Notes on Intuitionistic Fuzzy Sets
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A novel complex intuitionistic fuzzy analytic hierarchy process method and its application to electric vehicle selection problem

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Abstract: Multi-criteria decision-making (MCDM) methods often involve uncertainty and subjective judgment. The Analytic Hierarchy Process (AHP) has been widely used to structure and solve such problems, and its extension with intuitionistic fuzzy sets (IF-AHP) allows

for modeling both membership and non-membership degrees. However, ordinary intuitionistic fuzzy sets may not fully capture complex uncertainties that include phase or directional information. This paper proposes a Complex Intuitionistic Fuzzy AHP (CIF-AHP) approach, integrating complex intuitionistic fuzzy sets into the traditional AHP framework. The methodology is applied to capture consumers' purchasing preferences regarding electric vehicles (EVs) under uncertainty, and a comparative analysis is conducted between CIF-AHP and ordinary IF-AHP. The results demonstrate that CIF-AHP provides richer information and enhanced differentiation among alternatives, offering a more nuanced decision-making tool in uncertain environments. The contribution of this study to the literature lies in identifying and weighting the key factors influencing EV adoption through a method that explicitly accounts for vagueness and hesitation in decision-making.
Keywords: Complex intuitionistic fuzzy sets, Analytic hierarchy process, AHP, Intuitionistic fuzzy AHP, Multi-criteria decision-making, MCDM.

2020 Mathematics Subject Classification: 03E72, 90B50, 90C31.

1 Introduction

Fuzzy sets, introduced by Lotfi A. Zadeh in 1965, extend classical set theory to model situations involving uncertainty and imprecision [8]. Unlike classical sets, where an element either fully belongs or does not belong, fuzzy sets allow partial membership, represented by a value between 0 and 1. This framework provides a rigorous mathematical approach for representing the inherently ambiguous and imprecise nature of human reasoning and natural language. Building on Zadeh's concept, Atanassov extended the framework by introducing Intuitionistic Fuzzy Sets (IFS), which incorporate both membership and non-membership degrees for each element [2]. In contrast to ordinary fuzzy sets, IFS also account for hesitation ($\pi = 1 - \mu - \nu$), explicitly representing uncertainty and incomplete information. This enriched structure makes IFS particularly suitable for decision-making, and multi-criteria evaluation, providing a more flexible and information-rich framework than ordinary fuzzy sets.

Further extending these concepts, Complex Fuzzy Sets (CFS) were developed by Ramot and colleagues, allowing membership functions to take values on the unit circle in the complex plane rather than being restricted to the interval [0,1] (see [6]). In CFS, membership is represented in terms of both magnitude and phase, enabling the modeling of additional dimensions. By introducing novel operations, CFS provides a highly flexible framework for applications in multi-criteria decision making, and effectively addressing the limitations of ordinary fuzzy sets, which are restricted to real-valued memberships [4]. To address the increasing complexity of real-world decision-making problems, several extensions of Complex Fuzzy Sets (CFS) have been proposed, each enhancing the representation of uncertainty in different ways. Complex Intuitionistic Fuzzy Sets (CIFS) generalize CFS by adding a non-membership function alongside the membership function, allowing both degrees to take complex values on the unit circle and thereby expanding the range of uncertainty representation; basic operations such as complement, union, and intersection are defined for CIFS [1].

In many real-world decision-making scenarios, decision-makers needs to evaluate alternatives based on multiple, often conflicting criteria. This has led to the widespread adoption of Multi-Criteria Decision-Making (MCDM) methods, which provide structured approaches for systematically evaluating alternatives across diverse dimensions. The most commonly adopted MCDM techniques include the Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), ELimination Et Choice Translating REality (ELECTRE), VIKOR. Among these methods, AHP, originally developed by Saaty [7], is one of the most widely used methods due to its intuitive structure and hierarchical decomposition of complex problems. AHP enables decision-makers to perform pairwise comparisons between criteria and alternatives, converting qualitative judgments into quantitative weights through eigenvalue analysis. In classical AHP, evaluations of the decision makers are represented as crisp numbers which may not always reflect the ambiguity inherent in human preferences. To overcome these limitations AHP technique is later extended by fuzzy set theory.

The contribution of this study is twofold. First, it develops the theoretical framework for Complex Intuitionistic Fuzzy AHP (Complex IF-AHP), including the construction of complex intuitionistic fuzzy pairwise comparison matrices, aggregation operators. Second, the study provides a case study on electric vehicle selection problem to illustrate the practical application of the method and demonstrates its advantages in capturing complex, multi-dimensional uncertainties in the decision-making process.

