Notes on Intuitionistic Fuzzy Sets
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Integral equations with pentagonal intuitionistic fuzzy numbers

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Abstract: The paper presents an adaptation of pentagonal intuitionistic fuzzy numbers. The arithmetic operation of pentagonal intuitionistic fuzzy number is addressed here. Demonstration of a pentagonal intuitionistic fuzzy solution of intuitionistic fuzzy integral equation is carried out with the said numbers. Additionally, an illustrative example is also undertaken with a graph and a table to attain usefulness of the proposed concept.
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1 Introduction

1.1 Fuzzy sets and Intuitionistic fuzzy sets

In 1965, Lotfi A. Zadeh [28], a Professor of electrical engineering at the University of California in Berkeley, published the first of his papers on his new theory of fuzzy sets and systems. Since the 1980s, this mathematical theory of "unsharp amounts" has been applied with great success in many different fields. Later, Chang and Zadeh [6] introduced the concept of fuzzy numbers in 1972. Many mathematicians have been studying them (one-dimension or n-dimension fuzzy numbers, see for example [9, 10, 12, 14]). With the development of theories and applications of fuzzy numbers, this concept becomes more and more important. One of the generalizations of fuzzy sets [28] is the intuitionistic fuzzy sets (IFS). Out of several higher-order fuzzy sets, IFS was first introduced by Atanassov [4, 5] and have been found to be suitable to deal with unexplored areas. The fuzzy set considers only the degree of belongingness and non-belongingness. Fuzzy set theory does not incorporate the degree of hesitation, i.e., degree of non-determinacy defined as the complement of the sum of membership function and non-membership function to 1. To handle such situations, Atanassov explored the concept of fuzzy set theory through IFS theory. The degree of acceptance in fuzzy sets is only considered, while IFS is characterized by a membership function and a non-membership function so that the sum of both values can be less than one [4, 5]. Nowadays, IFSs are being studied extensively and being used in different fields of science and technology.

1.2 Pentagonal fuzzy numbers

Many researchers take pentagonal fuzzy numbers with different types of membership function. In this subsection we study some published work which is associated with pentagonal fuzzy numbers.

Authors Information	**Types of membership function**	**Main contribution**	**Application Area**
Panda and Pal [22]	Linear membership function of with symmetry	Define arithmetic operation and a exponent operation	Fuzzy matrix theory
Anitha and Parvathi [2]	Linear membership function	Find expected crisp value	Inventory control problem
Helen and Uma [13]	Linear membership function	Find the parametric form of pentagonal fuzzy number	Proof of all arithmetic operation using parametric form concept and find the ranking of pentagonal fuzzy number
Siji and Kumari [26]	Linear membership and non-membership function	Define all arithmetic operation Find the ranking of Intuitionistic fuzzy number	Application in network problem
VijinRaj and Karthik [27]	Linear membership function	Define all arithmetic operation	Application in Neural network problem
Dhanamand and Parimaldevi [7]	Linear membership function	Find the ranking of pentagonal fuzzy number using circumcenter of centroids and an index of modality	Apply in multi objective multi item inventory model
Pathinathan and Ponnivalavan [23]	Reverse order linear membership function	Define arithmetic operation	Define different type of reverse order fuzzy number
Ponnivalavan and Pathinathan [24],	Linear membership and non-membership function	Define arithmetic operation	Find score and accuracy function
Annie Christi and Kasthuri [3]	Linear membership and non-membership function	Define arithmetic operation and ranking	Transportation problem

From the above literature survey we see that linear fuzzy membership function with symmetry on both ends is only taken most of the cases. But what happen if we take the intuitionistic fuzzy cases where membership function and non-membership function is present, symmetry or asymmetry on both ends are present in different case? Obviously the formations are different. In this article we propose to show all types of possibility.

1.3 Review on fuzzy integral equation

Integral equations are very important in the theory of calculus and in particular, for practical applications. In this paper the concept of intuitionistic fuzzy integral equation is taken when the intuitionistic fuzzy number is taken as pentagonal intuitionistic fuzzy number. Before going to the main topic any one can study previous work related to fuzzy integral equation which is done by different researchers [1, 8, 11, 25]. The intuitionistic fuzzy differential and difference equation have been previously studied in [15–21].

