

Intuitionistic Fuzzy Sets Past, Present and Future

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Abstract

Remarks on history, theory, and applications of intuitionistic fuzzy sets are given. Some open problems are introduced.

Keywords: intuitionistic fuzzy sets.

1 Introduction, or The first steps of intuitionistic fuzziness

This paper discusses the origin, motivation, current state of research and open problems of an extension of Zadeh's fuzzy sets [22].

The author would like to ask the reader to let him use, whenever personal attitude or opinion is involved, the first person singular, reserving the usual 'we' for statements whose truth is not subjective.

The beginning of the idea of intuitionistic fuzziness was a happenstance: I was in a hospital and there read the Russian translation of Kaufmann's book [14]¹. It all began as a game: I added to the definition a second degree (degree of non-membership) and studied the properties of a set with both degrees. Of course, I saw that the new set is an extension of the ordinary fuzzy set, but I did not see immediately that it has essentially different properties. So the first research works on IFS followed step by step the existing results on fuzzy sets. Of course, it is not very

difficult to extend formally some concepts. It is interesting to show that the respective extension has specific properties, not available in the basic concept. Just when I convinced myself that the so-constructed sets really had worthy properties, I discussed them with my supervisor at the Mathematical Faculty of Sofia University - George Gargov (7 April 1947 - 9 Nov. 1996) - one of the most colourful Bulgarian mathematicians, and a person with various interests in science - mathematics, physics, biology, philosophy, linguistics, psychology, sociology etc., and arts - literature, music, theatre, cinema, art. He proposed the name "*Intuitionistic Fuzzy Set*" (*IFS*), because the way of fuzzification contains the intuitionistic idea (see, e.g. [13]).

Of course the question "*Are there adequate examples of the new definition?*" immediately arose. The answer is "*yes*". Here is an example (cf. [3]). Let E be the set of all countries with elective governments. Assume that we know for every country $x \in E$ the percentage of the electorate that have voted for the corresponding government. Denote it by $M(x)$ and let $\mu(x) = \frac{M(x)}{100}$ (*degree of membership, validity, etc.*). Let $\nu(x) = 1 - \mu(x)$. This number corresponds to the part of electorate who have not voted for the government. By fuzzy set theory alone we cannot consider this value in more detail. However, if we define $\nu(x)$ (*degree of non-membership, non-validity, etc.*) as the number of votes given to parties or persons outside the government, then we can show the part of electorate who have not voted at all or who have given bad voting-paper and the corresponding number will be $\pi(x) = 1 - \mu(x) - \nu(x)$ (*degree of indeterminacy, uncertainty, etc.*). Thus we can construct

¹In early 80's, only Russian translations of the books [22, 14, 12] were available in Bulgaria and for this reason just these books influenced the development of the first steps of IFS theory.

the set $\{\langle x, \mu(x), \nu(x) \rangle | x \in E\}$ and obviously,

$$0 \leq \mu(x) + \nu(x) \leq 1. \quad (1)$$

Obviously, for every ordinary fuzzy set $\pi_A(x) = 0$ for each $x \in E$ and these sets have the form $\{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle | x \in E\}$.

As it is well-known, in the beginning of the last century L. Brouwer introduced the concept of the *intuitionism*. He invited the mathematicians to remove Aristoteles' law of excluded middle. Therefore, if we have a proposition A , we can state that A is true, that A is false, or that we do not know whether A is true or false. On the level of first order logic, the proposition $A \vee \neg A$ is always valid. In the framework of a G. Boole's algebra this expression has truth value "*true*" (or 1). In the ordinary fuzzy logic of L. Zadeh, as well as in many-valued logics (starting with that of J. Lukasiewicz) the above expression *can* have value smaller than 1. The same is true in the case of IFS, but here this situation occurs on semantical as well as on estimations' level. Practically, we fuzzify our estimation in Brouwer's sense, accounting for the three possibilities. This was Gargov's reason to offer the name "IFS".

