Notes on Intuitionistic Fuzzy Sets
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(α,β)-Interval-valued intuitionistic fuzzy subgroups

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Abstract: In this paper, (α,β)-interval-valued intuitionistic fuzzy subgroups are studied. The definition of these structures are given by using (α,β)-interval-valued intuitionistic fuzzy sets. The structural properties of these subgroups are studied. Some examples are given about these structures to satisfy the conditions of propositions.
Keywords: Interval-valued intuitionistic fuzzy sets, (α,β)-interval-valued intuitionistic fuzzy sets, (α,β)-interval-valued intuitionistic fuzzy subgroups.
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1 Introduction

The definition of fuzzy sets was given by Zadeh in 1965 [29]. The concept of fuzzy subgroups was introduced by Rosenfeld in 1971 [22]. The structural properties of these groups are studied by same author.

The concept of interval-valued fuzzy set was studied by different authors in [12–15, 23, 28, 30–32]. The topological properties of interval valued fuzzy sets were studied by Mondal and

Samantha [19]. Inter valued fuzzy sets have many studies areas like as graphs in [21].

The definition of interval valued fuzzy subgroups was given by Biswas in 1994 [10]. Many authors studied interval valued fuzzy subgroups [10, 16–18].

Intuitionistic fuzzy sets are defined by Atanassov in 1983 [1]. The algebraic and fundamental properties of these sets are examined by same author [1, 4]. Algebraic studies about intuitionistic fuzzy sets are studied many authors in [24–27].

The concept of interval valued intuitionistic fuzzy set was introduced by Atanassov and Gargov in 1989 [2]. The topological properties of interval valued intuitionistic fuzzy sets were studied by Mondal and Samantha [20].

The definition of interval valued intuitionistic fuzzy subgroups was given by Aygunoglu, Varol, Cetkin and Aygun in 2012 [7].

α-interval valued fuzzy sets are introduced by Çuvalcıoğlu, Bal and Çitil in 2022 [11]. The algebraic properties of α-interval valued fuzzy sets were studied by same authors.

The definition of α-interval valued fuzzy subgroups is introduced by Bal, Çuvalcıoğlu and Tuğrul in 2022 [8]. The structural properties of these subgroups are studied by same authors.

The definition of (α,β)-interval valued intuitionistic fuzzy set is given by Bal, Çuvalcıoğlu and Altıncı in 2023 [9]. The fundamental properties of these sets are examined by same authors.

2 (α,β)-Interval-valued intuitionistic fuzzy sets

Interval valued intuitionistic fuzzy sets which is the generalization of intuitionistic fuzzy sets and interval valued fuzzy sets were introduced by Atanassov and Gargov in 1989 [2]. Membership and non-membership functions on interval valued intuitionistic fuzzy sets are closed sub-intervals whose the sum of supremums is equal to 1 or less than 1 of unit interval [2]. Other fundamental properties of these were studied in [2, 3, 5].

In this paper, $D(I)$ represents all closed sub-intervals of unit interval. The elements of $D(I)$ are shown with capital letters such as M, N, etc. In this place, M^L and M^U are called respectively lower end point and upper end point for interval $M=\left[M^L,M^U\right]$.

In this section, the set of $D(I_\alpha)$ that is all closed sub-intervals of unit interval including $\alpha\in[0,1]$ which is subset of $D(I)$ that mentioned above, is considered.

Definition 1. [9] For all $\alpha,\beta\in[0,1]$ and for all $M\in D(I_\alpha), N\in D(I_\beta)$, it holds that

$$\begin{aligned} D(I_\alpha)\times D(I_\beta) &= \left\{(M,N)\,|\,M\in D(I_\alpha), N\in D(I_\beta) \text{ and } M^U+N^U\le 1\right\} \\ &= \left\{(M,N)\,|\left[M^L,\alpha,M^U\right],\left[N^L,\beta,N^U\right] \text{ such that } M^U+N^U\le 1\right\} \end{aligned}$$

is called (α,β)-interval valued set.

The order relation on $D(I_\alpha)\times D(I_\beta)$ is defined below.