1.4 Motivation for that research

Intuitionistic fuzzy sets theory plays an important role in uncertainty modeling. Now the question is, if we wish to take a pentagonal intuitionistic fuzzy number, then how its geometrical representations look like. What are its membership and non-membership functions? So, if a decision maker takes an intuitionistic fuzzy number that can graphically look like a pentagon, then how its membership function and non-membership function can be defined? From this point of view, we try to define pentagonal intuitionistic fuzzy numbers, which can be a better choice for the decision makers in different situations.

1.5 Novelties

Although there are several papers where pentagonal fuzzy sets and numbers [2, 3, 7, 13, 22–24] are defined and applied to various fields, there still exists some work to be done, which is defined as follows:

(i) Formation of pentagonal intuitionistic fuzzy number in an easier manner.
(ii) The parametric form of the pentagonal intuitionistic fuzzy numbers.
(iii) Arithmetic operations on pentagonal intuitionistic fuzzy numbers.
(iv) The number is considered with integral equations, i.e., pentagonal intuitionistic fuzzy integral equation are defined and solved.

1.6 Structure of the paper

The paper is organized as follows. In Section 2, the basic concept on fuzzy number and intuitionistic fuzzy number are defined. In Section 3, we give a brief description and formation of pentagonal intuitionistic fuzzy number. In Section 4, we addressed some arithmetic operation on linear pentagonal intuitionistic fuzzy number with symmetry. In Section 5, solution of fuzzy integral equation with pentagonal intuitionistic fuzzy number is found with numerical example. The conclusions are written in Section 6.

2 Preliminaries

Definition 2.1. Fuzzy set [28]: A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)): x \in A, \mu_{\tilde{A}}(x) \in [0,1]\}$. In the pair $(x, \mu_{\tilde{A}}(x))$the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belong to the interval [0, 1], called membership function.

Definition 2.2. Intuitionistic fuzzy set [4, 5]: Let a set X be fixed. An IFS $\tilde{A}^i$ in X is an object having the form $\tilde{A}^i = \{\langle x, \mu_{\tilde{A}^i}(x), \vartheta_{\tilde{A}^i}(x)\rangle: x \in X\}$, where the $\mu_{\tilde{A}^i}(x): X \to [0,1]$ and $\vartheta_{\tilde{A}^i}(x): X \to [0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x \in X$ to the set $\tilde{A}^i$, which is a subset of X, for every element of $x \in X$, $0 \le \mu_{\tilde{A}^i}(x) + \vartheta_{\tilde{A}^i}(x) \le 1$.

Definition 2.3. Intuitionistic fuzzy number: An IFN $\tilde{A}^i$ is defined as follows

(i) an intuitionistic fuzzy subject of real line;

(ii) normal, i.e., there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = 1$(so $\vartheta_{\tilde{A}^i}(x_0) = 0$);

(iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e.,

$$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)) \forall\, x_1, x_2 \in R, \lambda \in [0,1];$$

(iv) a concave set for the non-membership function $\vartheta_{\tilde{A}^i}(x)$, i.e.,

$$\vartheta_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \ge \max(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)) \forall\, x_1, x_2 \in R, \lambda \in [0,1].$$

Definition 2.4. [20]: The intuitionistic fuzzy integral of intuitionistic fuzzy process $\tilde{u}(t)$, $\int_a^b \tilde{u}(t)dt$ for $a, b \in I$, is defined by

$$\left[\int_a^b \tilde{u}(t)dt\right]^{\alpha,\beta} = \left[\int_a^b u_1^\alpha(t)dt\,, \int_a^b u_2^\alpha(t)dt\,; \int_a^b u'^\beta_1(t)dt\,, \int_a^b u'^\beta_2(t)dt\right]$$

Provided that the Lebesgue integrals on the right exist.

3 Pentagonal intuitionistic fuzzy numbers

In this section we develop pentagonal fuzzy numbers from a different viewpoint.