Now, it is clear that IFS *can be* different from ordinary fuzzy sets.

In May 1983 it turned out that the new sets allow the definition of operators which are, in a sense, analogous to the modal ones (in the case of ordinary fuzzy sets such operators are meaningless, since they reduce to identity). It was then that the author realized that he had found a promising direction of research and published the results in [1].

In March 1991 the author learned also of the notion of an "IFS" proposed by Gaisi Takeuti and Satako Titani [20]. However, they just put a very different meaning in the same term. Therefore, clearly, the present author and the above two Japanese mathematicians proposed the concept in question independently. My first communication appeared in June 1983 in Bulgarian [1] and English (with some extensions, written together with S. Stoeva) in August 1983 [8], while by this time Takeuti and Titani's paper was in press.

About ten years ago, the question about the name

was asked again. Now there are concepts "*bi-fuzzy set*", "*vague set*", "*neutrosophic set*" and others that are other names of the same object. About 1990 I understood that the Russian mathematician Narin'jani introduced the concept of IFS and studied the same properties that I studied in the hospital several years before me. Unfortunately, up to now I only know this fact, but I had not seen the original Narin'jani's research and I do not know the name that he used.

About 1986 I saw for a first time the concept of "*Interval-Valued Fuzzy Set*" (IVFS), but for a long time I thought that it is introduced in 1984 and not by the original authors. Two years after this in [2] one and in [6] together with G. Gargov, we discussed the equipolence of this concept with IFS. From our construction it is seen that each IFS can be represented by an IVFS and each IVFS can be represented by an IFS. I write these years to emphasis that then I believed IFS were defined prior to IVIFS. Now, I know (merely as a fact, without having seen the original texts) that IVFS are essentially older. Therefore, the question "*what is IFS' justification of existence?*" may be asked. We shall discuss it below.

Finally, I would like to note that it is now quite late to change the name of the IFS, be it good or not.

2 What makes the difference between IFS and the other fuzzy set extensions

As we noted above, IFSs are extensions of the standard fuzzy sets. All results which hold of fuzzy sets can be transformed here, too. Also, any research based on fuzzy sets can be described in terms of IFS.

On the other hand, there have been defined over IFSs not only operations similar to the ordinary fuzzy set ones, but also operators that cannot be defined in case of fuzzy sets.

IFS have geometrical interpretations. The first of them (see Fig. 1) is a trivial modification of the fuzzy set one.

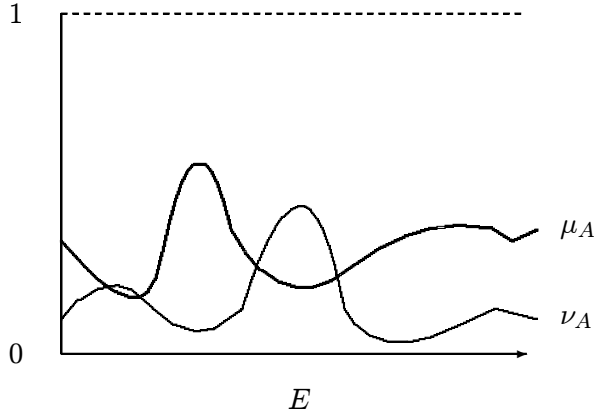


Figure 1

Its analogue is given in Figure 2.

