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From fuzzy to intuitionistic fuzzy: Easy and lazy

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Abstract: A simple approach is proposed for generating a single intuitionistic fuzzy set from two or more fuzzy sets, defined over the same universe. The membership and the non-membership functions of the constructed intuitionistic fuzzy set are obtained by calculating the minimum and the maximum functions of the given fuzzy sets, with all the rest values lying within the so formed 'belt' of uncertainty in the resultant intuitionistic fuzzy set.

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1 The proposed approach

The present short note proposes an easy and 'lazy' approach to i-fuzzification, i.e. for generating intuitionistic fuzzy sets (IFSs) from given fuzzy sets (FSs). The approach is applicable especially well when we have at our disposal uniformly structured data sets, with no missing or uncertain data, all data belonging in the [0, 1] interval. An idea in the sense of this approach has been presented in [2].

The approach is suitable for using in cases when we prefer to work with one single IFS, rather than with a bunch of many FSs. To construct an IFS from FSs, we are required to have at least two non-identical FSs, while there is no restriction upon the number of FSs in the bunch.

We discuss the cases when the fuzzy data are plottable:

- either linearly, along the x-axis, where the y-axis is confined in the [0, 1] interval,
- or cyclically, in a radar chart (also known as: spider chart, star chart, kiviat chart), which spokes have been normalized to the [0, 1] interval.

The linear graphic representation of a FS is classical in the fuzzy sets theory, while the cyclic data representation in radar charts, though intuitive, has only been proposed recently in [3] for the case of FSs and IFSs.

Let us have a universe X and a bunch of n FSs A_i, $i = 1, \ldots, n$, defined as follows:

$$A_i = \{\langle x, \mu_{A_i}(x)\rangle \mid x \in X\},$$

where $\mu_{A_i}(x)$ is the membership function that attributes an element x of X to the set A_i.

In our approach to i-fuzzification, we perform two operations over the bunch of FSs: intersection and union. We define them below, as follows:

$$\bigcap_{i=1}^{n} A_i = \left\{ \left\langle x, \min_j(\mu_{A_i}(x)) \right\rangle \mid x \in X \right\}$$

$$\bigcup_{i=1}^{n} A_i = \left\{ \left\langle x, \max_j(\mu_{A_i}(x)) \right\rangle \mid x \in X \right\}$$

It is trivial to observe, that $\forall x \in X$, and for $i, j = 1, \ldots, n$, hold

$$\min_j(\mu A_i(x)) \leq \max_j(\mu A_i(x))$$

and

$$\bigcap_{i=1}^{n} A_i \subseteq \bigcup_{i=1}^{n} A_i.$$

Then, we take the so calculated sets of the intersection and union and declare that in the so constructed by us IFS B, the intersection of the bunch of FSs will be assigned the role of the function of intuitionistic fuzzy membership function μ_B, while the union of the bunch of FSs will be assigned the role of the function $(1 - \nu_B)$, where ν_B is, as traditionally defined, the intuitionistic fuzzy non-membership function. Formally written, the IFS B has the form:

$$B = \left\{ \left\langle x, \min_j(\mu_{A_i}(x)), 1 - \max_j(\mu_{A_i}(x)) \right\rangle \mid x \in X \right\}$$

The reason to take the function $(1 - \nu_B)$, rather than the function ν_B, is our desire to conform with the modified form of the standard graphical representation of IFSs [1, p. 38], which visualizes the membership function μ_B bottom up from 0, the non-membership function ν_B top down from 1, thus leaving a belt, representing the IFS's uncertainty function π_B, calculated as $\pi_B = 1 - \mu_B - \nu_B$, where $0 \leq \pi_B \leq 1$.

