On a Game-Method for Modelling with Intuitionistic Fuzzy Estimations. Part 1

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Abstract. A new extension of Conway's Game of Life is introduced. It is based on a previous Conway's game extension, given by the authors. Now we use elements of intuitionistic fuzziness that give more detailed estimations of the degrees of existence and of the non-existence of the objects occuring the cells of the game plane. Rules for the motions and rules for the interactions among the objects are dicsussed.

1 Introduction

In a series of papers (see [7, 11]), the authors extended the standard Conway's Game of Life (CGL) (see, e.g., [17]), adding in it intuitionistic fuzzy estimations (for intuitionistic fuzziness see [5]). On the other hand, more than 30 years ago the authors introduced the idea for another extension of CGL, called "game-method for modelling" (GMM), and its application in astronomy and combinatorics (see [1–3, 8, 16]). This idea found particular application in growth and dynamics of forest stands (see, [12, 13]) and forest fires (see [14]).

Here we will introduce a new extension of the standard CGL on the basis of both modifications, discussed by us.

2 Description of the game-method for modelling from crisp point of view

The standard CGL has a "universe" which is an infinite two-dimensional orthogonal grid of square cells, each of which is in one of two possible states, alive or dead, or (as an equivalent definition) in the square there is an asterisk, or not. The first situation corresponds to the case when the cell is alive and the second – to the case when the cell is dead.

Now, following the ideas from [1, 2, 8], we will extend the CGL.

Let us have a set of symbols S and an n-dimensional simplex (in the sense of [15]) comprising of n-dimensional cubes (at $n = 2$, a two-dimensional net of squares). Let material points (or, for brief, objects) be found in some of the vertices of the simplex and let a set of rules A be given, containing:

1) rules for the motions of the objects along the vertices of the simplex;

2) rules for the interactions among the objects.

Let the rules from the i-th type be marked as i-rules, where $i = 1, 2$.

When $S = \{*\}$, we obtain the standard CGL.

To each object is associated its number, n-tuple of coordinates characterizing its location in the simplex and a symbol from S reflecting the peculiarity of the object (e.g. in physical applications – mass, charge, concentration, etc.). We shall call an *initial configuration* every ordered set of $(n+2)$-tuples with an initial component being the number of the object; the second, third, etc. until the $(n+1)$-st – its coordinates; and the $(n+2)$-nd – its symbol from S. We shall call a *final configuration* the ordered set of $(n+2)$-tuples having the above form and being a result of a (fixed) initial configuration, modified during a given number of times when the rules from A have been applied.

The single application of a rule from A over a given configuration K will be called an elementary step in the transformation of the model and will be denoted by $A_1(K)$. In this sense, if K is an initial configuration, and L is a final configuration derived from K through multiple application the rules from A, then configurations $K_0, K_1, ..., K_m$ will exist, for which $K_0 = K, K_{i+1} = A_1(K_i)$ for $0 \le i \le m-1, K_m = L$, (the equality "=" is used in the sense of coincidence in the configurations) and this will be denoted by

$$L = A(K) \equiv A_1(A_1(...A_1(K)...)).$$

Let a rule P be given, which juxtaposes to a combination of configurations M a single configuration $P(M)$ being the mean of the given ones. We shall call this rule a *concentrate rule.* The concentration can be made either over the values of the symbols from S for the objects, or over their coordinates, (not over both of them simultaneously).

For example, if k-th element of M $(1 \le k \le s$, where s is the number of elements of $M)$ is a rectangular with $p \times q$ squares and if the square staying on (i,j)-th place $(1 \le i \le p, 1 \le j \le q)$ contains number $d^k_{i,j} \in \{0, 1, ..., 9\}$, then on the (i,j)-th place of $P(M)$ stays:

– minimal number

$$d_{i,j} = [\frac{1}{s}\sum_{k=1}^{s} d^k_{i,j}],$$

– maximal number

$$d_{i,j} = \lceil \frac{1}{s}\sum_{k=1}^{s} d^k_{i,j} \rceil,$$

– average number

$$d_{i,j} = [\frac{1}{s}\sum_{k=1}^{s} d^k_{i,j} + \frac{1}{2}],$$

where for real number $x = a + \alpha$, where a is a natural number and $\alpha \in [0, 1)$: $[x] = a$ and

$$\lceil x \rceil = \begin{cases} a, & \text{if } \alpha = 0 \\ a + 1, & \text{if } \alpha > 0 \end{cases}$$

Let B be a criterion derived from physical or mathematical considerations. For two given configurations K_1 and K_2, it answers the question whether they are close to each other or not. For example, for two configurations K_1 and K_2 having the form from the above example,

$$B(K_1, K_2) = \frac{1}{p.q} \sum_{i=1}^{p} \sum_{j=1}^{q} |d^1_{i,j} - d^2_{i,j}| < C_1$$

or

$$B(K_1, K_2) = (\frac{1}{p.q} \sum_{i=1}^{p} \sum_{j=1}^{q} (d^1_{i,j} - d^2_{i,j})^2)^{\frac{1}{2}} < C_2,$$

where C_1 and C_2 are given constants.