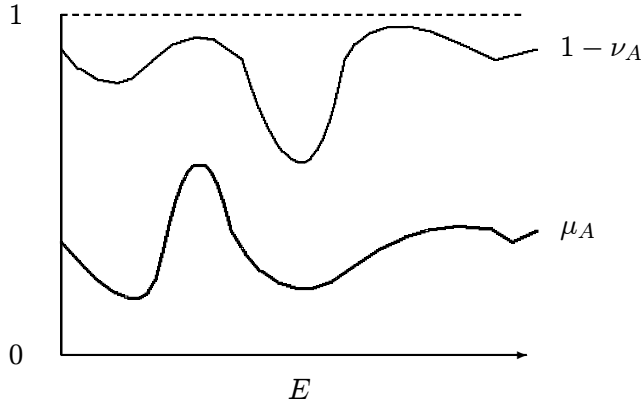


Figure 2

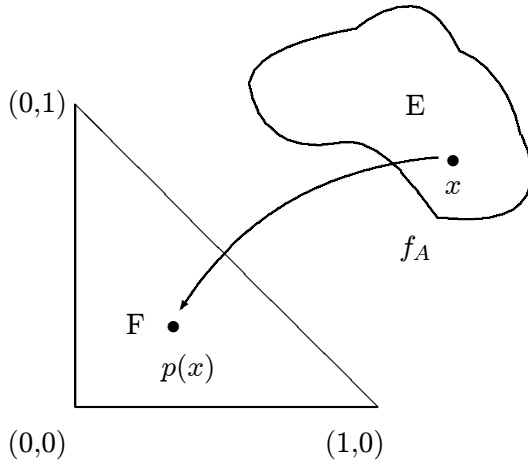


Figure 3

Let a universe E be given. Consider the figure F in the Euclidean plane with a Cartesian coordinate system. The IFS-interpretation in Fig. 3

does not have analogues in fuzzy set theory. Now, all elements of a given fuzzy set will be represented only by points of the hypotenuse.

Let $A \subset E$ be a fixed set. Then we can construct a function f_A from E to F such that if $x \in E$, then $p(x) = f_A(x) \in F$, the point p has coordinates $\langle a, b \rangle$ for which: $0 \leq a + b \leq 1$ and these coordinates are such that $a = \mu_A(x)$, $b = \nu_A(x)$.

We will note that there can exist two elements $x, y \in E$, $x \neq y$, for which $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$ with respect to some set $A \subset E$, i.e., for which $f_A(x) = f_A(y)$.

Similarly to the fuzzy set theory, a large number of relations and operations over IFSs are defined, but more interesting are the modal operators that can be defined over the IFSs. They do not have analogues in fuzzy set theory.

Let A be an IFS and let $\alpha, \beta \in [0, 1]$.

The simplest operators are

$$\begin{aligned} \Box A &= \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle | x \in E \}; \\ \Diamond A &= \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle | x \in E \}. \end{aligned}$$

They are analogous of the modal logic operators “necessity” and “possibility”. In the frameworks of the IFSs theory we can extend these operators, defining the following ones.

$$\begin{aligned} D_\alpha(A) &= \{ \langle x, \mu_A(x) + \alpha \cdot \pi_A(x), \nu_A(x) + (1 - \alpha) \cdot \pi_A(x) \rangle | x \in E \}; \\ F_{\alpha, \beta}(A) &= \{ \langle x, \mu_A(x) + \alpha \cdot \pi_A(x), \nu_A(x) + \beta \cdot \pi_A(x) \rangle | x \in E \}, \text{ where } \alpha + \beta \leq 1; \\ G_{\alpha, \beta}(A) &= \{ \langle x, \alpha \cdot \mu_A(x), \beta \cdot \nu_A(x) \rangle | x \in E \}; \\ H_{\alpha, \beta}(A) &= \{ \langle x, \alpha \cdot \mu_A(x), \nu_A(x) + \beta \cdot \pi_A(x) \rangle | x \in E \}, \\ H_{\alpha, \beta}^*(A) &= \{ \langle x, \alpha \cdot \mu_A(x), \nu_A(x) + \beta \cdot (1 - \alpha \cdot \mu_A(x) - \nu_A(x)) \rangle | x \in E \}, \\ J_{\alpha, \beta}(A) &= \{ \langle x, \mu_A(x) + \alpha \cdot \pi_A(x), \beta \cdot \nu_A(x) \rangle | x \in E \}, \\ J_{\alpha, \beta}^*(A) &= \{ \langle x, \mu_A(x) + \alpha \cdot (1 - \mu_A(x) - \beta \cdot \nu_A(x)), \beta \cdot \nu_A(x) \rangle | x \in E \}. \end{aligned}$$