Definition 3.1. Pentagonal fuzzy number: A pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ should satisfy the following conditions:

(1) $\mu_{\tilde{A}}(x)$ is a continuous function in the interval [0,1],

(2) $\mu_{\tilde{A}}(x)$ is strictly increasing and continuous function on $[a_1, a_2]$ and $[a_2, a_3]$,

(3) $\mu_{\tilde{A}}(x)$ is strictly decreasing and continuous function on $[a_3, a_4]$ and $[a_4, a_5]$.

3.1 Linear pentagonal intuitionistic fuzzy number with symmetry on both ends

Definition 3.2. Linear pentagonal intuitionistic fuzzy number with symmetry (LPIFNS):
A linear pentagonal fuzzy number is written as

$$\tilde{A}_{LS}^{i} = \big((a_1, a_2, a_3, a_4, a_5), (a_1', a_2, a_3, a_4, a_5'); r_1, r_2\big),$$

whose membership function and non-membership function are written as

$$\mu_{\tilde{A}_{LS}^{i}}(x) = \begin{cases} r_1 \dfrac{x - a_1}{a_2 - a_1} \ \ if \ \ a_1 \le x \le a_2 \\ 1 - (1 - r_1)\dfrac{x - a_2}{a_3 - a_2} \ \ if \ a_2 \le x \le a_3 \\ 1 \ \ if \ x = a_3 \\ 1 - (1 - r_1)\dfrac{a_4 - x}{a_4 - a_3} \ if \ a_3 \le x \le a_4 \\ r_1 \dfrac{a_5 - x}{a_5 - a_4} \ \ if \ a_4 \le x \le a_5 \\ 0 \ if \ x > a_5 \end{cases}$$

and

$$\vartheta_{\tilde{A}_{LS}^{i}}(x) = \begin{cases} 1 - (1 - r_2)\dfrac{x - a_1'}{a_2 - a_1'} \ \ if \ \ a_1' \le x \le a_2 \\ r_2 \dfrac{a_3 - x}{a_3 - a_2} \ \ if \ a_2 \le x \le a_3 \\ 0 \ \ if \ x = a_3 \\ r_2 \dfrac{x - a_3}{a_4 - a_3} \ \ if \ a_3 \le x \le a_4 \\ 1 - (1 - r_2)\dfrac{a_5' - x}{a_5' - a_4} \ \ if \ a_4 \le x \le a_5' \\ 0 \ if \ x > a_5 \end{cases}$$

Definition 3.3. $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-cut or parametric form of LPIFNS: (α, β)-cut or parametric form of LPIFNS is represented by the formulae

$$A_{(\alpha,\beta)} = \{x \in X | \mu_{\tilde{A}_{LS}}(x) \ge \alpha, \mu_{\tilde{A}_{LS}}(x) \le \beta\} =$$
$$\big[\big\{\big(A_{1L}(\alpha), A_{2L}(\alpha)\big), \big(A_{2R}(\alpha), A_{1R}(\alpha)\big)\big\}; \{(A_{1L}'(\beta), A_{2L}'(\beta)), (A_{2R}'(\beta), A_{1R}'(\beta))\}\big],$$

where

$$A_{1L}(\alpha) = a_1 + \tfrac{\alpha}{r_1}(a_2 - a_1) \ \ for \ \alpha \in [0, r_1], A_{2L}(\alpha) = a_2 + \tfrac{1-\alpha}{1-r_1}(a_3 - a_2) \ \ for \ \alpha \in [r_1, 1],$$

$$A_{2R}(\alpha) = a_4 - \tfrac{1-\alpha}{1-r_1}(a_4 - a_3) \ for \ \alpha \in [r_1, 1], A_{1R}(\alpha) = a_5 - \tfrac{\alpha}{r_1}(a_5 - a_4) \ for \ \alpha \in [0, r_1],$$

$$A_{1L}'(\beta) = a_1' + \tfrac{1-\beta}{1-r_2}(a_2 - a_1') for \ \beta \in [r_2, 1], A_{2L}'(\beta) = a_3 - \tfrac{\beta}{r_2}(a_3 - a_2) \ for \ \beta \in [0, r_2],$$

$$A_{2R}'(\beta) = a_3 + \tfrac{\beta}{r_2}(a_4 - a_3) for \ \beta \in [0, r_2], A_{1R}'(\beta) = a_5' - \tfrac{1-\beta}{1-r_2}(a_5' - a_4) \ \ for \ \beta \in [r_2, 1].$$

Note that$A_{1L}(\alpha)$, $A_{2L}(\alpha)$, $A_{2R}'(\beta)$, $A_{1R}'(\beta)$ are increasing functions and $A_{2R}(\alpha)$, $A_{1R}(\alpha)$, $A_{1L}'(\beta)$,$A_{2L}'(\beta)$ are decreasing functions.

