Seventh Int. Conf. on IFSs, Sofia, 23-24 August 2003

NIFS Vol. 9 (2003), 3, 40-45

Finite intuitionistic fuzzy machines

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Abstract. We define finite intuitionistic fuzzy machines and investigate their behavior.

Key words: Finite intuitionistic fuzzy machine.

1 Introduction

Fuzzy machines were proposed by Santos in [12, 13] and studied by him in [12 – 15], by Santos and Wee [16]. During the next two decades Adamek and Wechler [1], Wechler [19, 20], Peeva [6, 7], Peeva and co-authors [10, 18] were active in this area.

Now fuzzy sequential machines are subject of the monographs [3] by Malik and Mordeson, [4] by Mordeson and Malik.

All the above publications concern mainly max-min fuzzy sequential machines.

As soon as the concept of fuzzy sequential machine had been introduced [12, 13], Santos studied equivalence, reduction and minimization problems for finite max-product fuzzy machines [15] and for finite maximin fuzzy machines [14]. Most of the results presented in [14, 15] have counterparts in the theory of stochastic sequential machines [5, 17].

In order to investigate finite max-product or maximin fuzzy machines a type of algebra, called the max-product (maximin, respectively) algebra, has been developed by Santos [14, 15].

Next development of the theory for behavior, reduction and minimization for finite L-fuzzy machines is presented by Peeva [8]. It is valid in particular for Santos' maximin machines.

In this paper we introduce intuitionistic fuzzy machines and provide computing their complete behavior. In Section 2 we give the necessary preliminaries for direct problem resolution in intuitionistic fuzzy relational calculus. It provides computing the behavior of finite intuitionistic fuzzy machines (Section 3). The last Section 4 describes open problems (equivalence, reduction and minimization) that will be subject of next papers.

2 Direct Problem Resolution in Intuitionistic Fuzzy Relational Calculus

The terminology for intuitionistic fuzzy sets is according to [2].
Let $I = ([0,1], \max, \min, 0, 1)$ (shortly I) denote the real closed interval $[0, 1]$ with universal bounds 0 and 1 and with operations $\max : [0,1]\times[0,1] \to [0,1]$ and $\min : [0,1]\times[0,1] \to [0,1]$.

The matrix $A = \{(\mu_{ij}^A, \nu_{ij}^A)\}_{m\times n}$ with elements $\mu_{ij}^A, \nu_{ij}^A \in [0,1]$ such that $0 \le \mu_{ij}^A + \nu_{ij}^A \le 1$ for each i, j, is called an *intuitionistic fuzzy matrix* [11].

Two intuitionistic fuzzy matrices $A_{m\times p}$ and $B_{p\times n}$ are called *conformable*, if the number of the columns in $A_{m\times p}$ equals the number of the rows in $B_{p\times n}$. This means that the product of $A_{m\times p}$ and $B_{p\times n}$, in this order, makes sense.

Let $A = \{(\mu_{ij}^A, \nu_{ij}^A)\}_{m\times p}$ and $B = \{(\mu_{ij}^B, \nu_{ij}^B)\}_{p\times n}$ be given finite conformable intuitionistic fuzzy matrices. The matrix $C = \{(\mu_{ij}^C, \nu_{ij}^C)\}_{m\times n} = A * B$ is called ***standard-costandard product*** (shortly S-C product) of A and B [11] if $\mu_{ij}^C = \max\limits_{k=1}^{p} \{\min(a_{ik}, b_{kj})\}$ and $\nu_{ij}^C = \min\limits_{k=1}^{p} \{\max(a_{ik}, b_{kj})\}$ when $1 \le i \le m$, $1 \le j \le n$. (1)

It is proved in [9] that if $A = \{(\mu_{ij}^A, \nu_{ij}^A)\}_{m\times p}$ and $B = \{(\mu_{ij}^B, \nu_{ij}^B)\}_{p\times n}$ are finite conformable intuitionistic fuzzy matrices, their S-C product $C = \{(\mu_{ij}^C, \nu_{ij}^C)\}_{m\times n} = A * B$ is also intuitionistic fuzzy matrix. Computing the S-C product is called *direct problem resolution.* Direct problem for S-C product is solvable in polynomial time [11]. Computing S-C product is among the options of the software for direct problem resolution for various matrix operations, described in [11].

3 Finite intuitionistic fuzzy machines. Behavior of finite intuitionistic fuzzy machines

We introduce finite intutionistic fuzzy machines that are not defined in the literature.
A ***finite intuitionistic fuzzy machine*** (IFM) is a quadruple $A=(X,Q,Y,M)$, where:

i) X,Q,Y are finite nonempty sets of ***input letters***, ***states*** and ***output letters***, respectively;

ii) $M=\{M(x\,|\,y)|\ x\in X,\ y\in Y\}$ is the set of (the transition-output) intuitionistic fuzzy matrices of A, that determines its ***stepwise behavior***.