If we have an ordinary fuzzy set A , then

$$\Box A = A = \Diamond A,$$

while for a proper IFS A :

$$\Box A \subset A \subset \Diamond A$$

and

$$\Box A \neq A \neq \Diamond A.$$

Also the following equalities are valid for each IFS A :

$$\Box \bar{A} = \overline{\Diamond A},$$

$$\Diamond \bar{A} = \overline{\Box A}.$$

In modal logic both operators \Box and \Diamond are related to the last two connections, but no other connection between them is observed. In the IFS-case, we can see that operators D_α and $F_{\alpha,\beta}$ ($\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$) are their direct extensions, because:

$$\Box A = D_0(A) = F_{0,1}(A),$$

$$\Diamond A = D_1(A) = F_{1,0}(A).$$

These equalities show a deeper interconnection between the two ordinary modal logic operators.

The so defined modal operators allow for a more detailed estimation of information. For our electoral example above, using operators \Box and \Diamond , we obtain the same results as by ordinary fuzzy sets: by \Box we obtain that only the people, who voted for the government parties, are for the government and the rest are against, while, by \Diamond - that only the people who voted against government parties are against the government and all the rest support it. Of course, none of these estimations is correct. By opinion polls, using statistical data, or expert knowledge, we can change the actual people opinions with the help of the extended modal operators. For example, operator $F_{\alpha,\beta}$ increases both degrees “for” and “against” government on the basis of the opinion of the people who had not voted. On the other hand, the real estimations for a totalitarian state, where people voted 95 % “for” the “loved” government leaders, 3-4 % put white voting-papers and only 1-2 % are “against”, can be obtained on the basis of the results of the vote, but after their change, e.g., with operators $H_{\alpha,\beta}$ or $H_{\alpha,\beta}^*$. In democratic countries with estimations “for” the government (m) and “against it” (n), where, of course, $m > n$,

the two estimations correspond only to the people opinion on voting day and it will be changed afterwards. This change can be represented by some of the extended modal operators. For example, if the interest in politics falls, we shall use operator $G_{\alpha,\beta}$. In all these examples, the choice of the parameters α and β is important and their values are obtained by experts.

Two analogues of the topological operators can be defined over the IFSs, too: operator “closure” C and operator “intersection” I :

$$C(A) = \{ \langle x, \sup_{y \in E} \mu_A(y), \inf_{y \in E} \nu_A(y) \rangle | x \in E \},$$

$$I(A) = \{ \langle x, \inf_{y \in E} \mu_A(y), \sup_{y \in E} \nu_A(y) \rangle | x \in E \}.$$

It is very interesting to note that the IFS-interpretations of both operators coincide, respectively, with the IFS-interpretations of the logic quantifiers \exists and \forall (see, e.g. [3]).

Finally, it will be interesting to mention that IFSs can be represented in the form $\langle A, B \rangle$, where A and B are ordinary fuzzy sets (see, e.g. [9, 18]). For this reason it may be wrongly considered that IFSs are trivial extensions of ordinary fuzzy sets. We use the following argument against such a claim. To state this is analogous to asserting that the set of the complex numbers is a trivial extension of the set of the real numbers.

All operators discussed above can be transformed for the IVFS case, too, because they are not defined there. On the other hand, using IFS-form, we can work easier with interval data, than with IVFS-form. Also, we can easily interpret interval data as points of the IFS-interpretation triangle. For example, let us have the set of intervals $[a_1, b_1], [a_2, b_2], \dots, [a_n, b_n]$. Let $A \leq \min a_i < \max b_i \leq B$. Of course, $A < B$, because otherwise for all i : $a_i = b_i$. Now, for interval $[a_i, b_i]$ we can construct numbers

$$\mu_i = \frac{a_i - A}{B - A}$$

,

$$\nu_i = \frac{B - b_i}{B - A}$$

that satisfy the condition $0 \leq \mu_i + \nu_i \leq 1$ and have the geometrical interpretation in Fig. 3.