Definition 2. [9] For all $(M,N),(P,R)\in D(I_\alpha)\times D(I_\beta)$,

$$(M,N)\le(P,R):\Leftrightarrow M\le P \text{ and } N\ge R.$$

Here:

$$(M,N)<(P,R)\Leftrightarrow M<P,N\ge R \quad \text{or} \quad M\le P,N>R \quad \text{or} \quad M<P,N>R\ .$$

Proposition 1. [9] *The set* $\left(\mathrm{D}(\mathrm{I}_\alpha)\times\mathrm{D}(\mathrm{I}_\beta),\le\right)$ *is a partial ordered set.*

As a result of above proposition, the defined relation order on $D(I_\alpha)\times D(I_\beta)$ and

$$\inf\{(M,N),(P,R)\}=(\inf\{M,P\},\sup\{N,R\})$$
$$\sup\{(M,N),(P,R)\}=(\sup\{M,P\},\inf\{N,R\})$$

is a lattice because the above conditions are satisfied.

It is seen that the units of this lattice are $([0,1-\beta],[\beta,\beta])$ and $([\alpha,\alpha],[0,1-\alpha])$ from the lemma below.

Lemma 1. [9] $\left(D(I_\alpha)\times D(I_\beta),\wedge,\vee\right)$ *is a complete lattice with units* $([0,1-\beta],[\beta,\beta])$ *and* $([\alpha,\alpha],[0,1-\alpha])$.

Negation function on crisp sets and fuzzy sets is defined below.

Definition 3. [9] *For all* $(M,N)\in D(I_\alpha)\times D(I_\beta)$, the function

$$\mathcal{N}:D(I_\alpha)\times D(I_\beta)\to D(I_\alpha)\times D(I_\beta),$$
$$\mathcal{N}((M,N))=\left(\left[\alpha-M^L,\alpha-\beta+N^U\right],\left[\beta-N^L,\beta-\alpha+M^U\right]\right)$$

is the negation function.

Definition 4. [9] Let X be a universal set. For the functions $M_A:X\to D(I_\alpha)$ and $N_A:X\to D(I_\beta)$, for all $x\in X, M_A{}^U(x)+N_A{}^U(x)\le 1$,

$$A=\{x,M_A(x),N_A(x)\ |\ x\in X\}$$

is called (α,β)-interval-valued intuitionistic fuzzy set. The family of (α,β)-interval-valued intuitionistic fuzzy sets on X is denoted by (α,β)-IVIFS(X).

Some algebraic operations on (α,β)-IVIFS(X) are defined below.

Definition 5. [9] Let X be a universal set, $A,B\in(\alpha,\beta)$-IVIFS(X) and Λ is an index set for all $\lambda\in\Lambda$,

i. $A^c=\left\{\left\langle x,\left[\alpha-M_A{}^L(x),\alpha-\beta+N_A{}^U(x)\right],\left[\beta-N_A{}^L(x),\beta-\alpha+M_A{}^U(x)\right]\right\rangle \mid x\in X\right\};$

ii. $A \sqsubseteq B \Leftrightarrow \forall x \in X, M_A^L(x) \le M_B^L(x), M_A^U(x) \ge M_B^U(x)$ and

$$N_A^L(x) \ge N_B^L(x), N_A^U(x) \le N_B^U(x);$$

iii. $$A \sqcap B = \Big\{\Big\langle x, \big[\inf\{M_A^L(x), M_B^L(x)\}, \sup\{M_A^U(x), M_B^U(x)\}\big],$$
$$\big[\sup\{N_A^L(x), N_B^L(x)\}, \inf\{N_A^U(x), N_B^U(x)\}\big]\Big\rangle \mid x \in X\Big\};$$

iv. $$A \sqcup B = \Big\{\Big\langle x, \big[\sup\{M_A^L(x), M_B^L(x)\}, \inf\{M_A^U(x), M_B^U(x)\}\big],$$
$$\big[\inf\{N_A^L(x), N_B^L(x)\}, \sup\{N_A^U(x), N_B^U(x)\}\big]\Big\rangle \mid x \in X\Big\};$$

v. $$\sqcap_{\lambda\in\Lambda} A_\lambda = \Big\{x, \Big[\underset{\lambda\in\Lambda}{\wedge} M_{A\lambda}^L(x), \underset{\lambda\in\Lambda}{\vee} M_{A\lambda}^U(x)\Big], \Big[\underset{\lambda\in\Lambda}{\vee} N_{A\lambda}^L(x), \underset{\lambda\in\Lambda}{\wedge} N_{A\lambda}^U(x)\Big] \mid x \in X\Big\};$$

vi. $$\sqcup_{\lambda\in\Lambda} A_\lambda = \Big\{x, \Big[\underset{\lambda\in\Lambda}{\vee} M_{A\lambda}^L(x), \underset{\lambda\in\Lambda}{\wedge} M_{A\lambda}^U(x)\Big], \Big[\underset{\lambda\in\Lambda}{\wedge} N_{A\lambda}^L(x), \underset{\lambda\in\Lambda}{\vee} N_{A\lambda}^U(x)\Big] \mid x \in X\Big\}.$$