Key point 3.1. The basic concept of the above number is the left picked point and right picked points are same (see on Figure 1 the picked point for membership is r_1 and for non-membership is r_2).

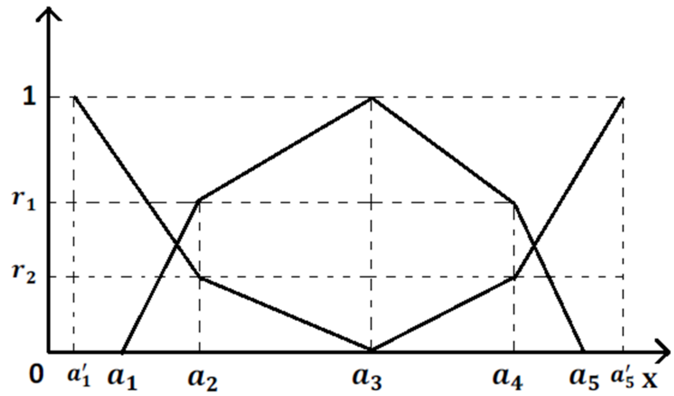

Figure 1. Linear pentagonal intuitionistic fuzzy number with symmetry

3.2 Linear pentagonal intuitionistic fuzzy number with asymmetry on both ends

Definition 3.4. Linear pentagonal intuitionistic fuzzy number with asymmetry (LPIFNAS): A linear pentagonal fuzzy number is written as

$$\tilde{A}^i_{LAS} = \big((a_1, a_2, a_3, a_4, a_5), (a_1', a_2, a_3, a_4, a_5'); (r_1, r_2; s_1, s_2)\big),$$

whose membership function and non-membership function are written as

$$\mu_{\tilde{A}^i_{LAS}}(x) = \begin{cases} r_1 \dfrac{x - a_1}{a_2 - a_1} \;\; if \;\; a_1 \le x \le a_2 \\ 1 - (1 - r_1)\dfrac{x - a_2}{a_3 - a_2} \;\; if \; a_2 \le x \le a_3 \\ 1 \;\; if \; x = a_3 \\ 1 - (1 - s_1)\dfrac{a_4 - x}{a_4 - a_3} \; if \; a_3 \le x \le a_4 \\ s_1 \dfrac{a_5 - x}{a_5 - a_4} \;\; if \; a_4 \le x \le a_5 \\ 0 \; if \; x > a_5 \end{cases}$$

and

$$\vartheta_{\tilde{A}^i_{LAS}}(x) = \begin{cases} 1 - (1 - r_2)\dfrac{x - a_1'}{a_2 - a_1'} \;\; if \;\; a_1' \le x \le a_2 \\ r_2 \dfrac{a_3 - x}{a_3 - a_2} \;\; if \; a_2 \le x \le a_3 \\ 0 \;\; if \; x = a_3 \\ s_2 \dfrac{x - a_3}{a_4 - a_3} \;\; if \; a_3 \le x \le a_4 \\ 1 - (1 - s_2)\dfrac{a_5' - x}{a_5' - a_4} \;\; if \; a_4 \le x \le a_5' \\ 0 \; if \; x > a_5 \end{cases}$$

Definition 3.5. $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-cut or parametric form of LPIFNAS: (α, β)-cut or parametric form of

