}(\gamma_A))$, and $f^{-1}(B)=(f^{-1}(\mu_B), f^{-1}(\gamma_B))\in\tau$. Hence it is clear that $\forall\, x_1, x_2 \in X, x_1 \neq x_2$ such that

$$\Longrightarrow \begin{cases} f^{-1}\mu_A(x_1)=1, f^{-1}\gamma_A(x_1)=0; f^{-1}\mu_A(x_2)=0, f^{-1}\gamma_A(x_2)=1 \text{ and} \\ f^{-1}\mu_B(x_1)=0, f^{-1}\gamma_B(x_1)=1; f^{-1}\mu_B(x_2)=1, f^{-1}\gamma_B(x_2)=0. \end{cases}$$

Hence (X,τ) is also $IL-T_1(i)$. In the same manner, (b), (c), and (d) can be easily proved. □

5 Conclusions

The four innovative notions were proposed base on the conception of intuitionistic lattice fuzzy topological spaces in this article. By careful inspection, the four ideas are more comprehensive than that of Atanassov and Stoeva [6]. Theorem 4.1 and Theorem 4.2 demonstrates that our definitions satisfy the “Good extension” and “Hereditary” properties, respectively. Moreover, (Theorem 4.3) having preserved under one-one, onto, and continuous mapping, these notions maintain the topological property. Consequently, scientific researchers will find significant applications for these theories in shortly.

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