Multi-objective intuitionistic fuzzy linear programming and its application in transportation model

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Abstract:

This paper presents a new Intuitionistic Fuzzy Optimization (IFO) approach to solve the a Multi-Objective Linear Programming Problem (MOLPP) under uncertainty. The idea is based on extension of fuzzy optimization. This approach is an application of the intuitionistic fuzzy set. First we have considered a multi-objective linear programming with equality and inequality constraints with Intuitionistic Fuzzy (IF) goals. Their fuzzy non-linear membership and non-membership function have been taken for the degree of rejection of objectives and constraints together with the degree of satisfaction. Then it converts the said problem into a conventional linear programming problem. Finally we have showed application of this approach in the Capacitated Transportation Problem. Numerical examples are provided to illustrate our new approach.

Key-Words: Fuzzy optimization, Intuitionistic fuzzy sets, pareto optimal, non-membership function, Capacitated Transportation Problem.

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1. Introduction: The classical Transportation Problem(TP) refers to a special class of Linear programming Problem(LPP). This crisp TP was developed very well but they are very limited and in many cases they do not represent exactly the real problem[13]. In general, the real life problems are modeled with multi-objective[1,2,4,11,12,14]. In the last twenty years, the multi-objective transportation problem have been investigated in the sense of fuzzy set theory[8,9,10]. This fuzzy programming technique is more flexible and allows to find the solutions which are more sufficient to the real problem. In fuzzy optimization, the degree of acceptance of objectives and constraints are considered only. Now a day, the fuzzy set theory has been also developed in a large area and its different modification and generalization form have appeared. Intuitionistic fuzzy sets (IFS) is one of the generalization of fuzzy set theory. Out of several higher-order fuzzy sets, intuitionistic fuzzy sets introduced by Atanassov [5,6,7] have been found to be well suited to dealing with vagueness. The concept of an IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy sets. Ingeneral, the theory of IFS is the generalization of fuzzy sets. Therefore, it is expected that, IFS could be used to simulate human decision-making process and any activities requiring human expertise and knowledge which are inevitably imprecise or not totally reliable.

Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than one[5].

In this paper, a solution procedure of multi-objective transportation problem with equality and inequality constraint in intuitionistic fuzzy environment is presented. First, we have considered a MOTP with equality and inequality constraint where constraints goals are fuzzy numbers. Here the degree of acceptance (satisfaction) of objectives and constraints are considered as a non-linear hyperbolic function and degree of rejection (non-acceptance) of objectives and constraints are considered as a non-linear parabolic function. Then this intuitionistic fuzzy optimization problem is converted into a crisp one. It gives the (μ-γ) pareto optimal solutions.

2. Definition: Let a set E be fixed. An intuitionistic fuzzy set or IFS $\overset{\sim i}{A}$ in E is an object having the form: $\overset{\sim i}{A} = \{< x, \mu_{\overset{\sim i}{A}}(x), \gamma_{\overset{\sim i}{A}}(x) > / x \in E\}$, where the $\mu_{\overset{\sim i}{A}}(x) : \mathrm{E} \to [0,1]$ and $\gamma_{\overset{\sim i}{A}}(x) : \mathrm{E} \to [0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x \in$ E to the set $\overset{\sim i}{A}$, which is a subset of E, for every element of $x \in$ E, $0 \le \mu_{\overset{\sim i}{A}}(x) + \gamma_{\overset{\sim i}{A}}(x) \le 1$.

Obviously, each ordinary fuzzy set may be written as $\{< x, \mu_{\overset{\sim i}{A}}(x), 1 - \mu_{\overset{\sim i}{A}}(x) > / x \in E\}$.

The amount $\pi_{\overset{\sim i}{A}}(x) = 1 - \mu_{\overset{\sim i}{A}}(x) - \gamma_{\overset{\sim i}{A}}(x)$ is called the hesitation part, which may cater to either membership value or non-membership value or both.

3. Definition: If $\overset{\sim i}{A}$ and $\overset{\sim i}{B}$ are two IFS of the set E. then

1. $\overset{\sim i}{A} \subset \overset{\sim i}{B}$ iff $\forall$ x$\in$ E [$\mu_{\overset{\sim i}{A}}(x) \le \mu_{\overset{\sim i}{B}}(x)$ and $\gamma_{\overset{\sim i}{A}}(x) \ge \gamma_{\overset{\sim i}{B}}(x)$]
2. $\overset{\sim i}{A} = \overset{\sim i}{B}$ iff $\overset{\sim i}{A} \subset \overset{\sim i}{B}$ and $\overset{\sim i}{B} \subset \overset{\sim i}{A}$.
3. $\overset{\sim i}{A} \cap \overset{\sim i}{B} = \{< x, \min(\mu_{\overset{\sim i}{A}}(x), \mu_{\overset{\sim i}{B}}(x)), \max(\gamma_{\overset{\sim i}{A}}(x), \gamma_{\overset{\sim i}{B}}(x)) / x \in E\}$.
4. $\overset{\sim i}{A} \cup \overset{\sim i}{B} = \{< x, \max(\mu_{\overset{\sim i}{A}}(x), \mu_{\overset{\sim i}{B}}(x)), \min(\gamma_{\overset{\sim i}{A}}(x), \gamma_{\overset{\sim i}{B}}(x)) / x \in E\}$.

Obviously, every fuzzy set has the form : $\overset{\sim i}{A} = \{< x, \mu_{\overset{\sim i}{A}}(x), \gamma_{\overset{\sim i}{A}}(x) > / x \in E\}$.