The remainder of this paper is organized as follows to guide the reader through the development and evaluation of the proposed methodology. Section 2 reviews Complex Fuzzy Sets in the literature, highlighting key concepts and developments. Section 3 introduces the preliminaries of Complex Intuitionistic Fuzzy Sets and the necessary mathematical foundations. Section 4 details the proposed Complex Intuitionistic Fuzzy Analytic Hierarchy Process
(CIF-AHP), explaining its formulation and implementation steps. In Section 5, an application of CIF-AHP on EV selection problem is provided. Finally, the conclusion is given in Section 6.

2 Complex fuzzy sets in literature

The concept of Complex Fuzzy Sets (CFSs) was introduced as an extension of fuzzy theory in which the membership function assumes values in the unit disk of the complex plane, rather than being restricted to the real interval [5, 6].

A complex fuzzy set is represented by Equation (1):

$$\tilde{A} = \left\{ x, \omega(x) e^{2\pi i \beta(x)} \middle| x \in X \right\}, \tag{1}$$

where $\omega(x)$ represents the amplitude membership degree and $\beta(x)$ represents the phase membership degree. It satisfies the conditions $\omega(x) \in [0, 1]$, $\beta(x) \in [0, 1]$. The hesitancy degree is given by Equation (2):

$$h(x) = \left(1 - \omega(x)\right) e^{2\pi i (1 - \beta(x))}. \tag{2}$$

To examine the growth and structure of research on Complex Fuzzy Sets (CFSs), a bibliometric analysis was performed using Scopus da retrieved with the query TITLE-ABS-KEY (‘complex fuzzy set’). The dataset yielded 356 documents, which were analyzed across years, authors, countries, sources, and subject areas. The annual distribution of publications in Figure 1 shows that interest in CFSs has grown steadily, with rapid acceleration in the last five years.

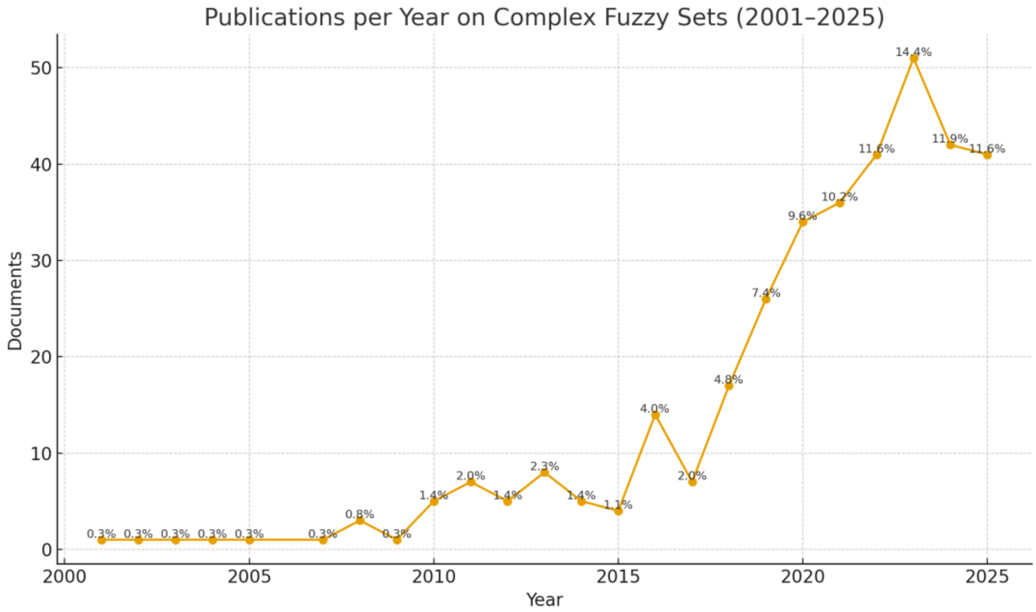

Figure 1. Annual publications on complex fuzzy sets
(2001–2025) with percentage shares.

The peak occurs in 2023–2025, reflecting the increasing recognition of CFSs in mathematical modeling and decision-making contexts. Notably, these recent years account for over 36% of all publications in the field. The field is dominated by several authors, Mahmood, T., Rehman U., Ali, Z., Dai, S. and Hu, B. Together, the top five authors produce nearly 45% of all publications (Figure 2).