For the set of configurations M and the set of rules A we shall define the set of configurations

$$A(M) = \{L | (\exists K \in M)(L = A(K))\}.$$

The rules A will be called statistically correct, if for a great enough (from a statistical point of view) natural number N:

$$(\forall m > N)(\forall M = \{K_1, K_2, ..., K_m\})$$

$$(B(A(P(M)), P(\{L_i | L_i = A(K_i), 1 \leq i \leq m\})) = 1). \qquad (*)$$

The essence of the method is in the following: the set of rules A, the proximity criterion B and the concentrate rule P are fixed preliminarily. A set of initial configurations M is chosen and the set of the corresponding final configurations is constructed. If the equation $(*)$ is valid we may assume that the rules from the set A are correct in the frames of the model, i.e. they are logically consistent. Otherwise, we replace a part (or all) of them with others. If the rules become correct, then we can add to the set some new ones or transform some of the existing and check permanently the correctness of the newly constructed system of rules. Thus, in consequtive steps, extending and complicating the rules in set A and checking their correctness, we construct the model of the given process. Afterwards we may check the temporal development (as regards the final system of rules A) of a particular initial configuration.

We initially check the correctness of the modelling rules and only then we proceed to the actual modelling. To a great deal this is due to the fact that we work over discrete objects with rules that are convenient for computer implementation. Thus, a series of checks of the equation $(*)$ can be performed only to construct the configuration $A(K)$ for a given configuration K and a set of rules A.

For example, if we would like to model (in a plane) the Solar System, we can mark the Sun by "9", Jupiter and Saturn – by "8", the Earth and Venus – by "7", the rest of the planets – by "6", the Moon and the rest bigger satelits – by "5", the smaller satelits – by "4", the bigger asteroids – by "3", the smaller asteroids – by "2" and the cosmic dust – by "1".

If we would like to model the forest dynamics, then the digits will correspond to the number of the territory trees in a square unit of the forest teritory. If a river flows through the forest, then we can mark its cells, e.g., by letter "R"; if there are stones without trees, we can mark them by cells with letter "S".

3 On the game-method for modelling with intuitionistic fuzzy estimations

The intuitionistic fuzzy propositional calculus has been introduced more than 20 years ago (see, e.g., [4, 5]). In it, if x is a variable, then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b \rangle,$$

so that $a, b, a + b \in [0, 1]$, where a and b are the degrees of validity (existence, membership, etc., and of non-validity, non-existence, etc.) of x and there the following definitions are given.

Below, we shall assume that for the two variables x and y the equalities: $V(x) = \langle a, b \rangle, V(y) = \langle c, d \rangle$ $(a, b, c, d, a + b, c + d \in [0, 1])$ hold.

For two variables x and y the operations "conjunction" (&), "disjunction" ($\vee$), "implication" ($\rightarrow$), and "(standard) negation" ($\neg$) are defined by:

$$V(x\&y) = \langle \min(a, c), \max(b, d) \rangle,$$

$$V(x \vee y) = \langle \max(a, c), \min(b, d) \rangle,$$

$$V(x \rightarrow y) = \langle \max(b, c), \min(a, d) \rangle,$$

$$V(\neg x) = \langle b, a \rangle.$$

In [6, 9, 10] the following two operations, which are analogues to operation "disjunction", are defined

$$V(x + y) = \langle a, b \rangle + \langle c, d \rangle = \langle a + c - ac, bd \rangle,$$

$$V(x \vee_L y) = \langle a, b \rangle \vee_L \langle c, d \rangle = \langle \min(1, a + c), \max(0, b + d - 1) \rangle,$$

$$V(x \vee_Z y) = \langle a, b \rangle \vee_Z \langle c, d \rangle = \langle \max(a, \min(b, c)), \min(b, d) \rangle.$$

The following relation of partial order is defined in IF logic (see, e.g., [5]) for every two variables x and y:

$$x \leq y \text{ if and only if } a \leq c \text{ and } b \geq d.$$

It can be easily seen that

$$V(x \vee_Z y) \leq V(x \vee y) \leq V(x + y) \leq V(x \vee_L y).$$

As we saw above, each asterisk from CGL corresponds to a symbol in the GMM. Now, we will perform the next step of extension. For this step, we have two possibilities that are equivalent in general case.