It is also worth noting that since in the fuzzy sets, there is only a membership function μ_A and its complement to 1, then it holds that $1 - \max(\mu_A) = \min(1 - \mu_A)$, then it is equally true to define the IFS B as

$$B = \left\{ \left\langle x, \min_j(\mu_{A_i}(x)), \ \min_j(1 - \mu_{A_i}(x)) \right\rangle \mid x \in X \right\}.$$

In our approach, all values of the bunch of FSs, which have not been extremal values, actually belong somewhere within this belt of uncertainty in the so constructed IFS. Thus, we opt to ignore these values, when we prefer to work with the single resultant IFS, rather than with the multitude of FSs that produce it along this approach.

2 Numerical example

We will illustrate the proposed approach for i-fuzzification by a simple example with the average monthly temperature in Sofia, Bulgaria in the period 2001–2012 year. We dispose of numerical data [4] as given in Table 1, and we will consider the months as the elements of our universe. We will first norm the available data to the [0; 1]-interval, where "0" can be considered "coldest" and "1" – "hottest" temperature, thus producing 12 FSs (in the rows).

Then we will generate the resultant IFS, based on these 12 FSs, by taking the minimum and maximum values per data indicator, i.e. per set element (in the columns).

Table 1. Average monthly temperatures in Sofia between 2001 and 2012 year, [4].

Month / Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2001	0.9	2.0	10.0	9.6	15.1	17.6	22.0	22.4	16.7	13.2	4.1	-5.2
2002	-2.0	5.2	7.5	9.1	15.8	19.9	22.0	19.2	19.2	19.2	6.5	-0.3
2003	0.3	-3.7	3.9	8.4	17.9	17.9	21.5	22.7	15.3	10.4	10.4	0.3
2004	-2.3	2.1	5.6	11.3	13.4	13.4	20.7	20.1	17.1	12.9	1.6	1.8
2005	0.6	-1.5	4.0	10.6	15.2	17.7	20.3	19.1	16.1	10.3	4.4	1.6
2006	1.8	-0.4	-0.4	-0.4	15.4	17.9	20.3	20.7	16.6	12.5	5.5	0.4
2007	4.5	3.9	6.9	11.6	16.5	20.5	23.7	21.5	14.6	10.8	2.8	-0.7
2008	0.4	3.2	3.2	11.6	15.6	19.7	21.1	23.1	15.6	12.1	6.4	2.5
2009	-0.8	0.7	5.1	11.3	16.3	19.1	21.4	20.8	16.4	11.2	7.5	2.8
2010	0.2	1.8	5.7	10.7	15.6	18.6	18.6	22.5	16.5	16.5	10.5	0.6
2011	-1.3	0.2	5.1	9.9	14.7	19.0	21.8	21.4	19.5	9.4	2.7	1.2
2012	-2.2	-3.6	6.6	11.9	14.8	21.5	24.7	22.9	24.8	14.0	7.7	-0.8

Table 2. Data from Table 1, as normed in the [0; 1]-interval.

Month / Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2001	0.20	0.24	0.51	0.49	0.68	0.76	0.91	0.92	0.73	0.61	0.31	0.00
2002	0.11	0.35	0.42	0.48	0.70	0.84	0.91	0.81	0.81	0.81	0.39	0.16
2003	0.18	0.05	0.30	0.45	0.77	0.77	0.89	0.93	0.68	0.52	0.52	0.18
2004	0.10	0.24	0.36	0.55	0.62	0.62	0.86	0.84	0.74	0.60	0.23	0.23
2005	0.19	0.12	0.31	0.53	0.68	0.76	0.85	0.81	0.71	0.52	0.32	0.23
2006	0.23	0.16	0.16	0.16	0.69	0.77	0.85	0.86	0.73	0.59	0.36	0.19
2007	0.32	0.30	0.40	0.56	0.72	0.86	0.96	0.89	0.66	0.53	0.27	0.15
2008	0.19	0.28	0.28	0.56	0.69	0.83	0.88	0.94	0.69	0.58	0.39	0.26
2009	0.15	0.20	0.34	0.55	0.72	0.81	0.89	0.87	0.72	0.55	0.42	0.27
2010	0.18	0.23	0.36	0.53	0.69	0.79	0.79	0.92	0.72	0.72	0.52	0.19
2011	0.13	0.18	0.34	0.50	0.66	0.81	0.90	0.89	0.82	0.49	0.26	0.21
2012	0.10	0.05	0.39	0.57	0.67	0.89	1.00	0.94	1.00	0.64	0.43	0.15

Due to the seasonal, i.e. cyclic nature of the data, we choose to visualize the data from Table 2 within a radar chart with 12 spokes for the 12 set elements (months). Thus, we obtain Figure 1 below with a bunch of 12 graphics, for each of the FSs (years in the period).