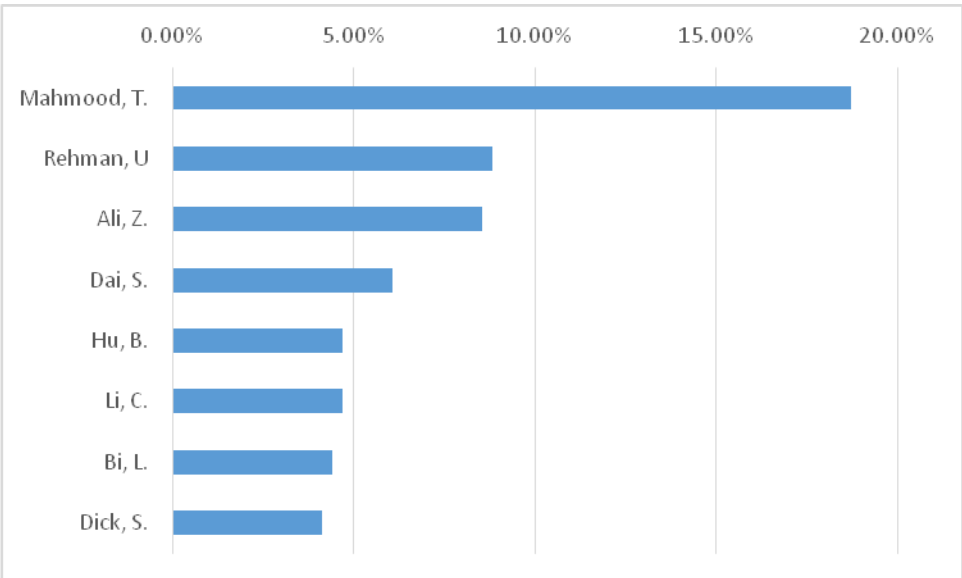

Figure 2. Top authors on complex fuzzy sets, with their percentage contributions.

The geographical distribution in Figure 3 highlights Asia as the central hub of CFS research. Pakistan leads with 138 publications (38.7%), followed by China (21.1%), India (18.5%), and Saudi Arabia (15.7%). The dominance of these countries collectively accounts for over 94% of the global research output in this area.

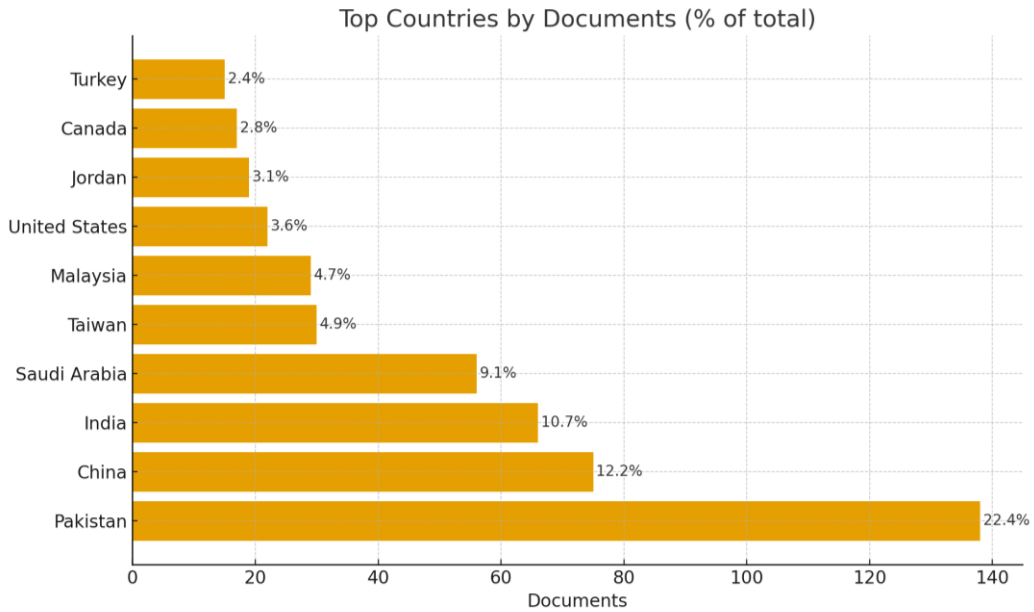

Figure 3. Top countries contributing to complex fuzzy sets, with percentage shares.

Research on CFSs is published mainly mathematics and computer science journals. The sources that publish complex fuzzy sets in Figure 4 include *IEEE Access* (21 documents, 5.9%), *Journal of Intelligent and Fuzzy Systems* (19, 5.3%), *Mathematics* (14, 3.9%), and *Symmetry* (12, 3.4%). This indicates a strong interdisciplinary anchoring, with applied mathematics, intelligent systems, and engineering.