If one focuses on previous works, it is easy to see that these lower end points and upper end points are intuitionistic fuzzy closer and interior operators [6].

Proposition 2. [9] *Let X be a universal set,* $A, B, C \in (\alpha, \beta)$*-IVIFS(X) and* Λ *is an index set* $\forall \lambda \in \Lambda$. *Then*

$$A \sqcap B = B \sqcap A;$$
$$A \sqcup B = B \sqcup A;$$
$$A \sqcap (B \sqcup C) = (A \sqcap B) \sqcup (A \sqcap C);$$
$$A \sqcup (B \sqcap C) = (A \sqcup B) \sqcap (A \sqcup C);$$
$$A \sqcap (\sqcup_\lambda B_\lambda) = \sqcup_\lambda (A \sqcap B_\lambda).$$

Proposition 3. [9] *Let X be a universal set. Functions* $0_X : X \to D(I) \times D(I)$ *and* $1_X : X \to D(I) \times D(I)$ *such that* $0_X : X \to ([0, 1-\beta], [\beta, \beta])$ *and* $1_X : X \to ([\alpha, \alpha], [0, 1-\alpha])$ are constants:

$$(0_X)^c = 1_X,$$
$$(1_X)^c = 0_X.$$

Definition 6. [9] Let X be a universal set and $A \in (\alpha, \beta)$-IVIFS(X). For all $([\lambda_1, \lambda_2], [\theta_1, \theta_2]) \in D(I_\alpha) \times D(I_\beta)$,

$$A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])} = \{x \in X \mid M_A(x) \ge [\lambda_1, \lambda_2] \text{ and } N_A(x) \le [\theta_1, \theta_2]\}.$$

$A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}$ is called $([\lambda_1, \lambda_2], [\theta_1, \theta_2])$-level subset of A.

It is easily seen from the definition, that $([\lambda_1,\lambda_2],[\theta_1,\theta_2])$-level subsets of A are crisp sets. Besides,

$$M_A(x)\geq[\lambda_1,\lambda_2]\Rightarrow M_A{}^L(x)\geq\lambda_1 \text{ and } M_A{}^U(x)\leq\lambda_2 \ ;$$
$$N_A(x)\leq[\theta_1,\theta_2]\Rightarrow N_A{}^L(x)\leq\theta_1 \text{ and } N_A{}^U(x)\geq\theta_2.$$

Proposition 4. [9] *Let X be a universal set*. For all $A,B\in(\alpha,\beta)$-IVIFS(X), for all $([\lambda_1,\lambda_2],[\theta_1,\theta_2])\in D(I_\alpha)\times D(I_\beta)$. *Then*

i. $x\in A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}\Leftrightarrow(M_A(x),N_A(x))\geq([\lambda_1,\lambda_2],[\theta_1,\theta_2])$;

ii. $A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}=M_{A[\lambda_1,\lambda_2]}\cap N_{A[\theta_1,\theta_2]}$;

iii. $(A\sqcup B)_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}=\left(M_{A[\lambda_1,\lambda_2]}\cup M_{B[\lambda_1,\lambda_2]}\cup\left(M_A{}^L{}_{\lambda_1}\cap M_B{}^U{}_{\lambda_2}\right)\cup\left(M_B{}^L{}_{\lambda_1}\cap M_A{}^U{}_{\lambda_2}\right)\right)\cap\left(N_{A[\theta_1,\theta_2]}\cup N_{B[\theta_1,\theta_2]}\cup\left(N_A{}^L{}_{\theta_1}\cap N_B{}^U{}_{\theta_2}\right)\cup\left(N_B{}^L{}_{\theta_1}\cap N_A{}^U{}_{\theta_2}\right)\right)$;

iv. $(A\sqcap B)_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}=A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}\cap B_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}$.