LPFNS is represented by the formulae

$$A_{(\alpha,\beta)} = \{x \in X | \mu_{\tilde{A}_{LS}}(x) \geq \alpha, \mu_{\tilde{A}_{LS}}(x) \leq \beta\} =$$
$$[\{(A_{1L}(\alpha), A_{2L}(\alpha)), (A_{2R}(\alpha), A_{1R}(\alpha))\}; \{(A'_{1L}(\beta), A'_{2L}(\beta)), (A'_{2R}(\beta), A'_{1R}(\beta))\}],$$

where

$$A_{1L}(\alpha) = a_1 + \frac{\alpha}{r_1}(a_2 - a_1) \;\; for\; \alpha \in [0, r_1],\; A_{2L}(\alpha) = a_2 + \frac{1-\alpha}{1-r_1}(a_3 - a_2) \;\; for\; \alpha \in [r_1, 1],$$

$$A_{2R}(\alpha) = a_4 - \frac{1-\alpha}{1-s_1}(a_4 - a_3)\; for\; \alpha \in [s_1, 1],\; A_{1R}(\alpha) = a_5 - \frac{\alpha}{s_1}(a_5 - a_4)\; for\; \alpha \in [0, s_1],$$

$$A'_{1L}(\beta) = a'_1 + \frac{1-\beta}{1-r_2}(a_2 - a'_1) for\; \beta \in [r_2, 1],\; A'_{2L}(\beta) = a_3 - \frac{\beta}{r_2}(a_3 - a_2)\; for\; \beta \in [0, r_2],$$

$$A'_{2R}(\beta) = a_3 + \frac{\beta}{s_2}(a_4 - a_3) for\; \beta \in [0, s_2],\; A'_{1R}(\beta) = a'_5 - \frac{1-\beta}{1-s_2}(a'_5 - a_4) \;\; for\; \beta \in [s_2, 1].$$

Note that $A_{1L}(\alpha)$, $A_{2L}(\alpha)$, $A'_{2R}(\beta)$, $A'_{1R}(\beta)$ are increasing functions and $A_{2R}(\alpha)$, $A_{1R}(\alpha)$, $A'_{1L}(\beta)$, $A'_{2L}(\beta)$ are decreasing functions.

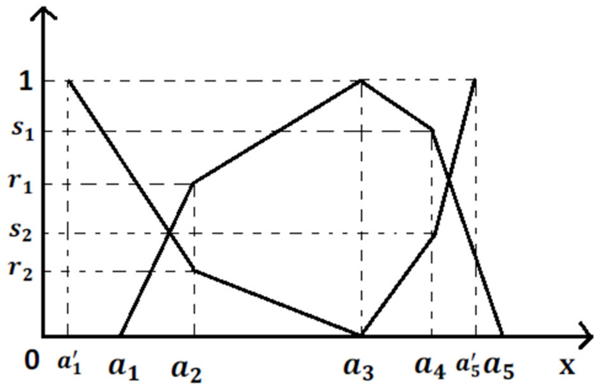

Figure 2. Linear pentagonal intuitionistic fuzzy number with asymmetry

Key point 3.2. The basic concept of the above number is the left picked point and right picked point are not same (See Fig. 2 the left picked point for membership value is r_1 and right picked point is s_1 whereas for non-membership value left picked point for membership value is r_2 and right picked point is s_2.

Note 3.1. If $r_1 = s_1$ and $r_2 = s_2$ then the linear pentagonal intuitionistic fuzzy number with asymmetry becomes linear pentagonal intuitionistic fuzzy number with symmetry.

4 Arithmetic operation on linear pentagonal intuitionistic fuzzy number with symmetry, i.e., the number of type $\tilde{A}^i_{LS} = ((a_1, a_2, a_3, a_4, a_5), (a'_1, a_2, a_3, a_4, a'_5); r_1, r_2)$

Here we define some arithmetic operations on pentagonal intuitionistic fuzzy numbers.