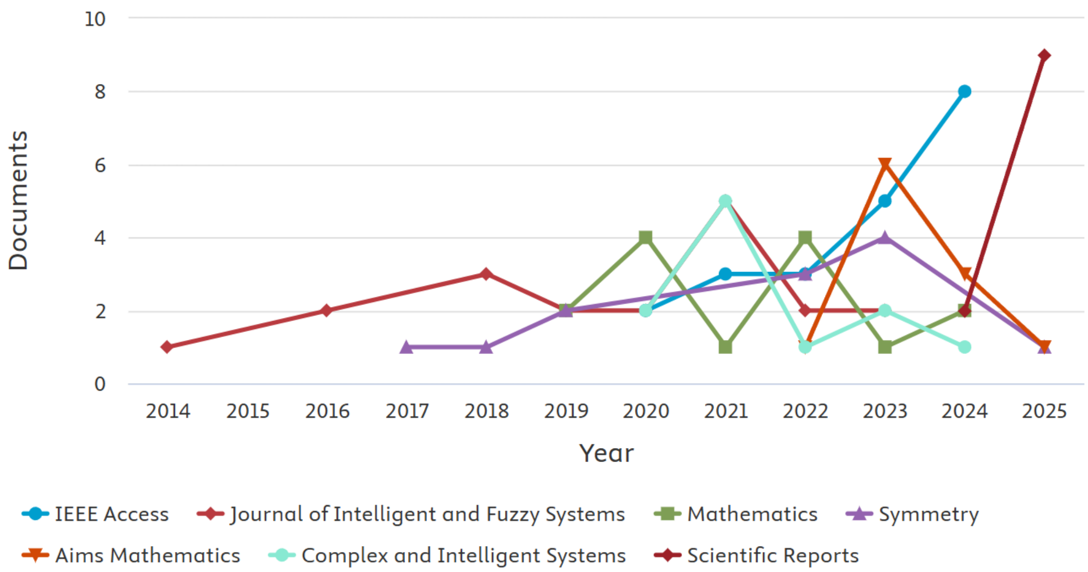

Figure 4. The sources frequently publish complex fuzzy sets

All these studies show a rapid growth since 2020, with recent years producing more than one-third of all publications. This multidisciplinary area is mainly published in mathematics and engineering, ensuring both theoretical and applied coverage. The co-occurrence map was generated using VOSviewer [3] to see the main connection between these multidisciplinary areas. A minimum occurrence threshold of five was applied, meaning that only keywords appearing at least five times in the dataset were included in the analysis. This step filters out less frequent and isolated terms, ensuring that the resulting map highlights the most relevant and recurring research themes. In the visualization, the size of each node corresponds to the frequency of a keyword, while the thickness of links between nodes represents the strength of their co-occurrence relationships.