In a general Multi-Objective Linear Programming Problem (MOLPP) all constraints goals are fixed. But in real life situation, these constraint goals can not be always fixed. So we can consider the constraint goals for the less than type constraints as at lest a_i and it may possible be extended to $a_i + a_i^0$. Similarly the constraint goals for the grater than type constraint is at most b_j and it may possible be diminished to $b_j - b_j^0$. These fact seems to take all the constraint goals as a intuitionistic fuzzy set and which will be more realistic description. Then the MOLPP becomes a multi-objective linear programming problem with intuitionistic fuzzy resources, which can be described as follows:

minimize Z =[Z^1, Z^2, Z^3, ---------- , Z^K] (3.1)

subject to $\sum_{j=1}^{n} x_{ij} \le \tilde{a}_i^{i}$ for i = 1,2,3,----------, m

$\sum_{i=1}^{m} x_{ij} \ge \tilde{b}_j^{i}$ for j = 1,2,3,----------, n

$x_{ij} \ge 0$, for all i, j

where

$Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}$, k = 1, 2, 3, ---------- , K

Here the constraints goals are characterized by the following intuitionistic fuzzy sets(IFS):

$\tilde{a}_i^{i} = (x_{ij}, \mu_{\tilde{a}_i^{i}}(x_{ij}), \gamma_{\tilde{a}_i^{i}}(x_{ij}))$, $\tilde{b}_j^{i} = (x_{ij}, \mu_{\tilde{b}_j^{i}}(x_{ij}), \gamma_{\tilde{b}_j^{i}}(x_{ij}))$, where $\mu_{\tilde{a}_i^{i}}(x_{ij})$, $\mu_{\tilde{b}_j^{i}}(x_{ij})$ are the degree of membership function of the IFS $\tilde{a}_i^{i}$ and $\gamma_{\tilde{a}_i^{i}}(x_{ij}), \gamma_{\tilde{b}_j^{i}}(x_{ij})$ are the degree of non-membership function of the IFS $\tilde{b}_j^{i}$

In the case when the degree of rejection(non-acceptance) is defined simultaneously with the degree of acceptance(membership) and when both these degrees are not complementary to each other them. Intuitionistic fuzzy(IF) set can be used as a more general and full tool for describing this fuzziness. It is possible to represent deeply existing nuances in problem formulation defining objectives and constraints(or part of them) by IF sets i.e a pairs of membership($\mu_s(x_{ij})$) and non-membership($\gamma_s(x_{ij})$) functions.

So to maximize the degree of acceptance of IF objectives and constraints and to minimize the degree of rejection of IF objectives and constraints we have the following:

$\max_{x_{ij}} \{ \mu_s(x_{ij}) \}$, s =1,2,3, ……,m+n+K

$\min_{x_{ij}} \{ \gamma_s(x_{ij}) \}$, s =1,2,3, ……,m+n+K

s.t $\mu_s(x_{ij}) + \gamma_s(x_{ij}) \le 1$,

$\mu_s(x_{ij}) \ge \gamma_s(x_{ij})$,

$\gamma_s(x_{ij}) \ge 0$

where $\mu_s(x_{ij})$ denotes the degree of membership of x_{ij} to the s-th IF sets and $\gamma_s(x_{ij})$ denotes the degree of rejection of x_{ij} to the s-th IF sets.

To construct the pay-off matrix we have the following two LPP with and without tolerance for each constraints:

minimize Z =[Z^1, Z^2, Z^3, ---------- , Z^K] (3.2)

subject to $\sum_{j=1}^{n} x_{ij} \leq a_i + a_i^0$ for i = 1,2,3,----------, m

$\sum_{i=1}^{m} x_{ij} \geq b_j - b_j^0$ for j = 1,2,3,----------, n

$x_{ij} \geq 0$, for all i, j

where

$Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}$, k = 1, 2, 3, ---------- , K

And minimize Z =[Z^1, Z^2, Z^3, ---------- , Z^K] (3.3)

subject to $\sum_{j=1}^{n} x_{ij} \leq a_i$ for i = 1,2,3,----------, m

$\sum_{i=1}^{m} x_{ij} \geq b_j$ for j = 1,2,3,----------, n

$x_{ij} \geq 0$, for all i, j

where

$Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}$, k = 1, 2, 3, ---------- , K

4. Intutionistic fuzzy programming technique to solve multi-objective linear programming problem with special type of hyperbolic membership and parabolic non-membership function:

In 1981, Leberling proposed a special non-linear membership function described with a hyperbolic function in MOLP problems by considering that the rate of increase in membership of satisfaction must not always be constant as in case of a linear membership function. Following the maximizing decision together with a hyperbolic membership function, he proved that there exist an equivalent linear programming problem. In this paper, intuitionistic fuzzy goals and objective value have been represented by hyperbolic membership and parabolic non-membership functions.

We first find the lower bound as L_k(least value) and upper bound as U_k(worst value) for the k-th objective function of the problem (3.3) & (3.4), k = 1,2,3,…. K where U_k is the highest acceptable level of achievement for kth object and L_k the aspired level of achievement for the objective k. When the aspiration levels for each objectives and constraints in both of membership and non-membership function have been specified, then we formed a intuitionistic fuzzy model and then convert the intuitionistic fuzzy model onto a crisp model.

Algorithm:

Step-1. Solve the MOLPs (3.2)&(3.3) as a single objective transportation problem K times for each problem by taking one of the objective at a time.
Step-2. From the result of Step-1. determine the corresponding values for every objective at each solutions derived and construct a payoff matrix as:

$$\begin{bmatrix} Z_1(X^{11*}) & Z_1(X^{12*}) & Z_2(X^{11*}) & Z_2(X^{12*}) \ldots\ldots\ldots\ldots & Z_K(X^{11*}) & Z_K(X^{12*}) \\ Z_1(X^{21*}) & Z_1(X^{22*}) & Z_2(X^{21*}) & Z_2(X^{22*}) \ldots\ldots\ldots\ldots & Z_K(X^{21*}) & Z_K(X^{22*}) \\ Z_1(X^{31*}) & Z_1(X^{32*}) & Z_2(X^{31*}) & Z_2(X^{32*}) \ldots\ldots\ldots\ldots & Z_K(X^{31*}) & Z_K(X^{32*}) \\ \ldots\ldots\ldots\ldots & & & & & \\ \ldots\ldots\ldots\ldots & & & & & \\ Z_1(X^{K-11*}) & Z_1(X^{K-12*}) & Z_2(X^{K-11*}) & Z_2(X^{K-12*}) \ldots\ldots\ldots & Z_K(X^{K-11*}) & Z_K(X^{K-12*}) \\ Z_1(X^{K1*}) & Z_1(X^{K2*}) & Z_2(X^{K1*}) & Z_2(X^{K2*}) \ldots\ldots\ldots & Z_K(X^{K1*}) & Z_K(X^{K2*}) \end{bmatrix}$$

Step-3. From step-2. We find the worst(U_k) and the best(L_k) values of each objectives for the degree of acceptance and rejection corresponding to the set of solutions as follows:
Step -3a.