(1) Multiplication by crisp number

(1.1) Multiplication by a positive crisp number

If $\tilde{A}^i_{LS} = ((a_1, a_2, a_3, a_4, a_5), (a'_1, a_2, a_3, a_4, a'_5); r_1, r_2)$ is a linear pentagonal intuitionistic fuzzy number and k is a positive crisp number, then

$$k\tilde{A}_{LS} = ((ka_1, ka_2, ka_3, ka_4, a_5), (ka_1', ka_2, ka_3, ka_4, ka_5'); r_1, r_2).$$

(1.2) Multiplication by a negative crisp number

If $\tilde{A}_{LS}^{i} = ((a_1, a_2, a_3, a_4, a_5), (a_1', a_2, a_3, a_4, a_5'); r_1, r_2)$ is a linear pentagonal intuitionistic fuzzy number and k is a negative crisp number, then

$$k\tilde{A}_{LS} = ((ka_5, ka_4, ka_3, ka_2, ka_1), (ka_5', ka_4, ka_3, ka_2, ka_1'); r_1, r_2).$$

(2) Addition of two pentagonal intuitionistic fuzzy numbers

Consider two pentagonal intuitionistic fuzzy numbers

$$\tilde{A}_{LS}^{i} = ((a_1, a_2, a_3, a_4, a_5), (a_1', a_2, a_3, a_4, a_5'); r_{11}, r_{12})$$

and

$$\tilde{B}_{LS}^{i} = ((b_1, b_2, b_3, b_4, b_5), (b_1', b_2, b_3, b_4, b_5'); r_{21}, r_{22}),$$

then the addition of the two numbers is given by

$$\tilde{C}_{LS} = ((a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5),$$
$$(a_1' + b_1', a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5' + b); r_1, r_2),$$

where $r_1 = \min\{r_{11}, r_{21}\}$ and $r_2 = \max\{r_{12}, r_{22}\}$.

(3) Subtraction of two pentagonal intuitionistic fuzzy numbers

Consider two pentagonal intuitionistic fuzzy numbers $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5; r_1)$ and $\tilde{B}_{LS} = (b_1, b_2, b_3, b_4, b_5; r_2)$, then the addition of the two numbers is given by

$$\tilde{D}_{LS} = (a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1; r),$$

where $r_1 = \min\{r_{11}, r_{21}\}$ and $r_2 = \max\{r_{12}, r_{22}\}$.

5 Solution of intuitionistic fuzzy integral equation with pentagonal intuitionistic fuzzy number

5.1 Intuitionistic fuzzy integral equation

Consider the linear Fredholm integral equation of second kind

$$u(x) = f(x) + \lambda \int_a^b k(x,t)u(t)dt,$$

where $x \in D$, $u(x)$ and $f(x)$ are functions on $D = [a, b]$ and $k(x,t)$ is an arbitrary kernel function over $T = [a, b] \times [a, b]$, and u is unknown on D.The above integral equation is said to be intuitionistic integral equation if:

(1) $f(x)$ is intuitionistic fuzzy valued function.

(2) Only $k(x,t)$ is intuitionistic fuzzy valued function.

(3) Both $f(x)$ and $k(x,t)$ are intuitionistic fuzzy valued functions.

5.2 Condition for existence for solving intuitionistic fuzzy integral equation

Consider the pentagonal intuitionistic fuzzy integral equation

$$u(x) = f(x) + \lambda \int_a^b k(x,t)u(t)dt.$$

Let the solution of the above PIFIE be $\tilde{u}(x)$ and its (α,β)-cut be $u(x)[\alpha,\beta] = [\{(u_{1L}(x,\alpha), u_{2L}(x,\alpha)), (u_{2R}(x,\alpha), u_{1R}(x,\alpha))\}; \{(u'_{1L}(x,\beta), u'_{2L}(x,\beta)), (u'_{2R}(x,\beta), u'_{1R}(x,\beta))\}]$

The solution is a strong solution if

(i) $\frac{\partial u_{1L}(x,\alpha)}{\partial \alpha} > 0, \frac{\partial u_{2L}(x,\alpha)}{\partial \alpha} > 0, \frac{\partial u_{2R}(x,\alpha)}{\partial \alpha} < 0\,, \frac{\partial u_{1R}(x,\alpha)}{\partial \alpha} < 0$ for α defined on a particular interval, and

(ii) $\frac{\partial u'_{1L}(x,\beta)}{\partial \beta} < 0, \frac{\partial u'_{2L}(x,\beta)}{\partial \beta} < 0, \frac{\partial u'_{2R}(x,\beta}{\partial \beta} > 0, \frac{\partial u'_{1R}(x,\beta)}{\partial \beta} > 0$ for β defined on a particular interval

Otherwise, the solution is a weak solution.