3 (α,β)-Interval-valued intuitionistic fuzzy subgroups

As a result of the above discussions about the (α,β)-interval valued sets, the definition of (α,β)-interval valued intuitionistic fuzzy subgroup is given below.

Definition 7. Let G be a group and $A\in(\alpha,\beta)$-IVIFS(G). If the following conditions hold, then for all $x,y\in G$, A is called (α,β)-interval valued intuitionistic fuzzy subgroup:

i. $A(xy)\geq\inf\{A(x),A(y)\}$;

ii. $A(x^{-1})\geq A(x)$.

If A is (α,β)-interval valued intuitionistic fuzzy subgroup, then it is shortly denoted by (α,β)-IVIFS(G, A).

Example 1. For $(\mathbb{Z},+)$ Abelian group, $\alpha=\frac{1}{6}$ and $\beta=\frac{2}{3}$. The function $A:\mathbb{Z}\to D\left(I_{\frac{1}{6}}\right)\times D\left(I_{\frac{2}{3}}\right)$,

$$A(0)=\left(\left[\frac{1}{6},\frac{1}{6}\right],\left[0,\frac{5}{6}\right]\right)$$

and

$$A(k)=\begin{cases}\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right), & k \text{ is even and } k\neq 0\\ \left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right), & k \text{ is odd}\end{cases}$$

is a $\left(\frac{1}{6},\frac{2}{3}\right)$-interval valued intuitionistic fuzzy subgroup.

Solution:

i. *k*, *m* are given arbitrary.

<u>Case 1.</u> Let *k* and *m* be even, then $k+m$ is even.

$$A(k+m)=\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right)\geq \inf\left\{\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right),\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right)\right\}=\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right)$$
$$=\inf\{A(k),A(m)\}$$

<u>Case 2.</u> Let *k* be even and *m* be odd, then $k+m$ is odd.

$$A(k+m)=\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right)\geq \inf\left\{\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right),\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right)\right\}=\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right)$$
$$=\inf\{A(k),A(m)\}$$

<u>Case 3.</u> Let *k* be odd and *m* be even, then $k+m$ is odd.

$$A(k+m)=\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right)\geq \inf\left\{\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right),\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right)\right\}=\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right)$$
$$=\inf\{A(k),A(m)\}$$

<u>Case 4.</u> Let *k* and *m* be odd, then $k+m$ is even.

$$A(k+m)=\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right)\geq \inf\left\{\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right),\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right)\right\}=\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right)$$
$$=\inf\{A(k),A(m)\}$$

ii. *k* is a given arbitrary.

<u>Case 1.</u> Let *k* be even, then –*k* is even.

$$A(-k)=\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right)\geq\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right)=A(k)$$

<u>Case 2.</u> Let *k* be odd, then –*k* is odd.

$$A(-k)=\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right)\geq\left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right)=A(k)$$

From the above results, $\left(\frac{1}{6},\frac{2}{3}\right)$-IVIFS($\mathbb{Z}$, *A*).

Proposition 5. *Let G be a group and* (α,β) - *IVIFS*(G,A)). *For all* $x\in G$,

$$A\left(x^{-1}\right)=A(x)$$
$$A(e)\geq A(x)$$

Proof. Let $x\in G$ be arbitrary.

$$(\alpha,\beta)\text{ - }IVIFS(G,A)\Rightarrow A\left(x^{-1}\right)\geq A(x) \quad (1)$$

and

$$A\left(\left(x^{-1}\right)^{-1}\right)\geq A\left(x^{-1}\right)\Rightarrow A(x)\geq A\left(x^{-1}\right) \quad (2)$$

inequalities from (1) and (2),

$$A\left(x^{-1}\right)=A(x)$$
$$A(e)=A\left(xx^{-1}\right)\geq\inf\left\{A(x),A\left(x^{-1}\right)\right\}=\inf\left\{A(x),A(x)\right\}=A(x).$$ □

Proposition 6. *Let G be a group and* $A\in(\alpha,\beta)$-IVIFS(*G*). *Then, for all* $x,y\in G$,

$$(\alpha,\beta)\text{ - }IVIFS(G,A)\Leftrightarrow A\left(xy^{-1}\right)\geq\inf\left\{A(x),A(y)\right\}$$

Proof. Let $x,y\in G$ be given arbitrary, then the proof is split in two cases.