$U_k^{acc} = \max \left\{ Z_k(X^{rs^*}) \right\}$ and $L_k^{acc} = \min \left\{ Z_k(X^{rs^*}) \right\}$

$1 \le r \le K$ $1 \le r \le K$

$s = \{1, 2\}$ $s = \{1, 2\}$

for degree of acceptance of objectives.

Step –3b. We presents a new upper bound for the degree of rejection of objectives as follows:

$L_k^{rej} = L_k^{acc} + t\,(U_k^{acc} - L_k^{acc})$ with $U_k^{rej} = U_k^{acc}$ where $0 < t < 1$

Step-4. the initial intuitionistic fuzzy model becomes (in term of aspiration levels with each objectives)

Find $\{ x_{ij}$, i = 1,2,3,----------, m ; j =1,2,3,….., n $\}$ (4.1)

so as to satisfy

$Z_k \le L_k^{acc}$ with tolerance $(U_k^{acc} - L_k^{acc})$ for the degree of acceptance, for k = 1,2,3, …, K

$Z_k \ge U_k^{rej}$ with tolerance $(U_k^{rej} - L_k^{rej})$ for the degree of rejection, for k = 1,2,3, ……., K

$\sum_{j=1}^{n} x_{ij} \le a_i$ with tolerance a_i^0 for the degree of acceptance, for i = 1,2,3,----------, m

$\sum_{j=1}^{n} x_{ij} \ge a_i + a_i^0$ with tolerance d_i^0 for the degree of rejection, for i = 1,2,3,----------, m

where $d_i^0 = t\, a_i^0$ with $0 < t < 1$

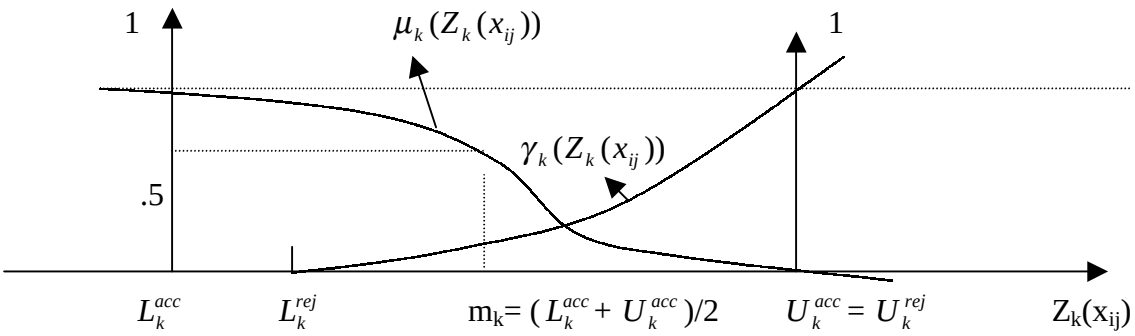
$\sum_{i=1}^{m} x_{ij} \ge b_j$ with tolerance b_j^0 for the degree of acceptance, for j = 1,2,3,---------, n

$\sum_{i=1}^{m} x_{ij} \leq b_j - b_j^0$ with tolerance d_j^0 for the degree of rejection, for j = 1,2,3,---------, n

where $d_j^0 = t b_j^0$ with $0 < t < 1$

Step-5. Define the membership(acceptance) and non-membership(rejection) functions of IF objectives and constraints(or part of them) as follows:

For the k-th(k =1,2,3,….,K) objectives functions,
a hyperbolic membership function ($\mu_k(Z_k(x_{ij}))$) is defined by



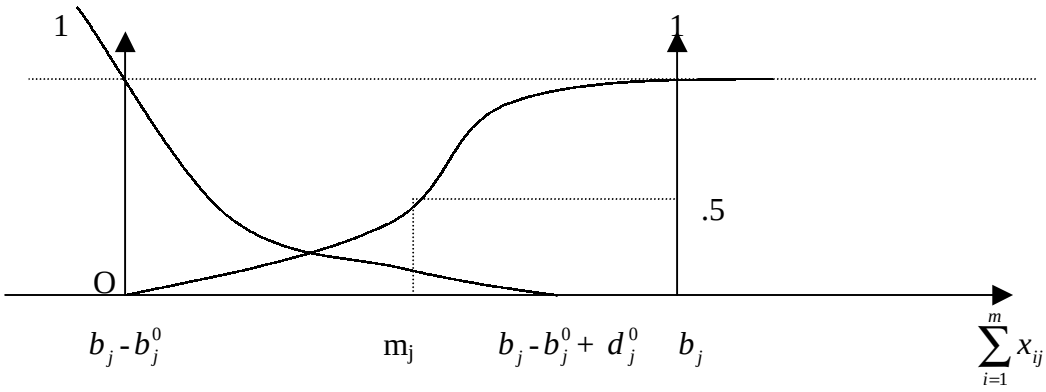
$$\mu_k(Z_k(x_{ij})) = \begin{cases} 1, & Z_k(x_{ij}) \leq L_k^{acc} \\ \frac{1}{2}\tanh(m_k - Z_k(x_{ij})) + \frac{1}{2}, & L_k^{acc} \leq Z_k(x_{ij}) \leq U_k^{acc} \\ 0, & Z_k(x_{ij}) \geq U_k^{acc} \end{cases}$$