5.3 Solution of intuitionistic fuzzy integral equation

Consider the integral equation$u(x) = \tilde{f}(x) + \lambda \int_a^b k(x,t)u(t)dt$. In this integral equation $\tilde{f}(x)$ isa linear pentagonal intuitionistic fuzzy function. Consider $k(x,t)$ is a positive function.

Solution: Taking the (α,β)-cut on the above integral equation we have

$$u_{1L}(x,\alpha) = f_{1L}(x,\alpha) + \lambda \int_a^b k(x,t)u_{1L}(x,\alpha)dt$$

$$u_{2L}(x,\alpha) = f_{2L}(x,\alpha) + \lambda \int_a^b k(x,t)u_{2L}(x,\alpha)dt$$

$$u_{2R}(x,\alpha) = f_{2R}(x,\alpha) + \lambda \int_a^b k(x,t)u_{2R}(x,\alpha)dt$$

$$u_{1R}(x,\alpha) = f_{1R}(x,\alpha) + \lambda \int_a^b k(x,t)u_{1R}(x,\alpha)dt$$

$$u'_{1L}(x,\beta) = f'_{1L}(x,\beta) + \lambda \int_a^b k(x,t)u'_{1L}(x,\beta)dt$$

$$u'_{2L}(x,\beta) = f'_{2L}(x,\beta) + \lambda \int_a^b k(x,t)u'_{2L}(x,\beta)dt$$

$$u'_{2R}(x,\beta) = f'_{2R}(x,\beta) + \lambda \int_a^b k(x,t)u'_{2R}(x,\beta)dt$$

$$u'_{1R}(x,\beta) = f'_{1R}(x,\beta) + \lambda \int_a^b k(x,t)u'_{1R}(x,\beta)dt$$

Note 5.1. The above integral equations are the crisp integral equations. Anyone can easily solve it.

Example 5.1. Consider the integral equation $u(x) = \tilde{f}(x) + \lambda \int_a^b k(x,t)u(t)dt$ where $\tilde{f}(x)$ is a pentagonal intuitionistic fuzzy valued function defined as

$$\tilde{f}(x) = ((2,2.5,3,3.5,4),(1,2.5,3,3.5,5);0.6,0.4)e^{-x}$$

and $\lambda = 1$, $a = 0$, $b = x$, $k(x,t) = \sin(x-t)$.

Solution: The solution is written as follows

$$u_{1L}(x,\alpha) = \left(2 + \frac{5\alpha}{6}\right)(2e^{-x} + x - 1)$$
$$u_{2L}(x,\alpha) = \left(2.5 + \frac{5}{4}(1-\alpha)\right)(2e^{-x} + x - 1)$$
$$u_{2R}(x,\alpha) = \left(3.5 - \frac{5}{4}(1-\alpha)\right)(2e^{-x} + x - 1)$$
$$u_{1R}(x,\alpha) = \left(4 - \frac{5}{6}\alpha\right)(2e^{-x} + x - 1)$$
$$u'_{1L}(x,\beta) = \left(1 + \frac{5}{2}(1-\beta)\right)(2e^{-x} + x - 1)$$
$$u'_{2L}(x,\beta) = \left(3 - \frac{5}{4}\beta\right)(2e^{-x} + x - 1)$$
$$u'_{2R}(x,\beta) = \left(3 + \frac{5}{4}\beta\right)(2e^{-x} + x - 1)$$
$$u'_{1R}(x,\beta) = \left(5 - \frac{5}{2}(1-\beta)\right)(2e^{-x} + x - 1)$$

Table 1. Value of $u_{1L}(x,\alpha)$,$u_{2L}(x,\alpha)$,$u_{2R}(x,\alpha)$,$u_{1R}(x,\alpha)$, $u'_{1L}(x,\beta)$, $u'_{2L}(x,\beta)$, $u'_{2R}(x,\beta)$ and $u'_{1R}(x,\beta)$at $x = 2$ for different α,β