First, we can change the symbols from S, with which the objects in GMM are marked, to IF-couples, determining the degree of existence and degree of non-existence of this object.

Second, we can keep the symbols from S and attach the same IF-couple to them.

Let us discuss the second case, because, obviously, we can obtain the first one directly.

As a first step of the research in the present direction, it is convenient to assume that the objects will not change their IF-parameters as a result of movement from one cell to another, that has a common side with the previous cell. In a future research, we will discuss the more complex situation, when in the result of the movement the IF-parameters changes (e.g., will decrease). Therefore, the criteria for existence of an object before and after its movement will be the same and we can use the conditions from [11], but here we will extend its list of criteria.

Let us assume that the square $\langle i, j \rangle$ is assigned a pair of real numbers $\langle \mu_{i,j}, \nu_{i,j} \rangle$, so that $\mu_{i,j} + \nu_{i,j} \leq 1$. We can call the numbers $\mu_{i,j}$ and $\nu_{i,j}$ degree of existence and degree of non-existence of an object, or (in CGL and its IF-extension), of a symbol "*" in square $\langle i, j \rangle$. Therefore, $\pi(i, j) = 1 - \mu_{i,j} - \nu_{i,j} \leq 1$ will correspond to the degree of uncertainty, e.g., lack of information about existence of an asterisk in the respective cell.

Below, we will formulate seven criteria for existence of an object in a cell, that will include as a particular case the standard game. We must note that the list below is longer than the list from [11], where six criteria are given.

We will suppose that there exists an object in square $\langle i, j \rangle$ if:

(**1**) $\mu_{i,j} = 1.0$. Therefore, $\nu_{i,j} = 0.0$. This is the situation in the standard CGL, when an asterisk exists in square $\langle i, j \rangle$.

(**2**) $\mu_{i,j} > 0.5$. Therefore, $\nu_{i,j} < 0.5$. In the partial case, when $\mu_{i,j} = 1 > 0.5$, we obtain $\nu_{i,j} = 0 < 0.5$, i.e., the standard existence of the object – case 1.

(**3**) $\mu_{i,j} \geq 0.5$. Therefore, $\nu_{i,j} \leq 0.5$. Obviously, if case 2 is valid, then case 3 also will be valid.

(**4**) $\mu_{i,j} > \nu_{i,j}$. Obviously, cases (1) and (2) are partial cases of the present one, but case (3) is not included in the currently discussed case for $\mu_{i,j} = 0.5 = \nu_{i,j}$.

(**5**) $\mu_{i,j} \geq \nu_{i,j}$. Obviously, all previous cases are partial cases of the present one.

(**6**) $\mu_{i,j} > 0$. Obviously, cases (1), (2), (3) and (4) are partial cases of the present one, but case (5) is not included in the currently discussed case for $\mu_{i,j} = 0.0 = \nu_{i,j}$.

(**7**) $\nu_{i,j} < 1$. Obviously, cases (1), (2), (3) and (4) are partial cases of the present one, but cases (5) and (6) are not included in the currently discussed case for $\mu_{i,j} = 0.0$.

From these criteria it follows that if one is valid – let it be s-th criterion ($1 \le s \le 7$) then we can assert that the object exists with respect to the s-th criterion and, therefore, it will exist with respect to all other criteria the validity of which follows from the validity of the s-th criterion.

On the other hand, if s-th criterion is not valid, then we will say that the object does not exist with respect to s-th criterion. It is very important that in this case the square may not be totally empty. We may tell that the square is *"s-full"* if it contains an object with respect to s-th criterion, or, that it is *"s-empty"* if it is empty or contains an object, that does not satisfy the s-th criterion.

For the aims of the GMM, it will be suitable to use (with respect to the type of the concrete model) one of the first four criteria for existence of an object. Let us say for each fixed square $\langle i, j\rangle$ that therein is an object by s-th criterion ($1 \le s \le 4$), whether this criterion confirms the existence of the object.

We must mention that in CGL there are no rules for object (asterisk) transfer. On the other hand, in GMM a new situation arises, which does not exist in CGL, but is an extension of the CGL-situations related to bourning and death of asterisks. In the present case, two or more objects can enter one cell and now we have to use the rules for interaction. Let us discuss the simpler case when only two objects can enter one cell, although the formulas below will be valid for $n \ge 2$ objects, as well because each of the discussed below operations is associatlve.