More interesting is the case (see [7]) when interval data are elements of two (or more) sets. Then we can obtain, e.g., IFS-geometrical interpretation in Fig. 4. and using topological operators, defined over IFS, we can separate these sets.

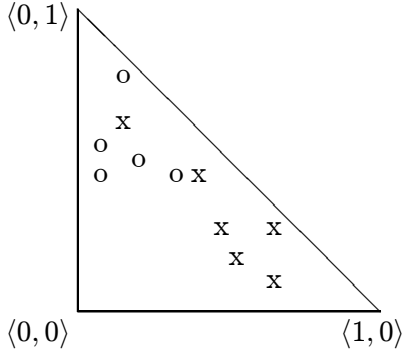


Fig. 4

The concept of IVFS was extended in the sense of the IFS to “interval-valued IFS”. For the new sets it was shown that they have some essentially new properties than the ordinary IFS.

Similar is the situation with the L -fuzzy sets. Really, each IFS can be interpreted as a L -fuzzy set, because the IFS-interpretation triangle can be interpreted as a complete lattice. On the other hand, many kinds of L -fuzzy sets have IFS-interpretation, while still many of them *cannot* represent a given IFS (see the paper of Chris Cornelis, Etienne Kerre and the author for the present conference). Following the idea of the L -fuzzy sets, in 1984 Stefka Stoeva and the author introduced the concept of “Intuitionistic L -Fuzzy Set”. Later, in [10] Dogan Coker proved that Pawlak’s fuzzy rough sets are intuitionistic L -fuzzy sets, while Guo-jun Wang and Ying-Yu He in [21], and Chris Cornelis, Etienne Kerre and Glad Deschrijver in [11] discussed relations between L -fuzzy sets and IFSs.

In [3] there are IFSs over different universes and IFSs of type 2, for which (1) is changed with $\mu_A(x)^2 + \nu_A(x)^2 \leq 1$. It can be easily seen that the latter inequality is a natural extension of the ordinary fuzzy set condition $\mu_A(x) \in [0, 1]$, as well as (1). Of course, we can continue in the direction of increasing the powers, but such sets will not be very applicable. However, the same is the situation with the L -fuzzy sets, when the lattice has a more complex form.

Some other extensions of the IFSs are introduced by S. Rizvi, H.J. Naqvi and D. Nadeem (see [17]), called “rough IFSs”, by P.K. Maji, R. Biswas and A.R. Roy (see [15]), called “intuitionistic fuzzy soft sets” and by S. Samanta and T.K. Mondal (see [19]), called “intuitionistic fuzzy rough sets” and “rough IFSs”.

The author thinks that one of the most useful extensions of the IFS are so called “temporal IFS” introduced in 1990 (see [3, 5]). All operations, relations and operators over IFS can be transferred to them, too. They have the form

$$A(T) = \{ \langle x, \mu_A(x, t), \nu_A(x, t) \rangle \mid \langle x, t \rangle \in E \times T \},$$

where E is a universe, T is a non-empty set and

- (a) $A \subset E$ is a fixed set,
- (b) $\mu_A(x, t) + \nu_A(x, t) \leq 1$ for every $\langle x, t \rangle \in E \times T$,
- (c) $\mu_A(x, t)$ and $\nu_A(x, t)$ are the degrees of membership and non-membership, respectively, of the element $x \in E$ at the time-moment $t \in T$.

The voting example could be much improved if we can test the society attitude to the respective parties and to the government at some time. Using temporal IFSs we can trace it at time.

Similarly to fuzzy sets’, IFS theory also has different aspects.