$\boldsymbol{\alpha,\beta}$	$\boldsymbol{u_{1L}(x,\alpha)}$	$\boldsymbol{u_{2L}(x,\alpha)}$	$\boldsymbol{u_{2R}(x,\alpha)}$	$\boldsymbol{u_{1R}(x,\alpha)}$	$\boldsymbol{u'_{1L}(x,\beta)}$	$\boldsymbol{u'_{2L}(x,\beta)}$	$\boldsymbol{u'_{2R}(x,\beta)}$	$\boldsymbol{u'_{1R}(x,\beta)}$
0	2.5413			5.0827		3.8120	3.8120	
0.1	2.6472			4.9768		3.6532	3.9708	
0.2	2.7531			4.8709		3.4943	4.1297	
0.3	2.8590			4.7650		3.3355	4.2885	
0.4	2.9649			4.6591	3.1767	3.1767	4.4473	4.4473
0.5	3.0708			4.5532	2.8590			4.7650
0.6	3.1767	3.1767	4.4473	4.4473	2.5413			5.0827
0.7		3.3355	4.2885		2.2237			5.4003
0.8		3.4943	4.1297		1.9060			5.7180
0.9		3.6532	3.9708		1.5883			6.0357
1		3.8120	3.8120		1.2707			6.3534

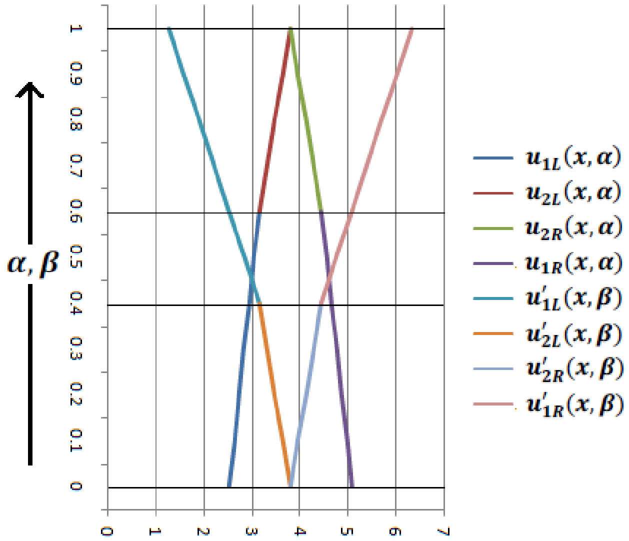

Figure 3. Plot of $u_{1L}(x,\alpha)$,$u_{2L}(x,\alpha)$,$u_{2R}(x,\alpha)$,$u_{1R}(x,\alpha)$, $u'_{1L}(x,\beta)$, $u'_{2L}(x,\beta)$, $u'_{2R}(x,\beta)$ and $u'_{1R}(x,\beta)$at $x = 2$ for α,β

Remark 5.1. Clearly from graph and table we see that $u_{1L}(x,\alpha)$, $u_{2L}(x,\alpha)$, $u'_{2R}(x,\beta)$, $u'_{1R}(x,\beta)$ are increasing functions and $u_{2R}(x,\alpha)$, $u_{1R}(x,\alpha)$, $u'_{1L}(x,\beta), u'_{2L}(x,\beta)$ are decreasing functions at $x = 2$. Hence for this particular point $x = 2$ the solution is a strong solution.

6 Conclusion

In this paper the concept of pentagonal intuitionistic fuzzy number is defined. The said number valued function is applied to elucidate the pentagonal intuitionistic fuzzy solutions of the integral equation. Arithmetic operations of a particular pentagonal intuitionistic fuzzy number are also addressed. Further a numerical example is illustrated with pentagonal intuitionistic fuzzy number with intuitionistic fuzzy integral equation. Comprehensively, the whole deliberation reaches its conclusion with the following remarks:

- Demonstrating pentagonal intuitionistic fuzzy numbers enabled to meet the imprecise parameters as well, which is approvingly advantageous for the decision makers to analyze the result in a more precise manner.
- By different situation the decision maker can take pentagonal intuitionistic fuzzy number as per the problem definition.

Thus in future we seek to apply these concepts to find the solution of different types of problem with different type of pentagonal intuitionistic fuzzy number and apply in various fields of engineering and sciences.

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