Case " $\Rightarrow$ ".

$$A\left(xy^{-1}\right)\geq\inf\left\{A(x),A\left(y^{-1}\right)\right\}=\inf\left\{A(x),A(y)\right\}$$

Case " $\Leftarrow$ ".

$$A(e)=A\left(xx^{-1}\right)\geq\inf\left\{A(x),A(x)\right\}=A(x)$$
$$\Rightarrow A\left(x^{-1}\right)=A\left(ex^{-1}\right)\geq\inf\left\{A(e),A(x)\right\}=A(x)$$
$$A(xy)=A\left(x\left(y^{-1}\right)^{-1}\right)\geq\inf\left\{A(x),A\left(y^{-1}\right)\right\}\geq\inf\left\{A(x),A(y)\right\}.$$

This completes the proof. □

Proposition 7. *Let G be a group and* $A,B\in(\alpha,\beta)$-IVIFS(*G*). *Then,*

$$(\alpha,\beta)-IVIFS(G,A)\text{ and }(\alpha,\beta)-IVIFS(G,B)\Rightarrow(\alpha,\beta)-IVIFS(G,A\sqcap B)$$

Proof. Let $x,y\in G$ be given arbitrary. Then,

$$(A\sqcap B)\left(xy^{-1}\right)=\inf\left\{A\left(xy^{-1}\right),B\left(xy^{-1}\right)\right\}\geq\inf\left\{\inf\left\{A(x),A(y)\right\},\inf\left\{B(x),B(y)\right\}\right\}$$
$$=\inf\left\{\inf\left\{\left(M_A(x),N_A(x)\right),\left(M_A(y),N_A(y)\right)\right\},\inf\left\{\left(M_B(x),N_B(x)\right),\left(M_B(y),N_B(y)\right)\right\}\right\}$$
$$=\inf\left\{\left(\inf\left\{M_A(x),M_A(y)\right\},\sup\left\{N_A(x),N_A(y)\right\}\right),\left(\inf\left\{M_B(x),M_B(y)\right\},\sup\left\{N_B(x),N_B(y)\right\}\right)\right\}$$
$$=\left(\inf\left\{\inf\left\{M_A(x),M_A(y)\right\},\inf\left\{M_B(x),M_B(y)\right\}\right\},\sup\left\{\sup\left\{N_A(x),N_A(y)\right\},\sup\left\{N_B(x),N_B(y)\right\}\right\}\right)$$
$$=\left(\inf\left\{\inf\left\{M_A(x),M_B(x)\right\},\inf\left\{M_A(y),M_B(y)\right\}\right\},\sup\left\{\sup\left\{N_A(x),N_B(x)\right\},\sup\left\{N_A(y),N_B(y)\right\}\right\}\right)$$
$$=\inf\left\{\left(\inf\left\{M_A(x),M_B(x)\right\},\sup\left\{N_A(x),N_B(x)\right\}\right),\left(\inf\left\{M_A(y),M_B(y)\right\},\sup\left\{N_A(y),N_B(y)\right\}\right)\right\}$$
$$=\inf\left\{(A\sqcap B)(x),(A\sqcap B)(y)\right\}.$$

This completes the proof. □

Proposition 8. *Let G be a group and I be an index set, then for all* $i \in I, A_{i\in I} \in (\alpha,\beta)$-IVIFS($G$), *it holds that*

$$(\alpha,\beta)\text{-}IVIFS(G, A_{i\in I}) \Rightarrow (\alpha,\beta)\text{-}IVIFS(G, \sqcap_{i\in I} A_i).$$