The algebraic research within IFS theory is aimed at defining intuitionistic fuzzy subgroups, constructing the category IFuz of IFS and other related categories. Intuitionistic fuzzy filters and ideals of lattices are also introduced.

There are different approaches to define IF numbers. P. Burillo, H. Bustince, V. Mohedano and M. Nikolova have worked in this area. T. Buhaescu has investigated interval valued real numbers.

Concepts of convexity and concavoconvexity for IFSs and temporal IFSs are introduced. The concept of intuitionistic (fuzzy) measure is defined. To this end, A. I. Ban introduces the limit of a sequence of IFSs. With the help of an abstract integral he gives a family of intuitionistic fuzzy entropies introduced by Burillo and Bustince. It is

proved that certain intuitionistic fuzzy entropies are intuitionistic fuzzy measures. On this theme there are papers of T. Gerstenkorn, J. Manko, P. Burillo, H. Bustince, and others.

The notion of intuitionistic fuzzy metric space is presented by O. Ozbakir and D. Coker. The solution concept for a semi linear equation with the fuzzy parameters is studied by Keti Peeva and Said Melliani.

A lot of research is devoted to *Intuitionistic Fuzzy Logic (IFL)*. There are a lot of papers by G. Gargov, A. Ban, H. Bustince, E. Kerre, C. Cornelis, N. Nikolov, and the author in which intuitionistic fuzzy propositional and predicate calculus, intuitionistic fuzzy modal and temporal logic are discussed. Norms and metrics over intuitionistic fuzzy logics and relations between the quantifiers and the modal type of operators in intuitionistic fuzzy logics are studied. Rules of inference and the notion of intuitionistic fuzzy deductive closure are investigated.

Intuitionistic fuzzy model of the axioms of the paraconsistent set theory NF_1 , intuitionistic logic and others are presented. Intuitionistic fuzzy interpretation of the conditional logic VW and Kun's axiom are given. It is proved that the Hauber's law is an intuitionistic fuzzy tautology.

In the last ten years IFS were applied in different areas. The IF-approach to artificial intelligence includes treatment of decision making and machine learning, neural networks and pattern recognition, expert systems database, machine reasoning, logic programming and IF Prolog, Petri nets and generalized nets.

In the last ten years IFS were used in the process of decision making. Eulalia Szmidt and Janusz Kacprzyk, Humberto Bustince and Pedro Burillo, Adrian Ban and Cecilia Temponi, Gabriella Pasi, Ronald Yager and the author obtained interesting results in this direction. E. Szmidt and J. Kacprzyk extend the classical Bellman and Zadeh's general approach to decision making under fuzziness, originally termed decision making in a fuzzy environment, to the case of IFS.

Intuitionistic fuzzy versions of one of the basic statistical nonparametrical methods and the k-NN method, are proposed by Ludmila Kouncheva,

Stefan Hadjitodorov, Ognian Asparoukhov and others.

Currently, IFSs have applications in various areas. There are applications of IFSs in medical diagnosis and in decision making in medicine, developed by Anthony Shannon, Soon Ki-Kim, Eulalia Szmidt, Janusz Kacprzyk, Humberto Bustince, Joseph Sorsich and others.

Plamen Angelov has solved some optimization problems by means of intuitionistic fuzzy sets and also has worked on optimization in an intuitionistic fuzzy environment.

There are many applications of IFS in chemistry. Some more interesting of them are following: a method for simulation of complex technological system by use of IF generalized nets, an IF generalized net approach for optimal scheduling of iron ore delivering, discharge and blending yards creation and others.

IFS approach in credit risk assessment is proposed in a series of works by Dinko Dimitrov.

Olympia Georgieva has described the key process variable and corrective actions of the waste water treatment plant with biosorption using the theory of IFSs. There are also IF generalized nets models of the gravitational field, in astronomy, sociology, biology, musicology, controllers, and others.

Intuitionistic fuzzy systems and IF abstract systems are defined and studied by Valentina Radeva, Hristo Aladjov and the author.