Parabolic non-membership function ($\gamma_k(Z_k(x_{ij}))$) is defined as

$$\gamma_k(Z_k(x_{ij})) = \begin{cases} 0, & Z_k(x_{ij}) \leq L_k^{rej} \\ \left(\frac{z_k(x_{ij}) - L_k^{rej}}{U_k^{rej} - L_k^{rej}}\right)^2, & L_k^{rej} \leq Z_k(x_{ij}) \leq U_k^{rej} \\ 1, & Z_k(x_{ij}) \geq U_k^{rej} \end{cases}$$

For the j-th (j =1,2,3, ….., n) constraint,

the hyperbolic membership function ($\mu_j(\sum_{i=1}^{m} x_{ij})$ is defined by



$= 1,$ $\sum_{i=1}^{m} x_{ij} \geq b_j$

$\mu_j(\sum_{i=1}^{m} x_{ij}) = \frac{1}{2}\tanh(\sum_{i=1}^{m} x_{ij} - m_j) + \frac{1}{2}$ $b_j - b_j^0 \leq \sum_{i=1}^{m} x_{ij} \leq b_j$

$= 0\ ,$ $\sum_{i=1}^{m} x_{ij} \leq b_j - b_j^0$

here $m_j = (2b_j - b_j^0)/2$

Parabolic non-membership function($\gamma_j(\sum_{i=1}^{m} x_{ij})$) is defined as

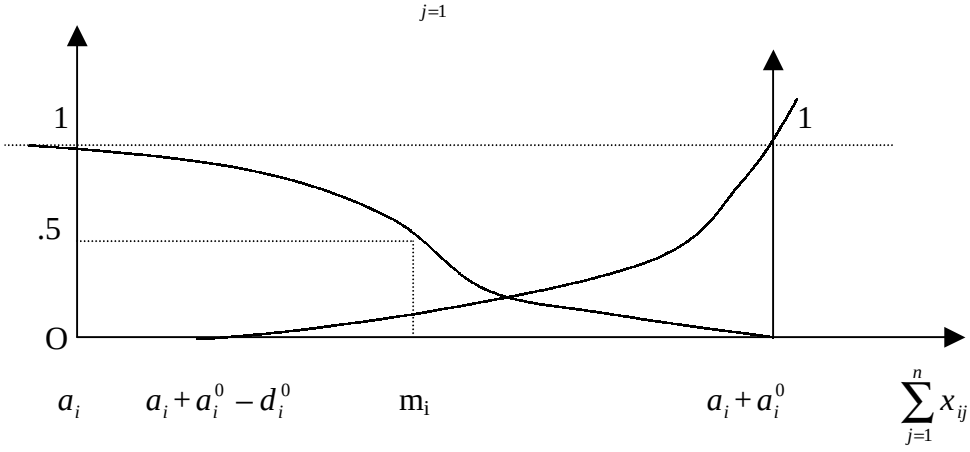
$= 0\ ,$ $b_j - b_j^0 + d_j^0 \leq \sum_{i=1}^{m} x_{ij}$

$\gamma_j(\sum_{i=1}^{m} x_{ij}) = \left(\frac{b_j - b_j^0 + d_j^0 - \sum_{i=1}^{m} x_{ij}}{d_j^0} \right)^2$, $b_j - b_j^0 \leq \sum_{i=1}^{m} x_{ij} \leq b_j - b_j^0 + d_j^0$

$= 1\ ,$ $\sum_{i=1}^{m} x_{ij} \leq b_j - b_j^0$

For the i-th (i =1,2,3, ….., m) constraint,

the hyperbolic membership function ($\mu_i(\sum_{j=1}^{n} x_{ij})$ is defined by



$$= 1 , \qquad \sum_{j=1}^{n} x_{ij} \le a_i$$

$$\mu_i(\sum_{j=1}^{n} x_{ij}) = \frac{1}{2}\tanh(m_i - \sum_{j=1}^{n} x_{ij}) + \frac{1}{2} , \qquad a_i \le \sum_{j=1}^{n} x_{ij} \le a_i + a_i^0 ,$$

$$= 0 , \qquad a_i + a_i^0 \le \sum_{j=1}^{n} x_{ij}$$

here $m_i = (2a_i + a_i^0)/2$

Parabolic non-membership function ($\gamma_i(\sum_{j=1}^{n} x_{ij})$) is defined as

$$= 0 , \qquad \sum_{j=1}^{n} x_{ij} \le a_i + a_i^0 - d_i^0$$

$$\gamma_i(\sum_{j=1}^{n} x_{ij}) = \left(\frac{\sum_{j=1}^{n} x_{ij} - a_i}{d_i^0} \right)^2 , \qquad a_i + a_i^0 - d_i^0 \le \sum_{j=1}^{n} x_{ij} \le a_i + a_i^0$$

$$= 1 , \qquad a_i + a_i^0 \le \sum_{j=1}^{n} x_{ij}$$

Step-6. Find an equivalent crisp model by using the membership and non-membership functions of objectives, constraints by IF as follows:

$$\max \{ \sum_{1}^{m+n+K} (\mu_s(x_{ij}) - \gamma_s(x_{ij})) \} \qquad (4.2)$$

x_{ij}

subject to

$\mu_s(x_{ij}) + \gamma_s(x_{ij}) \le 1$

$\mu_s(x_{ij}) \ge \gamma_s(x_{ij})$

$\gamma_s(x_{ij}) \ge 0$

$x_{ij} \ge 0$, for all i, j , s =1,2,3, ……,m+n+K

Step-7. Solve the above crisp model by an appropriate mathematical programming algorithm.

Intuitionistic fuzzy optimization (IFO) problem such as fuzzy optimization problem can be represented as a two-stage process, which includes aggregation of objectives and constraints and defuzzification (maximization of aggregation function). Usually the applied Bellman-Zadeh's approach for fuzzy optimization problem solving realizes min-aggregation.