Proof. Let $x, y \in G$ be given arbitrary. Then,

$$\begin{aligned}
&(\alpha,\beta)\text{-}IVIFS(G, A_{i\in I})\\
&\Rightarrow A_{i\in I}(xy^{-1}) \geq \inf\{A_{i\in I}(x), A_{i\in I}(y)\}\\
&\Rightarrow A_{i\in I}(xy^{-1}) \geq \inf\{(M_{A_{i\in I}}(x), N_{A_{i\in I}}(x)), (M_{A_{i\in I}}(y), N_{A_{i\in I}}(y))\}\\
&\Rightarrow A_{i\in I}(xy^{-1}) \geq (\inf\{M_{A_{i\in I}}(x), M_{A_{i\in I}}(y)\}, \sup\{N_{A_{i\in I}}(x), N_{A_{i\in I}}(y)\})\\
&\Rightarrow A_{i\in I}(xy^{-1}) \geq ([\inf\{M_{A_{i\in I}}{}^L(x), M_{A_{i\in I}}{}^L(y)\}, \sup\{M_{A_{i\in I}}{}^U(x), M_{A_{i\in I}}{}^U(y)\}],\\
&\qquad [\sup\{N_{A_{i\in I}}{}^L(x), N_{A_{i\in I}}{}^L(y)\}, \inf\{N_{A_{i\in I}}{}^U(x), N_{A_{i\in I}}{}^U(y)\}])\\
&\Rightarrow \sqcap_{i\in I} A_i(xy^{-1}) = ([\underset{i\in I}{\wedge} M_{A_i}{}^L(xy^{-1}), \underset{i\in I}{\vee} M_{A_i}{}^U(xy^{-1})], [\underset{i\in I}{\vee} N_{A_i}{}^L(xy^{-1}), \underset{i\in I}{\wedge} N_{A_i}{}^U(xy^{-1})])\\
&\Rightarrow \sqcap_{i\in I} A_i(xy^{-1}) \geq ([\underset{i\in I}{\wedge} \inf\{M_{A_i}{}^L(x), M_{A_i}{}^L(y)\}, \underset{i\in I}{\vee} \sup\{M_{A_i}{}^U(x), M_{A_i}{}^U(y)\}],\\
&\qquad [\underset{i\in I}{\vee} \sup\{N_{A_i}{}^L(x), N_{A_i}{}^L(y)\}, \underset{i\in I}{\wedge} \inf\{N_{A_i}{}^U(x), N_{A_i}{}^U(y)\}])\\
&\quad = ([\inf\{\underset{i\in I}{\wedge} M_{A_i}{}^L(x), \underset{i\in I}{\wedge} M_{A_i}{}^L(y)\}, \sup\{\underset{i\in I}{\vee} M_{A_i}{}^U(x), \underset{i\in I}{\vee} M_{A_i}{}^U(y)\}],\\
&\qquad [\sup\{\underset{i\in I}{\vee} N_{A_i}{}^L(x), \underset{i\in I}{\vee} N_{A_i}{}^L(y)\}, \inf\{\underset{i\in I}{\wedge} N_{A_i}{}^U(x), \underset{i\in I}{\wedge} N_{A_i}{}^U(y)\}])\\
&\quad = (\inf\{[\underset{i\in I}{\wedge} M_{A_i}{}^L(x), \underset{i\in I}{\vee} M_{A_i}{}^L(y)], [\underset{i\in I}{\wedge} M_{A_i}{}^U(x), \underset{i\in I}{\vee} M_{A_i}{}^U(y)]\}\\
&\qquad \sup\{[\underset{i\in I}{\vee} N_{A_i}{}^L(x), \underset{i\in I}{\wedge} N_{A_i}{}^L(y)], [\underset{i\in I}{\wedge} N_{A_i}{}^U(x), \underset{i\in I}{\wedge} N_{A_i}{}^U(y)]\})\\
&\quad = \inf\{\sqcap_{i\in I} A_i(x), \sqcap_{i\in I} A_i(y)\} \Rightarrow (\alpha,\beta)\text{-}IVIFS(G, \sqcap_{i\in I} A_i).
\end{aligned}$$

Proposition 9. *Let G be a group. Then,*

$$(\alpha,\beta)\text{-}IVIFS(G,A) \Leftrightarrow \forall([\lambda_1,\lambda_2],[\theta_1,\theta_2]) \in D(I_\alpha)\times D(I_\beta).$$

$$A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])} \neq \varnothing, ([\lambda_1,\lambda_2],[\theta_1,\theta_2])\text{-}\textit{level subset of A is subgroup of G.}$$

Proof.