As a next step, we perform the intersection and union of the 12 FSs by calculating the minimum and maximum values, see Table 3 and the resultant Figure 2.

Table 3. Taking the minimal and maximal values of the FS' membership functions for all elements of the universe.

Month / Min, Max	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
min(2001÷2012)	0.10	0.05	0.16	0.16	0.62	0.62	0.79	0.81	0.66	0.49	0.23	0.00
max(2001÷2012)	0.32	0.35	0.42	0.57	0.77	0.89	1.00	0.94	1.00	0.81	0.52	0.27

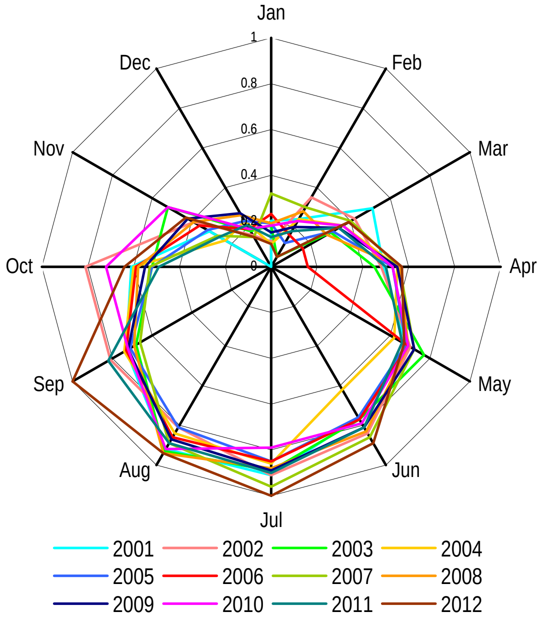

Figure 1. Membership functions of the 12 FS, as plotted on a radar chart.

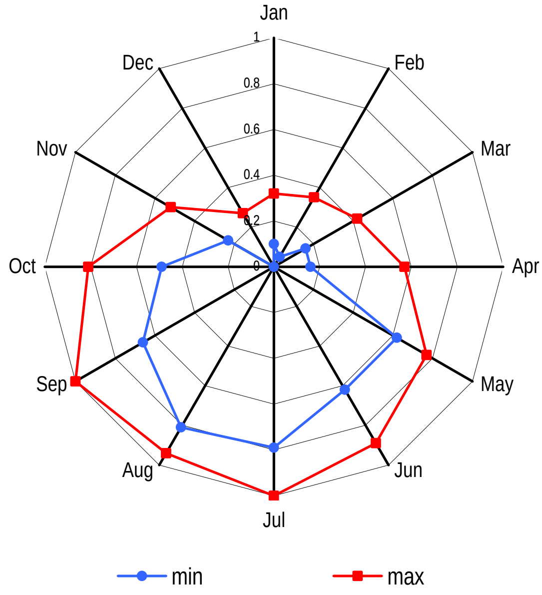

Figure 2. The IFS' membership function μ_B (min) and the non-membership function ν_B (max).

For the sake of completeness, we will also demonstrate the visualization of the same 12 FSs and the resultant single IFS, as plotted on a linear chart, see respectively Figures 3 and 4.