Conjunction of IF set is defined as
$\tilde{G} \cap \tilde{C} = \{< x, \mu_{\tilde{G}}(X) \cap \gamma_{\tilde{G}}(X), \mu_{\tilde{C}}(X) \cup \gamma_{\tilde{C}}(X) >, x \in E\}$ where $\tilde{G}$ denotes an IF objective(gain) and $\tilde{C}$ denotes an IF constraint.

This application can be easily generalized and applied to the IFO problem.
Appling the above to the IFO problem (4.2), we have the following:

$\alpha \leq \mu_s(x_{ij})$, s = { k, i, j / k=1,2,3, ….,K; i = 1,2,3, …,m; j=1,2,3, …,m }

$\beta \geq \gamma_s(x_{ij})$, s = { k, i, j / k=1,2,3, ….,K; i = 1,2,3, …,m; j=1,2,3, …,m }

$\alpha + \beta \leq 1$

$\alpha \geq \beta$, $\beta \geq 0$, $x_{ij} \geq 0$, for all i, j

where α denotes the minimal degree of acceptance of objective(s) and constraint(s) and β denotes the maximal degree of rejection of objective(s) and constraint(s).
Now the IFO problem can be transformed to the following crisp optimization problem:

max (α - β)
subject to

$\alpha \leq \mu_s(x_{ij})$, s = { k, i, j / k=1,2,3, ….,K; i = 1,2,3, …,m; j=1,2,3, …,m }

$\beta \geq \gamma_s(x_{ij})$, s = { k, i, j / k=1,2,3, ….,K; i = 1,2,3, …,m; j=1,2,3, …,m }

$\alpha + \beta \leq 1$

$\alpha \geq \beta$, $\beta \geq 0$, $x_{ij} \geq 0$, for all i, j

It is equivalent to

max (α - β) (4.3)
subject to

$\tanh(m_k - Z_k(x_{ij})) \geq 2\alpha - 1$,

$\tanh(\sum_{i=1}^{m} x_{ij} - m_j) \geq 2\alpha - 1$,

$\tanh(m_i - \sum_{j=1}^{n} x_{ij}) \geq 2\alpha - 1$,

$Z_k(x_{ij}) - L_k^{rej} \leq (U_k^{rej} - L_k^{rej})\sqrt{\beta}$,

$b_j - \sum_{i=1}^{m} x_{ij} \leq d_j^0 \sqrt{\beta}$,

$\sum_{j=1}^{n} x_{ij} - a_i \leq d_i^0 \sqrt{\beta}$,

$\alpha + \beta \leq 1$

$\alpha \geq \beta$, $\beta \geq 0$, $x_{ij} \geq 0$, for all i, j

In the above formulation (4.3), however all the membership and non-membership function are non-linear functions and hence we can not directly apply the linear programming method. To circumvent such difficulty, we have transformed the problem in the following way:

we define, $\tanh^{-1}(2\alpha - 1) = \alpha'$ and $\sqrt{\beta} = \beta'$

such that $\alpha = \frac{1}{2}\tanh(\alpha') + \frac{1}{2}$ and $\beta = \beta'^2$

since $\tanh^{-1}(x)$ is strictly increasing function with respect to x then maximization of α is equivalent to the maximization of α'. Also since $\sqrt{\beta}$ is a strictly increasing function as that of β, the minimization of β is equivalent to the minimization of β'. Hence the above problem can be transformed to the following ordinary linear programming problem:

$$\max.(\alpha' - \beta') \qquad (4.4)$$

subject to

$$Z_k(x_{ij}) + \alpha' \le m_k$$

$$\sum_{i=1}^{m} x_{ij} - \alpha' \ge m_j$$

$$\sum_{j=1}^{n} x_{ij} + \alpha' \le m_i,$$

$$Z_k(x_{ij}) - L_k^{rej} \le (U_k^{rej} - L_k^{rej})\beta',$$

$$b_j - \sum_{i=1}^{m} x_{ij} \le d_j^0 \beta',$$

$$\sum_{j=1}^{n} x_{ij} - a_i \le d_i^0 \beta',$$

$\alpha' + \beta' \le 1$, $\alpha' \ge \beta'$, $\beta' \ge 0$, $x_{ij} \ge 0$, for all i, j

Step-8. Determine if the decision maker is satisfied with the solution identified in step-7.
Step-8a. If the decision maker is satisfied, STOP.
Step-8b. If the decision maker is not satisfied, continue with step-3b. again. And define a new upper bound for the degree of rejection of objectives and constraints.

This iteration will continue until and unless the decision maker is satisfied with the solutions.

Numerical Example 1:

$$\text{Min } Z_1 = 3x_1 + 2x_2 \qquad (1)$$

$$\text{Min } Z_2 = x_1 + 5x_2$$

such that

$$x_1 + x_2 \le \overset{\sim i}{18}$$

$$8x_1 + 6x_2 \ge \overset{\sim i}{112}$$

$$5x_1 + 7x_2 \ge \overset{\sim i}{96}$$

$$x_1, x_2 \ge 0$$

where the constraint intuitionistic fuzzy goals are characterized by the following way