A first step to describe a theory of the IF-graphs and temporal IF-graphs is made by Anthony Shannon and the author. Application of IF-graphs and IF-relation methods are also developed.

Of course, the list of the authors and their research is essentially longer and it will be an object of a new research, continuation of [16].

3 About the IFS-future

Like all young theories, the theory of IFSs contains a lot of open problems. While well-established theories contain famous problems with solutions that seem a matter of distant future, a number of "technical" problems persist in

new theories, perhaps not so hard but requiring plenty of time and research effort. IFS theory is now at its beginning, and most of its problems are of the latter type - nevertheless, there are some mathematical challenges of great interest to every researcher.

In [3, 4] the author gave a list of open problems. Here we shall introduce some of the most important.

All notations used here are described in [22]. A list of open problems is introduced there, but the present one is substantially updated and completed.

Problem 1: Construct an axiomatic system of the IFSs.

Problem 2: Develop efficient algorithms for construction of degree of membership and non-membership of a given IFS.

Problem 3: The IFS modal operators \Box and \Diamond are analogues of the same operators from ordinary modal logic. Is it possible to construct in ordinary modal logic analogues of the extensions of the IFS modal operators – $D_\alpha, F_{\alpha,\beta}, G_{\alpha,\beta}$, etc. For example, the fact that $\Box = D_0$ and $\Diamond = D_1$ shows the unity of the two ordinary modal logic operators.

Problem 4: What other norms, distances and metrics (essential from the standpoint of the IFS applications) can be defined over IFSs and over their extensions and what properties will they have?

Problem 5: Develop a theory of IF numbers (and IF complex numbers) and study their properties.

Problem 6: Develop IF-geometry, IF topology, IF analysis - in particular, IF differential and IF integral.

Problem 7: Algorithms for solving IF equations and inequalities and systems of IF equations and inequalities of algebraic, differential and other types.

Problem 8: What other extensions and modifications of the IFSs can be introduced and what properties will they have?

Problem 9: What are the connections between the IFSs (and their modifications) and the other fuzzy set extensions?

Problem 10: Develop an axiomatic systems for the IF logics (propositional, modal. temporal, etc.).

Problem 11: Develop IF Prolog and IF constraint logic programming.

Problem 12: Develop an IF approach to computational linguistics, including an approach to natural language semantics based on IF logic.

Problem 13: Develop IF preference theory and IF utility theory.

Problem 14: Define and study the properties of IF Boolean algebras.

Problem 15: Investigate the concepts of IF-information and IF-entropy.

Problem 16: Develop IF interpretations of abductive and approximate reasoning and possibilistic logic.

Problem 17: Develop statistical and probabilistical tools for IFS and IFL.

Problem 18: Develop algorithms for defuzzification and comparison of IFSs.

Problem 19: Develop an IF interpretation of quantum logic.

Problem 20: Develop an IF interpretation of many-sorted logic.

Solving any of the above problems (as well as a lot of other unformulated here problems) will promote the development of the IFS theory.

For twenty years and especially in the last eight ones, the IFS research increased essentially.

Since 1997, annual conferences on IFS are organized in Bulgaria and since 2001 - another in Poland.

The journal “*Notes on Intuitionistic Fuzzy Sets*” (NIFS, ISSN-1310-4926) is regularly published since 1995.

In [16] Mariana Nikolova, Nikolai Nikolov, Chris Cornelis, and Glad Deschrijver, prepared after an invitation of the Editor-in-Chief Prof. T. Kim and

published in “*Advanced Studies in Contemporary Mathematics*” a survey of the research on intuitionistic fuzzy sets. Now, two years after this the number of papers on IFSs increased by at least a hundred. Following the references of [16] we can see that there are authors from more than 30 countries in the world who work successfully on the theory and applications of the IFSs.

Having in mind all this, the author is an optimist for the future of the IFSs.

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