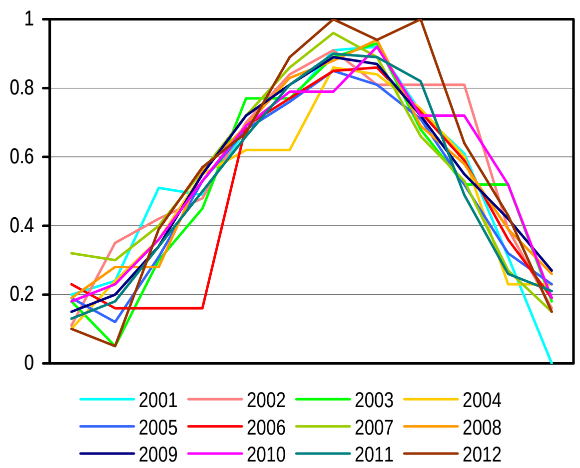

Figure 3. Membership functions of the 12 FS, as plotted on a linear chart.

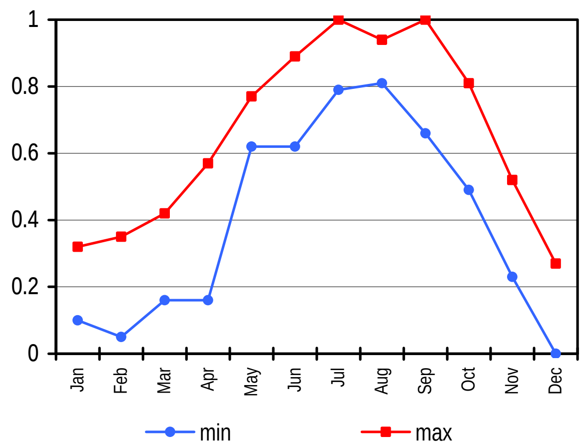

Figure 2. The IFS' membership function μ_B (min) and the non-membership function ν_B (max).

3 Discussions and conclusion

It is noteworthy that the proposed approach works best when there are no missing data along any of the data indicators, i.e. no missing values of the membership functions for the respective elements of the FSs.

However, if we have n FSs A_i, $i = 1, \ldots, n$, sharing the same universe of elements, and if for some element of the universe z we have no information about the value of the membership function in the k-th FS, $\mu_{A_k}(z)$, then there are two possibilities. The first possibility is that we dispose of all the values of

$$\mu_{A_1}(z), \ldots, \mu_{A_{k-1}}(z), \mu_{A_{k+1}}(z), \ldots \mu_{A_n}(z),$$

and at least one of these values equals 1, and, simultaneously, at least one another equals 0. In this very case, regardless of the missing value of $\mu_{A_k}(z)$, it would be possible to construct the maximal and the minimal values across all FSs for that element, and they will be 1 and 0, respectively, i.e. it would be possible to generate the IFS B.

The second possibility is that $\forall j = \{1, \ldots, k-1, k+1, \ldots, n\}$, $0 < \mu_{A_j}(z) < 1$. Then, no categorical conclusion can be reasonably made about the minimum and maximum functions, due to the missing value of $\mu_{A_k}(z)$. In this case, when producing the IFS B, we shall consider that the uncertainty value for this element z, $\pi_B(z) = 1$, leaving no other option for $\mu_B(z)$ and $\nu_B(z)$, but have them both equal to 0. This means that despite of the missing data, the IFS B can again be created, and still be meaningful with respect of the definition of an IFS.

Furthermore, in certain cases, additional considerations can be made about features of the FSs' membership functions' like convergence and monotonicity, thus leading to more categorical conclusions about the form of the resultant IFS, even in cases of partially missing data in the input FSs, which do not require us to consider the case of complete uncertainty, as in the discussion above.

In another line of research, it would be interesting to consider how this approach can be modified to conform to the triangular graphic representation of IFSs, [1, p. 39], which is also a popular and used one.

In conclusion, the herewith presented approach to i-fuzzification is not the sole one, as shown in [2]. It is considered to be best applicable to relatively tight bunches of different fuzzy sets, exhibiting membership functions with similar behaviour and ranges of values. However, its simplicity, together with the proposed usage of radar chart visualization, can make it well applicable in practice.

References

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