Let two objects have IF-estimations $\langle a, b\rangle$ and $\langle c, d\rangle$. Then, in the cell a new object with IF-estimations will be generated. The values of these estimations can be different in respect of the user preferences:

1 - strongly high: $\langle \min(1, a + c), \max(0, b + d - 1)\rangle$,
2 - high: $\langle a + c - ac, bd\rangle$,
3 - standard: $\langle \max(a, c), \min(b, d)\rangle$.
4 - weak: $\langle \max(a, c), \min(b, d)\rangle$.

For example, if we would like to model a situation of entering some people in a lift cage, then we can use the first formula, if a, b, c, d are the people's parameters of weight, normalized by the lift capacity. If modelling forest dynamics we would like to describe a case when trees from two regions (represented by squares) enter one and the same cell, then we can use the second formula. The fact that a planet in the Solar System swallows cosmic dust, of course, must be interpreted by the third formula.

4 Conclusion

In the next research in this direction of extension of CGL, we will discuss the possibility for constructing different aggregation procedures for obtaining a single configuration $P(M)$, that is juxtaposed to a combination of configurations M. Therefore, we plan to construct an intuitionistic fuzzy concentrate rule. Also, we will introduce different intuitionistic fuzzy criteria for proximity between two configurations. The concept of the statistical correctness of some rules also will obtain an intuitionistic fuzzy interpretation.

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References

1. Atanassov K.: On a combinatorial game-method for modelling, Advances in Modelling & Analysis, AMSE Press, Vol. 19, No. 2, 41-47 (1994)
2. Atanassov K.: Application of a combinatorial game-method in combinatorial geometry. Part 1: Combinatorial algorithms for solving variants of the Steiner-Rosenbaum's problem, Advances in Modelling & Analysis, Vol. 2, No. 1-2, 23-29 (1998)
3. Atanassov K.: Application of a combinatorial game-method in combinatorial geometry. Part 2: Algorithm for grouping and transferring of points and a general algorithm, Advances in Modelling & Analysis, Vol. 2, No. 1-2, 31-36 (1998)
4. Atanassov, K.: Two variants of intuitonistic fuzzy propositional calculus. Preprint IM-MFAIS-5-88, Sofia (1988)
5. Atanassov, K.: Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Heidelberg (1999)
6. Atanassov, K.: Remarks on the conjunctions, disjunctions and implications of the intuitionistic fuzzy logic Int. J. of Uncertainty, Fuzziness and Knowledge-Based Systems Vol. 9, No. 1, 55-65 (2001)
7. Atanassov, K., Atanassova, L.: A game-method for modelling, Proc. of the 3-rd International School "Automation and Scientific Instrumentation" Varna, Oct. 229-232 (1984)
8. Atanassov, K., Atanassova, L., Sasselov, D.: On the combinatorial game-metod for modelling in astronomy. Comptes Rendus de l'Academie bulgare des Sciences, Tome 47, No. 9, 5-7 (1994)
9. Atanassov, K., Tcvetkov, R.: On Zadeh's intuitionistic fuzzy disjusnction and conjunction. Notes on Intuitionistic Fuzzy Sets, Vol. 17, No. 1, 1-4 (2011)
10. Atanassov, K., Tcvetkov, R.: On Lukasiewicz's intuitionistic fuzzy disjusnction and conjunction. Annual of "Informatics" Section Union of Scientists in Bulgaria, Vol. 3 (2010) (to appear).
11. Atanassova, V., Atanassov, K.: Ant colony optimization approach to tokens'movement within generalized nets. Lecture Notes in Computer Science, Vol. 6046, Springer, Berlin, 240-247 (2011)
12. Dimitrov D.: Modelling the growth and dynamics of forest stands by game-method. Advances in Modelling & Analysis, Vol. 2, No. 1-2, 11-22 (1998)

13. Dimitrov D.: Modelling the growth and dynamics of forest stands by extended game-method. Advances in Modelling & Analysis, Vol. 4, No. 1-2, 7-21 (1999)
14. Dobrinkova, N., Fidanova, S., Atanassov, K.: Game-method model for field fires. Lecture Notes in Computer Science, Vol. 5910, 173-179 (2010)
15. Kuratovski K.: Topology, New York, Academic Press (1966)
16. Sasselov D., Atanassov K.: On the generalized nets realization of a combinatorial game-method for modelling in astronomy. Advances in Modelling & Analysis, AMSE Press, Vol. 23, No. 4, 59-64 (1995)
17. Wikipedia contributors: Conway's Game of Life. Wikipedia, The Free Encyclopedia. March 18, 2011. Available at: http://en.wikipedia.org/w/index.php?title=Conway's_Game_of_Life&oldid=419530565. Accessed March 20, 2011.