Each $M(x\,|\,y)\in M$ is a square matrix of order $|Q|$ with elements $(\mu_{qq'}(x\,|\,y), \nu_{qq'}(x\,|\,y))$, where $\mu_{qq'}(x\,|\,y), \nu_{qq'}(x\,|\,y)\in[0,1]$ and $0\le\mu_{qq'}(x\,|\,y)+\nu_{qq'}(x\,|\,y)\le 1$. We interpret $\mu_{qq'}(x\,|\,y)$ as degree of membership and $\nu_{qq'}(x\,|\,y)$ as degree of non-membership that the finite intuitionistic fuzzy machine will enter state q' and produce output y given that the present state is q and the input is x.

In this section we compute the input-output behavior of IFMs as provided by the direct problem resolution, cf. Section 2. An infinite matrix describes the complete input-output behavior of any IFM.

The ***free monoid of the words*** over the input set X (output set Y) is denoted by X^* (by Y^*, respectively) with the ***empty word*** e as the identity element. The ***length of the word*** u is denoted by $|u|$. By definition $|e|=0$. If $X\neq\varnothing$ and $Y\neq\varnothing$ then X^*, or Y^*, respectively, are countably infinite.

We denote by $(X|Y)^*=\{(u\,|\,v)|\ u\in X^*, v\in Y^*, |u|=|v|\}$ the set of all input-output pairs of words of the same length. For $u\in X^*$, $v\in Y^*$, if $|u|=|v|$, we write $(u\,|\,v)\in(X\,|\,Y)^*$.

Let $A=(X,Q,Y,M)$ be an IFM. We extend its stepwise behavior to the extended input-output behavior: for any pair $(u\,|\,v)\in(X\,|\,Y)^*$ we multiply the corresponding stepwise behavior matrices from the set M and compute $M(u\,|\,v)$ as S-C product according to (1):

Let $A=(X,Q,Y,M)$ be an IFM. The ***extended input-output behavior of*** A for $(u\,|\,v)\in(X\,|\,Y)^*$ is defined by the square matrix $M(u\,|\,v)$ of order $|Q|$:

$$M(u\,|\,v)=\begin{cases}M(x_1\,|\,y_1)*\cdots*M(x_k\,|\,y_k), \text{ if } (u\,|\,v)=(x_1\ldots x_k\,|\,y_1\ldots y_k),\ k\ge 1;\\ U, \text{if } (u\,|\,v)=(e\,|\,e)\end{cases}, \qquad (2)$$

where $U=(\delta_{ij})$ is the square matrix of order $|Q|$ with elements $\delta_{ij}=\begin{cases}(1,0) \text{ if } i=j;\\ (0,1) \text{ if } i\neq j\end{cases}$.
The interpretation of the elements of $M(u\,|\,v)$ is as follows: each element $m_{qq'}(u\,|\,v)$ in $M(u\,|\,v)$ determines the operation of A under the input word u beginning at state q, producing the output word v and reaching the state q' after $|u|=|v|$ consecutive steps.

Example 1

Let the finite intuitionistic fuzzy machine $A = (X, Q, Y, M)$ over $\mathbf{I} = ([0,1], \max, \min, 0, 1)$ be given with the following data:

$$X = \{x\},\ Y = \{y_1, y_2\},\ Q = \{q_1, q_2\},$$

$$M(x \mid y_1) = \begin{pmatrix} (0.1, 0.8) & (0.5, 0.5) \\ (0.2, 0.7) & (0.3, 0.6) \end{pmatrix},\ M(x \mid y_2) = \begin{pmatrix} (0, 0.9) & (0.15, 0.6) \\ (0.4, 0.5) & (1, 0) \end{pmatrix}.$$

Its extended input-output behavior for $(u \mid v) = (xx \mid y_1 y_2)$ according to (2) is:

$$M(u \mid v) = M(xx \mid y_1 y_2) = M(x \mid y_1) \bullet M(x \mid y_2) = \begin{pmatrix} (0.4, 0.5) & (0.5, 0.5) \\ (0.3, 0.6) & (0.3, 0.6) \end{pmatrix}.$$

If we are not interested in the next state q', we consider the input-output behavior of A:

The ***input-output behavior*** of A for $(u \mid v) \in (X \mid Y)^*$, is defined by the matrix

$$T(u \mid v) = \left(t_q(u \mid v)\right)_{\mid Q \mid \times 1} = \begin{cases} M(u \mid v) * E, \text{ if } (u \mid v) \neq (e \mid e); \\ E, \text{ if } (u \mid v) = (e \mid e) \end{cases},$$

where $E = \left((1,0)\ \ (1,0)\ \ \cdots (1,0)\right)^t_{\mid Q \mid \times 1}$. Each element $t_q(u \mid v)$ of $T(u \mid v)$ determines the operation of A under the input word u beginning at state q and producing the output word v after $|u| = |v|$ consecutive steps. This is the way of achieving maximal degree of membership and minimal degree of non-membership.