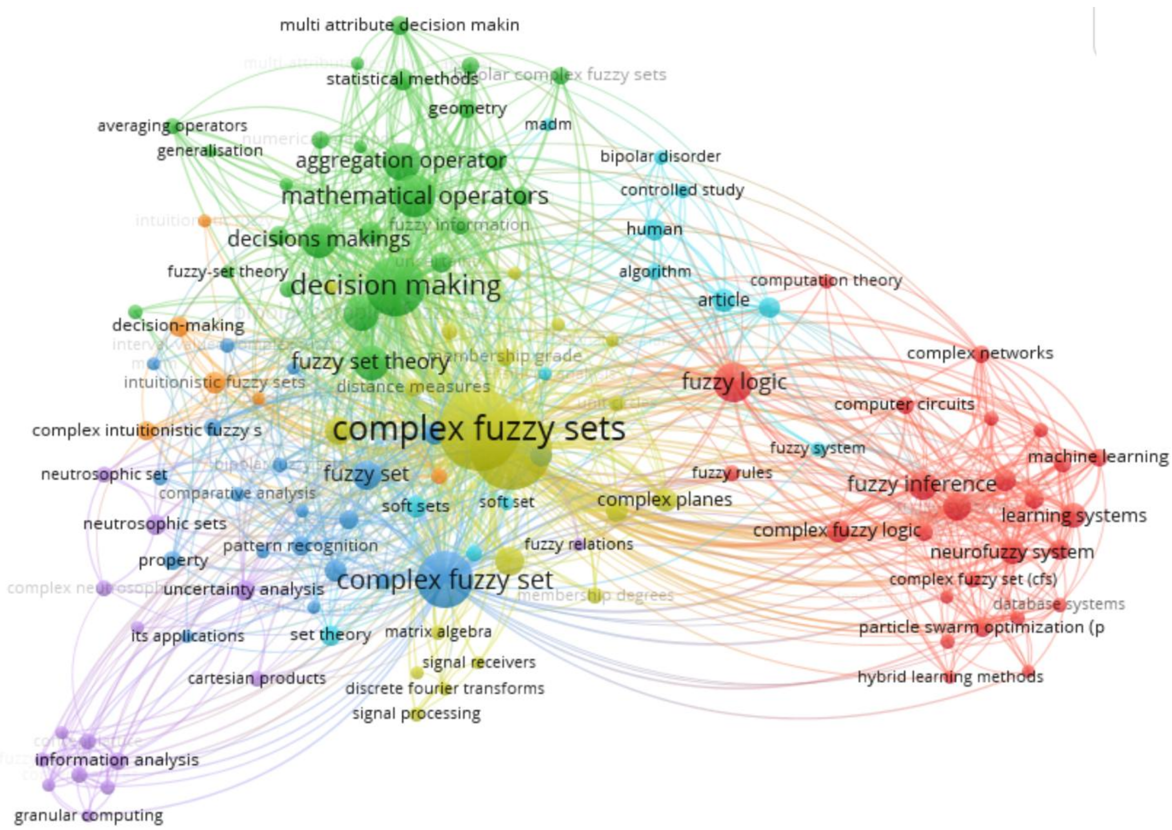

Figure 5. Keyword co-occurrences of complex fuzzy sets

The analysis identifies several distinct clusters. The yellow cluster is centered on complex fuzzy sets and includes terms such as ‘fuzzy set theory’, ‘aggregation operators’, and ‘mathematical operators’. The green cluster highlights the decision-making applications. Keywords like ‘decision making’, ‘multi-attribute decision making’, ‘statistical methods’, and ‘distance measures’ indicate that complex fuzzy sets are widely used in multi-criteria decision-making. The red cluster shows the connection between fuzzy logic and intelligent systems, containing terms such as ‘fuzzy inference’, ‘neuro-fuzzy systems’, ‘machine learning’, ‘particle swarm optimization’, and ‘hybrid learning methods’. The purple cluster represents extensions of fuzzy sets, including neutrosophic sets, complex intuitionistic fuzzy sets, soft sets, and granular computing. The blue cluster connects to engineering and signal processing.

3 Preliminaries on complex intuitionistic fuzzy sets

Let a CIFS be defined as in Equation (3).

$$(\mu,v)_I = \left(\omega e^{2\pi i\beta}, \tau e^{2\pi i\delta}\right)_I, \tag{3}$$

where $0 \le \mu \le 1$, $0 \le v \le 1$, $0 \le \omega \le 1$, $0 \le \tau \le 1$, $0 \le \beta \le 1$, $0 \le \delta \le 1$; $0 \le \omega_I + \tau_I \le 1$; $0 \le \beta_I + \delta_I \le 1$. The hesitancy degree is given as $h_I = \left(1-(\omega+\tau)\right)e^{2\pi i(1-(\beta+\delta))}$ [1].

4 Complex intuitionistic fuzzy AHP

The weights vector $\{w_1, w_2, \ldots, w_i, \ldots, w_n\}$, i = 1, 2, …, n is obtained by the following CIF-AHP method. Consider the following hierarchy in Figure 6. The main criteria are first pairwise-compared with respect to the goal and then the alternatives are pairwise-compared with respect to each main-criterion by Ξ experts, $\xi = 1, 2, \ldots, \Xi$. The associated weight vector of the experts is represented by $\rho_\xi = (\rho_1, \rho_2, \rho_3, \ldots, \rho_\Xi)$, $\xi = 1, 2, \ldots, \Xi$.

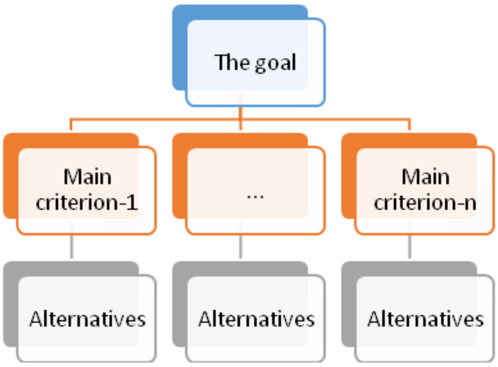

Figure 6. Hierarchy

The linguistic scale in Table 1 is used for the pairwise comparisons.