$\overset{\sim i}{18} = (x_{ij}, \mu_{\overset{\sim i}{18}}(x_{ij}), \gamma_{\overset{\sim i}{18}}(x_{ij}))$ with

$$= 1, \qquad x_1 + x_2 \le 18$$

$$\mu_{\underset{18}{\sim i}}(x_1 + x_2) = \frac{1}{2}\tanh(m_1 - x_1 - x_2) + \frac{1}{2}, \qquad 18 \le x_1 + x_2 \le 20,$$

$$= 0, \qquad 20 \le x_1 + x_2$$

and

$$= 0, \qquad x_1 + x_2 \le 18.5$$

$$\gamma_{\underset{18}{\sim i}}(x_1 + x_2) = \left(\frac{x_1 + x_2 - 18.5}{1.5}\right)^2, \qquad 18.5 \le x_1 + x_2 \le 20$$

$$= 1, \qquad 20 \le x_1 + x_2$$

$\overset{\sim i}{112} = (x_{ij}, \mu_{\underset{112}{\sim i}}(x_{ij}), \gamma_{\underset{112}{\sim i}}(x_{ij}))$ with

$$= 1, \qquad 8x_1 + 6x_2 \ge 112$$

$$\mu_{\underset{112}{\sim i}}(8x_1 + 6x_2) = \frac{1}{2}\tanh(8x_1 + 6x_2 - m_1) + \frac{1}{2}, \qquad 107 \le 8x_1 + 6x_2 \le 112,$$

$$= 0, \qquad 8x_1 + 6x_2 \le 107,$$

and

$$= 0, \qquad 110 \le 8x_1 + 6x_2$$

$$\gamma_{\underset{112}{\sim i}}(8x_1 + 6x_2) = \left(\frac{110 - 8x_1 - 6x_2}{3}\right)^2, \qquad 107 \le 8x_1 + 6x_2 \le 110$$

$$= 1, \qquad 8x_1 + 6x_2 \le 107$$

$\overset{\sim i}{96} = (x_{ij}, \mu_{\underset{96}{\sim i}}(x_{ij}), \gamma_{\underset{96}{\sim i}}(x_{ij}))$ with

$$= 1, \qquad 5x_1 + 7x_2 \ge 96$$

$$\mu_{\underset{96}{\sim i}}(5x_1 + 7x_2) = \frac{1}{2}\tanh(5x_1 + 7x_2 - m_1) + \frac{1}{2}, \qquad 90 \le 5x_1 + 7x_2 \le 96,$$

$$= 0, \qquad 5x_1 + 7x_2 \le 90,$$

and

$$= 0, \qquad 95 \le 5x_1 + 7x_2$$

$$\gamma_{\underset{96}{\sim i}}(5x_1 + 7x_2) = \left(\frac{95 - 5x_1 - 7x_2}{5}\right)^2, \qquad 90 \le 5x_1 + 7x_2 \le 95$$

$$= 1, \qquad 5x_1 + 7x_2 \le 90$$

To form a payoff table we have consider the following two problems with and without tolerance:

$\min Z_1 = 3x_1 + 2x_2$ (2) $\min Z_1 = 3x_1 + 2x_2$ (3)

$\min Z_2 = x_1 + 5x_2$ $\min Z_2 = x_1 + 5x_2$

such that such that

$x_1 + x_2 \leq 18$ $x_1 + x_2 \leq 20$

$8x_1 + 6x_2 \geq 112$ $8x_1 + 6x_2 \geq 107$

$5x_1 + 7x_2 \geq 96$ $5x_1 + 7x_2 \geq 90$

$x_1, x_2 \geq 0$ $x_1, x_2 \geq 0$

solving (2) & (3), we have the following payoff matrix:

$$\text{Payoff matrix} = \begin{bmatrix} 38 & 82 \\ 35.67 & 89.17 \\ 51 & 30 \\ 54 & 18 \end{bmatrix}$$

Therefore,

$L_1^{acc} = 35.67$, $U_1^{acc} = 54$, $L_2^{acc} = 18$, $U_2^{acc} = 89.17$ and we consider

$L_1^{rej} = 37$, $U_1^{rej} = 54$, $L_2^{rej} = 20$, $U_2^{rej} = 89.17$

Defining the hyperbolic membership(acceptance) and parabolic non-membership(rejection) functions of IF objectives and constraints(or part of them) as step-5, we have the following:

max.$(\alpha' - \beta')$ (4)

s.t

$$Z_1 + \alpha' \leq \frac{35.67 + 54}{2}$$

$$Z_2 + \alpha' \leq \frac{18 + 89.17}{2}$$

$$x_1 + x_2 + \alpha' \leq \frac{18 + 20}{2}$$

$$8x_1 + 6x_2 - \alpha' \geq \frac{112 + 107}{2},$$

$$5x_1 + 7x_2 - \alpha' \geq \frac{96 + 90}{2},$$

$Z_1 - 37 \leq (54 - 37)\beta'$,

$Z_2 - 20 \leq (89.17 - 20)\beta'$,

$x_1 + x_2 - 18.5 \leq (20 - 18.5)\beta'$,

$110 - 8x_1 - 6x_2 \leq (110 - 107)\beta'$,

$95 - 5x_1 - 7x_2 \leq (95 - 90)\beta'$,

$\alpha' + \beta' \leq 1$, $\alpha' \geq \beta'$, $\beta' \geq 0$, and $x_1, x_2 \geq 0$

Any linear algorithm or any simplex method can easily solve the above problem. The optimal solution satisfies the objective with degree $\alpha = 0.7986218$($\alpha' = 0.6888514$) and dissatisfies the objective with degree $\beta = 0.0968134$($\beta' = 0.3111486$) and $x_1^* =$

9.877180, $x_2^* = 6.328995$, $z_1^* = 42.29$, $z_2^* = 41.52$ Sum of the objective values = 42.29 + 41.52 = 83.81 The solution of the analogous fuzzy linear programming (FLP) problem and crisp linear programming (LP) problem lead to objective value of $z_1^* = 39.52$, $z_2^* = 46.42$, $x_1^* = 8.057692$, $x_2^* = 7.673077$ with total objective value = 39.52 + 46.42 = 85.94 and $z_1^* = 80.33$, $z_2^* = 95.33$ with total objective value = 80.33 + 95.33 = 175.66 respectively. It is noted that the total objective value, obtained by the IFO is better solution than the results, obtained from FO.

