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AN INTUITIONISTIC FUZZY INTERPRETATION OF THE BASIC AXIOM OF THE RESOLUTION

| George K. Gargov | and Krassimir T. Atanassov*
* CLBME - Bulg. Academy of Sci., Acad. G. Bonchev str., Bl. 105, Sofia-1113, Bulgaria,
e-mail: krat@bgcict.bitnet

The expression $((a \vee b))\&((\neg a \vee c)) \supset (b \vee c)$ is called "the basic axiom of the resolution" (see [1]). Obviously, it is a tautology in the first order logic sense (see, e.g., [2]). Here we shall discuss its interpretation in the terms of the Intuitionistic Fuzzy Logic (IFL).

First, following [3-6], we shall introduce some IFL definitions.

To each proposition (in the classical sense) we can assign its truth value: truth – denoted by 1, or falsity – 0. In the case of fuzzy logic this truth value is a real number in the interval $[0, 1]$ and may be called "truth degree" of a particular proposition. Here we add one more value – "falsity degree" – which will be in the interval $[0, 1]$ as well. Thus two real numbers, $\mu(p)$ and $\nu(p)$, are assigned to the proposition p with the following constraint to hold:

$$\mu(p) + \nu(p) \leq 1.$$

Let this assignment be provided by an evaluation function V defined over a set of propositions S in such a way that:

$$V(p) = \langle \mu(p),\ \nu(p) \rangle.$$

When the values $V(p)$ and $V(q)$ of the propositions p and q are known, the evaluation function V can be extended also for the operations "$\neg$", "&", "$\vee$", "$\supset$","$\equiv$", through the definitions:

$$\neg V(p) = V(\neg p) = \langle 1 - \mu(p), \mu(p) \rangle,$$

$$V(p) \wedge V(q) = V(p\&q) = \langle min(\mu(p), \mu(q)), max(\nu(p), \nu(q)) \rangle,$$

$$V(p \vee q) = \langle max(\mu(p), \mu(q)), min(\nu(p), \nu(q)) \rangle.$$

$$V(p) \rightarrow V(q) = V(p \supset q) = \langle max(\nu(p), \mu(q)), min(\mu(p), \nu(q)) \rangle$$

$$V(A \equiv B) = (V(p) \rightarrow V(q)) \wedge (V(q) \rightarrow V(p)).$$

A propositional form A (cf. [2]: each proposition is a propositional form; if A is a propositional form then $\neg A$ is a propositional form; if A and B are propositional forms, then $A\&B$, $A \vee B$,$A \supset B$ are propositional forms) is called a *tautology* if $V(A) = \langle 1, 0 \rangle$, for all valuation functions V, and an *intuitionistic fuzzy tautology* (IFT) [3] iff, if $V(A) = \langle a, b \rangle$, then $a \geq b$.

<u>THEOREM</u>: For every three propositional forms A, B and C,

$$((A \vee B)\&(\neg A \vee C)) \supset (B \vee C) \tag{1}$$

is an IFT.

<u>Proof</u>: Let $V(A) =< a, b >, V(B) =< c, d >, V(C) =< e, f >$, where $a, b, c, d, e, f \in [0, 1], a+ b1, c + d \leq 1, e + f \leq 1$. Then

$$V(((A \vee B))\&((\neg A \vee C)) \supset (B \vee C))$$

$$= (< max(a, c), min(b, d) > \wedge < max(b, e), min(a, f) >) \rightarrow < max(c, e), min(d, f) >$$

$$=< min(max(a, c), max(b, e)), max(min(b, d), min(a, f)) > \rightarrow < max(c, e), min(d, f) >$$

$$=< max(c, e, min(b, d), min(a, f)), min(d, f, max(a, c), max(b, e)) > .$$

Then

$$max(c, e, min(b, d), min(a, f)) - min(d, f, max(a, c), max(b, e))$$
$$\geq max(c, min(a, f)) - min(f, max(a, c)) \geq 0,$$

i.e., (1) is an IFT.

We must note that in the first order logic sense, the expression (1) is equivalent with the expression

$$((A \vee B)\&(\neg A \vee C)) \equiv (B \vee C),$$

but this is not an IFT, because, if

$$V(A) =< 1/3, 1/2 >,$$

$$V(B) =< 1/4, 2/3 >$$

and

$$V(C) =< 3/4, 1/4 >,$$

then

$$V(((A \vee B)\&(\neg A \vee C)) \equiv (B \vee C)) =< 1/3, 2/3 > .$$

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REMARK by K.A.: This is the last text, which G. Gargov and I have discussed. We planned it to be only a small part of a large research, but the fate broke our plans. I decided to publish the text in the form up to which we went together.

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