Table 1. Linguistic scale

Linguistic terms (l)	**Abbr.**	**CIF values ($\tilde{x}_{ij}$).**
Absolutely less importanthan	ALI	((0.1,0.15), (0.9,0.85))
Much less important than	MLI	((0.2,0.25), (0.8,0.75))
Less important than	LI	((0.3,0.35), (0.7,0.65))
Slightly less important than	SLI	((0.4,0.45), (0.6,0.55))
Equally important	EI	((0.5,0.5), (0.5,0.5))
Slightly more important than	SMI	((0.6,0.55), (0.4,0.45))
More important than	MI	((0.7,0.65), (0.3,0.35))
Much more important than	MMI	((0.8,0.75), (0.2,0.25))
Absolutely more important than	AMI	((0.9,0.85), (0.1,0.15))

Step 1. Rank and pairwise-compare the criteria with respect to the goal. Each expert ranks the criteria from the most important to the least important to satisfy the consistency condition with a higher possibility. In this case, the most important criterion must be called C_1 while the least important criterion must be called C_n where n is the number of criteria. Then, each expert fills in the matrix as in Table 2 by considering the stated ranking. This provides values larger than 1.0 above the diagonal whereas values smaller than 1.0 below the diagonal.

Table 2. Linguistic pairwise-comparison matrix with respect to the goal

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline C_1 & l_{11} & l_{12} & \cdots & l_{1n} \\ C_2 & l_{21} & l_{22} & \cdots & l_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_n & l_{n1} & l_{n2} & \cdots & l_{nn} \end{array}$$

Linguistic pairwise-comparison matrices are converted to CIF pairwise-comparison matrices by substituting the corresponding CIF values $(\tilde{x}_{ij})$ in the linguistic scale as in Table 3.

Table 3. Pairwise-comparisons of the main-criteria with respect to the goal

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline C_1 & \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ C_2 & \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_n & \tilde{x}_{n1} & \tilde{x}_{n2} & \cdots & \tilde{x}_{nn} \end{array}$$

Step 2. Measure the consistency ratio of each matrix by defuzzifying the CIF numbers by Equation (4).

$$\boldsymbol{S}_S = \begin{cases} 1, & \text{if } i = j;\ i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, n \\ 10 \times \sqrt{(\mu - \nu)^2 + (\rho - \tau)^2}, & \text{if } i \neq j;\ i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, n \end{cases} \tag{4}$$

The consistency ratio is equal to consistency index divided by random index (CI/RI) and should be at most equal to 0.10 to be a consistent matrix. Random index (RI) is determined from a readily given table. The consistency index (CI) is computed by Equation (5):

$$CI = \frac{\lambda_{\max} - n}{n - 1}. \tag{5}$$

Step 3. Aggregate the pairwise-comparison matrices filled by the experts by using the complex intuitionistic fuzzy Aczel–Alsina weighted averaging (CIFAAWA) operator given by Equation (6). Ω can be taken as 2.

$$\operatorname{CIFAAWA}\left(\zeta_1,\zeta_2,\zeta_3,\ldots,\zeta_\Xi\right)$$

$$=\left(\sqrt{1-e^{-\left(\sum_{\xi=1}^{\Xi}\left(\rho_\xi\left(-\log(1-\omega_\xi^2)\right)^{\Omega}\right)\right)^{1/\Omega}}}\; e^{2\pi i\sqrt{1-e^{-\left(\sum_{\xi=1}^{\Xi}\left(\rho_\xi\left(-\log(1-\beta_\xi^2)\right)^{\Omega}\right)\right)^{1/\Omega}}}},\right.$$
$$\left.\sqrt{1-e^{-\left(\sum_{\xi=1}^{\Xi}\left(\rho_\xi\left(-\log(1-\tau_\xi^2)\right)^{\Omega}\right)\right)^{1/\Omega}}}\; e^{2\pi i\sqrt{1-e^{-\left(\sum_{\xi=1}^{\Xi}\left(\rho_\xi\left(-\log(1-\delta_\xi^2)\right)^{\Omega}\right)\right)^{1/\Omega}}}}\right) \tag{6}$$

Step 4. Defuzzify the aggregated values in the CIF pairwise comparison matrix by using Equation (7). In the defuzzified matrix, if there is a number less than 1 above the diagonal or a number greater than 1 below the diagonal, the inverse of these numbers is taken.