<u>Case " $\Rightarrow$ ".</u> $x \in G, \exists([\lambda_1,\lambda_2],[\theta_1,\theta_2]) \in D(I_\alpha)\times D(I_\beta), A(x) = ([\lambda_1,\lambda_2],[\theta_1,\theta_2])$

$$\Rightarrow \forall x \in G, A(e) \geq A(x) = ([\lambda_1,\lambda_2],[\theta_1,\theta_2])$$

$$\Rightarrow e \in A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])} \Rightarrow A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])} \neq \varnothing$$

Let $x, y \in A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}$ be given arbitrary. Then,

$$A(x) \geq ([\lambda_1,\lambda_2],[\theta_1,\theta_2]) \text{ and } A(y) \geq ([\lambda_1,\lambda_2],[\theta_1,\theta_2])$$

$$\Rightarrow A(xy^{-1}) \geq \inf\{A(x), A(y)\} \geq ([\lambda_1,\lambda_2],[\theta_1,\theta_2]) \Rightarrow xy^{-1} \in A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}.$$

Case " $\Leftarrow$ ". Assume $\exists x_0, y_0 \in G, A\left(x_0 y_0^{-1}\right) < \inf\left\{A(x_0), A(y_0)\right\}$.

$A(x_0) = \left([a_1, a_2],[s_1, s_2]\right), A(y_0) = \left([b_1, b_2],[t_1, t_2]\right)$ is taken,

$\left([\lambda_1, \lambda_2],[\theta_1, \theta_2]\right) = \inf\left\{\left([a_1, a_2],[s_1, s_2]\right),\left([b_1, b_2],[t_1, t_2]\right)\right\}$ is chosen,

$$
\begin{aligned}
A(x_0) &= \left([a_1, a_2],[s_1, s_2]\right) \\
&\geq \inf\left\{\left([a_1, a_2],[s_1, s_2]\right),\left([b_1, b_2],[t_1, t_2]\right)\right\} \\
&= \left([\lambda_1, \lambda_2],[\theta_1, \theta_2]\right) \\
&\Rightarrow x_0 \in A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])} \\
A(y_0) &= \left([b_1, b_2],[t_1, t_2]\right) \\
&\geq \inf\left\{\left([a_1, a_2],[s_1, s_2]\right),\left([b_1, b_2],[t_1, t_2]\right)\right\} \\
&= \left([\lambda_1, \lambda_2],[\theta_1, \theta_2]\right) \\
&\Rightarrow y_0 \in A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}
\end{aligned}
$$

$$
A\left(x_0 y_0^{-1}\right) < \inf\left\{A(x_0), A(y_0)\right\} = \inf\left\{\left([a_1, a_2],[s_1, s_2]\right),\left([b_1, b_2],[t_1, t_2]\right)\right\}
$$

$$
= \left([\lambda_1, \lambda_2],[\theta_1, \theta_2]\right) \Rightarrow x_0 y_0^{-1} \notin A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])}
$$

is contradiction, then

$$
\forall x_0, y_0 \in G, A\left(x_0 y_0^{-1}\right) \geq \inf\left\{A(x_0), A(y_0)\right\}.
$$

This completes the proof. □

Proposition 10. *Let G be a group and (α, β)-IVIFS (G, A). Then,*

$$
\forall\left([\lambda_1, \lambda_2],[\theta_1, \theta_2]\right) \in D(I_\alpha) \times D(I_\beta),
$$

$$
A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])} \textit{ is a subgroup of } G \Rightarrow M_{A[\lambda_1,\lambda_2]} \textit{ is a subgroup of } G.
$$

Proof.

$$
\forall\left([\lambda_1, \lambda_2],[\theta_1, \theta_2]\right) \in D(I_\alpha) \times D(I_\beta), e \in A_{([\lambda_1,\lambda_2],[\theta_1,\theta_2])} = M_{A[\lambda_1,\lambda_2]} \cap N_{A[\theta_1,\theta_2]}
$$

$$
\Rightarrow e \in M_{A[\lambda_1,\lambda_2]} \textit{ and } e \in N_{A[\theta_1,\theta_2]} \Rightarrow M_{A[\lambda_1,\lambda_2]} \neq \varnothing
$$

$x, y \in M_{A[\lambda_1,\lambda_2]}$ are given arbitrary.

$$
M_A(x) \geq [\lambda_1, \lambda_2] \text{ and } M_A(y) \geq [\lambda_1, \lambda_2]
$$

$$
\Rightarrow M_A\left(xy^{-1}\right) \geq \inf\left\{M_A(x), M_A(y)\right\} \geq [\lambda_1, \lambda_2] \Rightarrow xy^{-1} \in M_{A[\lambda_1,\lambda_2]}
$$