5. Application in Capacitated Transportation Model

A transportation problem with capacity restriction is a linear programming problem and can be solved by a L.P algorithm. In many particular application, it is realistic to assume that the amount which can be sent on any particular route is restricted by tha capacity of that route. Further, whenever a route is altogether excluded, this can be expressed by limiting its capacity zero, this an alternative to attach a very high cost to that route. This type of transportation problem becomes a Capacitated Transportation Problem (CTP).

Consider m origin O_i (i=1,2, …,m) and n destination D_j (j=1,2,….,n). At each origin O_i, let a_i be the quantity of homogeneous product which we want to transport to n destination D_j to satisfy the demands b_j there. The sources may be production facilities, warehouses, supply points, etc. and destination may be consumption facilities, demand points, rtc. A penalty c_{ijp} is associated with transportation of a unit of the products from i-th source to j-th destination by means of the p-th conveyance for three dimensional Transportation Problem (TP). The penalty could represent transportation cost, delivery time, the product deterioration during transportation, under-used capacity, etc. A variable x_{ijp} represent the unknown quantity to be transported from i-th source to j-th destination by means of the p-th conveyance for three dimensional CTP. Let e_p be the capacity of the p-th conveyance. In real world, all the CTP are not a single objective in nature. We may have more than one objective function in TP with capacity restrictions. Let r_{ijp} be the capacity restriction on route i, j by mean of the p-th conveyance for three dimensional CTP. The cost of transporting a unit of product may be energy cost consumed in transportation, the transportation time, or the product deterioration during transportation, etc.

In the CTP, it is well known that in reality, all the constraints goals(sources & demands) fluctuates on both seasonal and situational bases. So these constraints goals can not be well defined always in real life. So if we introduce the constraint goals as a intuitionistic fuzzy set in placing the crisp (fixed) goals of the constraints, then the CTP becomes more realistic.

Mathematical model of multi-objective three dimensional capacitated transportation model with intuitionistic fuzzy goals can be represented as follows:

$$\text{Minimize } Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{P} c_{ijp}^{1} x_{ijp} \qquad (5.1)$$

$$\text{Minimize } Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{P} c_{ijp}^{2} x_{ijp}$$

……
……

Minimize $Z_K = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{P} c_{ijp}^{K} x_{ijp}$

Subject to,

$\sum_{j=1}^{n}\sum_{p=1}^{P} x_{ijp} \le \tilde{a}_i^{\,i}$ (i =1,2,…,m), $\sum_{i=1}^{m}\sum_{p=1}^{P} x_{ijp} \le \tilde{b}_j^{\,i}$ (j=1,2,…,n), $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijp} \ge \tilde{e}_p^{\,i}$ (p=1,2,…,P),

$0 \le x_{ijp} \le r_{ijp} \quad \forall$ i,j,p,

where intuitionistic fuzzy goals are characterized by

$\tilde{a}_i^{\,i} = (x_{ij}, \mu_{\tilde{a}_i^{\,i}}(x_{ij}), \gamma_{\tilde{a}_i^{\,i}}(x_{ij}))$, $\tilde{b}_j^{\,i} = (x_{ij}, \mu_{\tilde{b}_j^{\,i}}(x_{ij}), \gamma_{\tilde{b}_j^{\,i}}(x_{ij}))$, $\tilde{e}_p^{\,i} = (x_{ij}, \mu_{\tilde{e}_p^{\,i}}(x_{ij}), \gamma_{\tilde{e}_p^{\,i}}(x_{ij}))$,

, $a_i >0$, $b_j>0$, $e_p>0$, $r_{ijp} \ge 0 \ \forall$ i,j,p .

This is a MOLPP with intuitionistic fuzzy goals in constraints and can be solved by previous algorithm.

Example-2

We have considered a three-dimensional Multi-Objective Capacitated Transportation Problem (MOCTP) with intuitionistic fuzzy goals , having three objective functions, three sources, three demand points and three different modes of transportation. All the necessary data are given below:

C_{ijp}^{2} =

capacities	$\tilde{e}_1^{\,i} = 17$			$e_2 = 25$			$\tilde{e}_2^{\,i} = 9$			Demands
	conv1	conv2	conv3	conv1	conv2	conv3	conv1	conv2	conv3	
	9	12	9	6	9	7	3	7	7	$\tilde{b}_1^{\,i} = 11$
	5	6	5	9	11	3	6	8	6	$b_2 = 19$
	2	2	1	2	7	7	1	9	3	$\tilde{b}_3^{\,i} = 22$
supplies	$\tilde{a}_1^{\,i} = 24$			$\tilde{a}_2^{\,i} = 9$			$a_3 = 18$			

C^{2}_{ijp} =

capacities	$\tilde{e}_1 = 17$	$e_2 = 25$	$\tilde{e}_2 = 9$	Demands
	conv1 conv2 conv3	conv1 conv2 conv3	conv1 conv2 conv3	
	2 9 8	1 4 1	9 9 5	$\tilde{b}_1 = 11$
	2 8 1	4 5 2	8 6 9	$b_2 = 19$
	5 2 7	8 9 7	5 2 5	$\tilde{b}_3 = 22$
supplies	$\tilde{a}_1 = 24$	$\tilde{a}_2 = 9$	$a_3 = 18$	

C^{3}_{ijp} =

capacities	$\tilde{e}_1 = 17$	$e_2 = 25$	$\tilde{e}_2 = 9$	Demands
	conv1 conv2 conv3	conv1 conv2 conv3	conv1 conv2 conv3	
	2 4 6	3 6 4	8 4 9	$\tilde{b}_1 = 11$
	2 5 3	5 6 6	9 6 3	$b_2 = 19$
	1 9 1	8 3 9	5 7 11	$\tilde{b}_3 = 22$
supplies	$\tilde{a}_1 = 24$	$\tilde{a}_2 = 9$	$a_3 = 18$	

Capacity restriction of the routes are given as:

$0 \le x_{111} \le 25$, $0 \le x_{112} \le 45$, $0 \le x_{113} \le 50$, $0 \le x_{121} \le 60$, $0 \le x_{122} \le 40$, $0 \le x_{123} \le 85$,
$0 \le x_{131} \le 48$, $0 \le x_{132} \le 50$, $0 \le x_{133} \le 30$, $0 \le x_{211} \le 20$, $0 \le x_{212} \le 25$, $0 \le x_{213} \le 35$,
$0 \le x_{221} \le 38$, $0 \le x_{222} \le 40$, $0 \le x_{223} \le 27$, $0 \le x_{231} \le 38$, $0 \le x_{232} \le 44$, $0 \le x_{233} \le 12$,
$0 \le x_{311} \le 19$, $0 \le x_{312} \le 55$, $0 \le x_{313} \le 65$, $0 \le x_{321} \le 27$, $0 \le x_{322} \le 26$, $0 \le x_{323} \le 18$,
$0 \le x_{331} \le 29$, $0 \le x_{332} \le 39$, $0 \le x_{333} \le 15$.

The above MOCTP with TFN cost can be presented as follows:

$$\text{Min } Z_1=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{p=1}^{3} c^{1}_{ijp}\, x_{ijp}\ ,\ \text{Min } Z_2=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{p=1}^{3} c^{2}_{ijp}\, x_{ijp}\ ,\ \text{Min } Z_3=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{p=1}^{3} c^{3}_{ijp}\, x_{ijp} \qquad (1)$$

subject to,

$$\sum_{j=1}^{3}\sum_{p=1}^{3} x_{ijp} \le \tilde{a}_i \ (i=1,2,3),\ \sum_{i=1}^{3}\sum_{p=1}^{3} x_{ijp} \le \tilde{b}_j \ (j=1,2,3),\ \sum_{i=1}^{3}\sum_{j=1}^{3} x_{ijp} \ge \tilde{e}_p \ (p=1,2,3),\ 0 \le x_{ijp} \le r_{ijp}$$

$\forall$ i,j,p,

where intuitionistic fuzzy constraint goals are characterized as follows:

$\sum_{j=1}^{3}\sum_{p=1}^{3} x_{1jp} \le 24$ with tolerance 2 for the degree of acceptance,

$\sum_{j=1}^{3}\sum_{p=1}^{3} x_{1jp} \ge 26$ with tolerance 1.5 for the degree of rejection

$\sum_{j=1}^{3}\sum_{p=1}^{3} x_{2jp} \le 9$ with tolerance 3 for the degree of acceptance,

$\sum_{j=1}^{3}\sum_{p=1}^{3} x_{2jp} \ge 12$ with tolerance 2 for the degree of rejection

$\sum_{i=1}^{3}\sum_{p=1}^{3} x_{i1p} \le 11$ with tolerance 3 for the degree of acceptance,

$\sum_{i=1}^{3}\sum_{p=1}^{3} x_{i1p} \ge 14$ with tolerance 2 for the degree of rejection,

$\sum_{i=1}^{3}\sum_{p=1}^{3} x_{i3p} \le 22$ with tolerance 2 for the degree of acceptance,

$\sum_{i=1}^{3}\sum_{p=1}^{3} x_{i3p} \ge 24$ with tolerance 1.5 for the degree of rejection

$\sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij1} \ge 17$ with tolerance 3 for the degree of acceptance,

$\sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij1} \le 14$ with tolerance 2 for the degree of rejection,

$\sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij3} \ge 9$ with tolerance 2 for the degree of acceptance,

$\sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij3} \le 7$ with tolerance 1.5 for the degree of rejection,

$a_i > 0$, $b_j > 0$, $e_p > 0$, $r_{ijp} \ge 0 \ \forall$ i,j,p and equality sign holds for i =3; j = 2; p = 2 .

Solving the above MOCTP (1) with and without tolerance of constraint goals, we have following payoff matrix:

$$\text{Payoff matrix} = \begin{bmatrix} 197 & 297 & 351 \\ 180 & 223 & 340 \\ 390 & 101 & 244 \\ 307 & 87 & 239 \\ 293 & 340 & 149 \\ 281 & 294 & 132 \end{bmatrix}$$

Therefore,

$L_1^{acc} = 180$, $U_1^{acc} = 390$, $L_2^{acc} = 87$, $U_2^{acc} = 340$, $L_3^{acc} = 132$, $U_3^{acc} = 351$ and we consider

$L_1^{rej} = 190$, $U_1^{rej} = 390$, $L_2^{rej} = 100$, $U_2^{rej} = 340$, $L_3^{rej} = 140$, $U_3^{rej} = 351$

Taking all membership functions as hyperbolic function and all non-membership function as parabolic function as described in example 1, we have the following optimal solutions.

The optimal solution satisfies the objective with degree α =.7680425(α' =0.5986456) and dissatisfies the objective with degree β = .1610853 (β' =.4013544) and z_1^*= 270.27, z_2^*= 196.32, z_3^*= 224.68. The solution of the analogous fuzzy linear programming (FLP) problem and crisp linear programming (LP) problem lead to objective value of z_1^*= 284.00, z_2^*= 212.50, z_3^*= 240.50 and z_1^*= 293.333, z_2^*= 220.33, z_3^*= 249.83 respectively.

Conclusion: The new concept to the optimization problem in an IF environment is introduced in the paper. This concept allows one to define a degree of rejection which may not simply a complement of degree of acceptance. In this paper, two special type of membership and non-membership functions have been used to solve the MOTP. When we use the hyperbolic membership and parabolic non-membership functions then the crisp model becomes linear by giving suitable algebraic transformation. This gives the optimal solution which shows that the solution of IFO can satisfy the objective functions with higher degree than solution of analogous fuzzy and crisp problem. Moreover, we conclude that for a multi-objective probabilistic TP if demand parameters are gamma random variables, then the deterministic problem becomes non-linear. To solve this type of problem, these non-linear membership and non-membership functions can be used. Apart from the TP for the multi-objective non-linear problem, non-linear membership and non-membership functions in IF environment are very useful.

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