$$(S_S)_{ij}=\begin{cases}1, & \text{if } i=j, i=1,2,\ldots,n,\ j=1,2,\ldots,n\\ RHS, & \text{if } \omega>\tau,\ i\neq j\end{cases} \tag{7}$$

where

$$RHS=10\times\left(\left(\sqrt{1-e^{-\left(\sum_{\xi=1}^{\Xi}\left(\rho_\xi\left(-\log(1-\omega_{\xi ij}^2)\right)^{\Omega}\right)\right)^{1/\Omega}}}-\sqrt{1-e^{-\left(\sum_{\xi=1}^{\Xi}\left(\rho_\xi\left(-\log(1-\tau_{\xi ij}^2)\right)^{\Omega}\right)\right)^{1/\Omega}}}\right)^2+\right.$$
$$\left.\left(\sqrt{1-e^{-\left(\sum_{\xi=1}^{\Xi}\left(\rho_\xi\left(-\log(1-\omega_{\xi ij}^2)\right)^{\Omega}\right)\right)^{1/\Omega}}}-\sqrt{1-e^{-\left(\sum_{\xi=1}^{\Xi}\left(\rho_\xi\left(-\log(1-\tau_{\xi ij}^2)\right)^{\Omega}\right)\right)^{1/\Omega}}}\right)^2\right)^{\frac{1}{2}}$$

Step 5. Compute the weights of criteria and the priorities of alternatives from the defuzzified matrix obtained in Step 4. The crisp pairwise comparison matrix is given by Table 4.

Table 4. Defuzzified pairwise comparison matrix
with respect to the goal or with respect to the criteria

	C_1 or A_1	C_2 or A_2	$\cdots$	C_n or A_n
C_1 or A_1	1	$(S_S)_{12}$	$\cdots$	$(S_S)_{1n}$
C_2 or A_2	$(S_S)_{21}$	1	$\cdots$	$(S_S)_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n or A_n	$(S_S)_{n1}$	$(S_S)_{n2}$	$\cdots$	1

Step 6. Follow the same procedure for computing weights or priorities as in the classical AHP.

5 Application

The adoption of electric vehicles (EVs) plays a crucial role in promoting sustainable transportation and reducing environmental impacts. However, consumers' purchasing decisions regarding EVs are influenced by a wide range of criteria, including purchasing cost, battery cost, operating cost, and maintenance cost, driving range, charging duration, brand reliability, and battery life. The hierarchy of the criteria is given in Figure 7. The coexistence of subjective judgments, uncertainties, and sometimes conflicting evaluations among these criteria makes the decision-making process highly complex. In this context, the Complex Intuitionistic Fuzzy Analytic Hierarchy Process (CIF-AHP) provides a robust methodological framework to better capture consumer preferences under uncertainty.

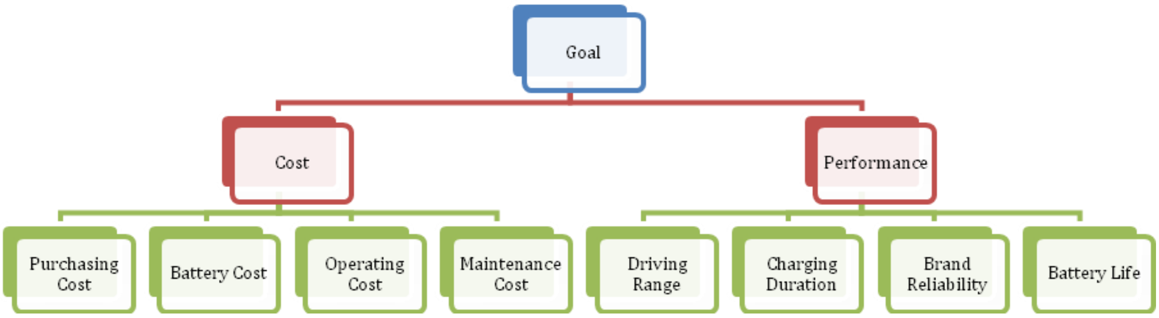

Figure 7. Hierarchy of the EV evaluation criteria

Step 1. The methodology commenced with the prioritization of criteria relative to the defined research objective. During the evaluation phase, the opinions of three domain experts ($k = 3$) were gathered through face-to-face interviews. To enhance the probability of achieving a high consistency ratio in the subsequent analysis, three domain experts established a consensus-based ranking of all criteria. This expert consensus deemed the Cost (**C**) criteria to be of greater importance than the Performance (**P**) criteria. The specific rank order from the best to worst for the Cost criteria was determined as follows: Purchasing cost (**C1**), Battery cost (**C2**), Operating cost (**C3**), and Maintenance cost (**C4**). Similarly, the rank order for the Performance criteria was defined as Driving range (P1), Charging duration (**P2**), Brand reliability (**P3**), and Battery life (**P4**).

Following the established hierarchy, each expert independently performed pairwise comparisons between the criteria at their respective levels. The individual expert judgments derived from these comparisons are systematically presented in Tables 5–7.