We use the matrices $T(u \mid v)$, $(u \mid v) \in (X \mid Y)^*$ to describe the complete input-output behavior of A. To this aim we suppose that $(X \mid Y)^*$ is ordered lexicographically [3, 14]. Let T be the semi-infinite matrix with $|Q|$ rows and with columns $T(u \mid v)$, ordered according to the lexicographical order in $(X \mid Y)^*$. The ***complete behavior matrix*** T describes the ***complete input-output behavior*** of A, cf. Table 1.

	$T(e \mid e)$	$T(x_1 \mid y_1)$	...	$T(u \mid v)$	...
q_1	(1,0)	$t_{q_1}(x_1 \mid y_1)$	...	$t_{q_1}(u \mid v)$	...
q_2	(1,0)	$t_{q_2}(x_1 \mid y_1)$	...	$t_{q_2}(u \mid v)$	...
...	...	...	...	...	...
q_n	(1,0)	$t_{q_n}(x_1 \mid y_1)$	...	$t_{q_n}(u \mid v)$	...
length l	$l = 0$	$l = 1$	...	$l = \|u\|$	...

Table 1: The complete behavior matrix T

Example 2

Let the IFM $A = (X, Q, Y, M)$ over $\mathbf{I} = ([0, 1], \max, \min, 0, 1)$ be given with the following data:

$$X = \{x_1, x_2\},\ Y = \{y_1, y_2\},\ Q = \{q_1, q_2, q_3\},$$

$$M(x_1 \mid y_1) = \begin{pmatrix} (0.3,\ 0.7) & (0,\ 1) & (0.2, 0.7) \\ (0,\ 0.9) & (0.1,\ 0.8) & (0.5, 0.5) \\ (0.5,\ 0.5) & (0,\ 1) & (0.6, 0.3) \end{pmatrix},\ M(x_1 \mid y_2) = \begin{pmatrix} (0,\ 1) & (0,\ 1) & (0.1,\ 0.8) \\ (0.2,\ 0.7) & (0,\ 1) & (0.3,\ 0.7) \\ (0.6,\ 0.3) & (0,\ 1) & (0.6,\ 0.3) \end{pmatrix},$$

$$M(x_2 \mid y_1) = \begin{pmatrix} (0.8, 0.1) & (0, 1) & (0, 1) \\ (0, 1) & (0.3, 0.6) & (0, 1) \\ (0.2, 0.7) & (0.1, 0.7) & (0.5, 0.4) \end{pmatrix},\ M(x_2 \mid y_2) = \begin{pmatrix} (0.9, 0) & (0.7, 0.3) & (0, 1) \\ (0.1, 0.7) & (0.6, 0.3) & (0, 1) \\ (0.4, 0.6) & (0.3, 0.7) & (0.2, 0.7) \end{pmatrix}.$$

For convenience we write

$$T_{1/1} = T(x_1 \mid y_1),\ T_{1/2} = T(x_1 \mid y_2),\ T_{2/1} = T(x_2 \mid y_1),\ T_{2/2} = T(x_2 \mid y_2).$$

We multiply the matrices and according to (1) and (2) we obtain:

$$M(x_1x_1 \mid y_1y_1) = \begin{pmatrix} (0.3, 0.7) & (0, 1) & (0.2, 0.7) \\ (0.5, 0.5) & (0.1, 0.8) & (0.5, 0.5) \\ (0.5, 0.5) & (0, 1) & (0.6, 0.3) \end{pmatrix},\qquad T_{11/11} = T(x_1x_1 \mid y_1y_1) = \begin{pmatrix} (0.3, 0.7) \\ (0.5, 0.5) \\ (0.6, 0.3) \end{pmatrix};$$

$$M(x_1x_1 \mid y_1y_2) = \begin{pmatrix} (0.2, 0.7) & (0, 1) & (0.2, 0.7) \\ (0.5, 0.5) & (0, 1) & (0.5\ 0.5) \\ (0.6, 0.3) & (0, 1) & (0.6, 0.3) \end{pmatrix},\qquad T_{11/12} = T(x_1x_1 \mid y_1y_2) = \begin{pmatrix} (0.2, 0.7) \\ (0.5, 0.5) \\ (0.6, 0.3) \end{pmatrix},$$

etc.

	$T_{e/e}$	$T_{1/1}$	$T_{1/2}$	$T_{2/1}$	$T_{2/2}$	$T_{11/11}$	$T_{11/12}$	...
q_1	(1, 0)	(0.3, 0.7)	(0.1, 0.8)	(0.8, 0.1)	(0.9, 0)	(0.3, 0.7)	(0.2, 0.7)	...
q_2	(1, 0)	(0.5, 0.5)	(0.3, 0.7)	(0.3, 0.6)	(0.6, 0.3)	(0.5, 0.5)	(0.5, 0.5)	...
q_3	(1, 0)	(0.6, 0.3)	(0.6, 0.3)	(0.5, 0.4)	(0.4, 0.6)	(0.6, 0.3)	(0.6, 0.3)	...
l	$l = 0$	$l = 1$				$l = 2$		

Table 2: An initial fragment from T

4 Conclusions

The next conventional problems concern equivalence, reduction and minimization of IFMs. They need methods and software for inverse problem resolution [9, 11] and will be subject of next papers.

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