This completes the proof. □

Example 2. For $(\mathbb{Z}, +)$ Abelian group, $\alpha = \frac{1}{6}$ and $\beta = \frac{2}{3}$. Function $A: \mathbb{Z} \to D\left(I_{\frac{1}{6}}\right) \times D\left(I_{\frac{2}{3}}\right)$,

$$
\mathrm{A}(0) = \left(\left[\frac{1}{6}, \frac{1}{6}\right],\left[0, \frac{5}{6}\right]\right)
$$

and

$$A(k)=\begin{cases}\left(\left[\frac{1}{7},\frac{1}{6}\right],\left[\frac{1}{5},\frac{5}{6}\right]\right); & k \text{ is even and } k\neq 0\\ \left(\left[\frac{1}{8},\frac{1}{5}\right],\left[\frac{1}{2},\frac{3}{4}\right]\right); & k \text{ is odd}\end{cases}$$

is a $\left(\frac{1}{6},\frac{2}{3}\right)$-interval valued intuitionistic fuzzy subgroup.

$$\left(\left[\frac{15}{112},\frac{1}{5}\right],\left[\frac{1}{4},\frac{4}{5}\right]\right)\in D\left(I_{\frac{1}{6}}\right)\times D\left(I_{\frac{2}{3}}\right),$$

$A_{\left(\left[\frac{15}{112},\frac{1}{5}\right],\left[\frac{1}{4},\frac{4}{5}\right]\right)}=2\mathbb{Z}$ is a subgroup of $\mathbb{Z}$. $M_{A\left[\frac{15}{112},\frac{1}{5}\right]}=2\mathbb{Z}$ is a subgroup of $\mathbb{Z}$.

Definition 8. Let X be a universal set and $A\in(\alpha,\beta)$-IVIFS(X). Then,

$$A^{*}=\left\{x\in X \mid A(x)>\left([0,1-\beta],[\beta,\beta]\right)\right\}$$

is called support of A.

Definition 9. Let G be a group and $A\in(\alpha,\beta)$-IVIFS(X). Then the set A_{*} is defined as follows,

$$A_{*}=\left\{x\in G \mid A(x)=A(e)\right\}.$$

Proposition 11. Let G be a group and (α,β)-$IVIFS(G,A)$. Then,

i. A^{*} is subgroup of G.

ii. A_{*} is subgroup of G.

Proof. i. $x,y\in A^{*}$ are given arbitrary.

$$A(x)>\left([0,1-\beta],[\beta,\beta]\right) \text{ and } A(y)>\left([0,1-\beta],[\beta,\beta]\right)$$
$$\Rightarrow \inf\left\{A(x),A(y)\right\}>\left([0,1-\beta],[\beta,\beta]\right)$$
$$\Rightarrow A\left(xy^{-1}\right)\geq \inf\left\{A(x),A(y)\right\}>\left([0,1-\beta],[\beta,\beta]\right)$$
$$\Rightarrow xy^{-1}\in A^{*}.$$

ii. $x,y\in A_{*}$ are given arbitrary.

$$A(x)=A(e) \text{ and } A(y)=A(e)\Rightarrow \inf\left\{A(x),A(y)\right\}=A(e)$$
$$\Rightarrow A\left(xy^{-1}\right)\geq \inf\left\{A(x),A(y)\right\}=A(e) \tag{3}$$

and

$$(\alpha,\beta)\text{-}IVIFS(G,A)\Rightarrow A(e)\geq A\left(xy^{-1}\right) \tag{4}$$

From (3) and (4), the below equality is obtained:

$$A\left(xy^{-1}\right)=A(e)\Rightarrow xy^{-1}\in A_{*}.$$

4 Conclusion

In this study, the definition of (α, β)-interval valued intuitionistic fuzzy subgroups is introduced. An example of these structures is given. It is shown that this example satisfies the conditions of (α, β)-interval valued intuitionistic fuzzy subgroups. Other fundamental properties these groups are examined.

The relations between (α, β)-interval valued intuitionistic fuzzy subgroups and the $([\lambda_1, \lambda_2], [\theta_1, \theta_2])$-level subset of (α, β)-interval valued intuitionistic fuzzy sets are studied.

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