Table 5. Expert preferences on the main criteria

	Expert 1		**Expert 2**		**Expert 3**	
	C	**P**	**C**	**P**	**C**	**P**
C	EI	SMI	EI	MI	EI	SMI
P		EI		EI		EI

Table 6. Expert preferences on the cost dimension of electric vehicles

	Expert 1				Expert 2				Expert 3			
	C1	**C2**	**C3**	**C4**	**C1**	**C2**	**C3**	**C4**	**C1**	**C2**	**C3**	**C4**
C1	EI	MI	MMI	AMI	EI	MMI	MMI	AMI	EI	MI	MMI	AMI
C2		EI	MI	MMI		EI	MMI	AMI		EI	SMI	MI
C3			EI	SMI			EI	SMI			EI	MI
C4				EI				EI				EI

Table 7. Expert preferences on the performance dimension of electric vehicles

	Expert 1				Expert 2				Expert 3			
	P1	**P2**	**P3**	**P4**	**P1**	**P2**	**P3**	**P4**	**P1**	**P2**	**P3**	**P4**
P1	EI	EI	SMI	MI	EI	SMI	MI	MMI	EI	EI	MI	MMI
P2		EI	MI	MMI		EI	MMI	AMI		EI	MI	MI
P3			EI	MI			EI	MI			EI	MI
P4				EI				EI				EI

In order to perform mathematical operations on the linguistic evaluations given in Tables 8–10, the linguistic evaluations are converted into numerical values using the CIF values given in Table 1.

Table 8. CIF values for expert preferences on main criteria

		C				P			
		ω	β	τ	δ	ω	β	τ	δ
Expert 1	**C**	0.50	0.50	0.50	0.50	0.60	0.55	0.40	0.45
	P	0.40	0.45	0.60	0.55	0.50	0.50	0.50	0.50
Expert 2	**C**	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35
	P	0.30	0.35	0.70	0.65	0.50	0.50	0.50	0.50
Expert 3	**C**	0.50	0.50	0.50	0.50	0.60	0.55	0.40	0.45
	P	0.40	0.45	0.60	0.55	0.50	0.50	0.50	0.50

Table 9. CIF values for expert preferences on cost criteria

		C1				C2				C3				C4			
		ω	β	τ	δ	ω	β	τ	δ	ω	β	τ	δ	ω	β	τ	δ
Expert1	**C1**	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35	0.80	0.75	0.20	0.25	0.90	0.85	0.10	0.15
	C2	0.30	0.35	0.70	0.65	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35	0.80	0.75	0.20	0.25
	C3	0.20	0.25	0.80	0.75	0.30	0.35	0.70	0.65	0.50	0.50	0.50	0.50	0.60	0.55	0.40	0.45
	C4	0.10	0.15	0.90	0.85	0.20	0.25	0.80	0.75	0.40	0.45	0.60	0.55	0.50	0.50	0.50	0.50
Expert2	**C1**	0.50	0.50	0.50	0.50	0.80	0.75	0.20	0.25	0.80	0.75	0.20	0.25	0.90	0.85	0.10	0.15
	C2	0.20	0.25	0.80	0.75	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.60	0.55	0.4	0.45
	C3	0.20	0.25	0.80	0.75	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.60	0.55	0.4	0.45
	C4	0.10	0.15	0.90	0.85	0.40	0.45	0.60	0.55	0.40	0.45	0.6	0.55	0.50	0.50	0.50	0.50
Expert3	**C1**	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35	0.80	0.75	0.20	0.25	0.90	0.85	0.10	0.15
	C2	0.30	0.35	0.70	0.65	0.50	0.50	0.50	0.50	0.60	0.55	0.40	0.45	0.70	0.65	0.30	0.35
	C3	0.20	0.25	0.80	0.75	0.40	0.45	0.60	0.55	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35
	C4	0.10	0.15	0.90	0.85	0.30	0.35	0.70	0.65	0.30	0.35	0.70	0.65	0.50	0.50	0.50	0.50

Table 10. CIF values for expert preferences on performance criteria

		P1				P2				P3				P4			
		ω	β	τ	δ	ω	β	τ	δ	ω	β	τ	δ	ω	β	τ	δ
Expert1	**P1**	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.60	0.55	0.40	0.45	0.70	0.65	0.30	0.35
	P2	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35	0.80	0.75	0.20	0.25
	P3	0.40	0.45	0.60	0.55	0.30	0.35	0.70	0.65	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35
	P4	0.30	0.35	0.70	0.65	0.20	0.25	0.80	0.75	0.30	0.35	0.70	0.65	0.50	0.50	0.50	0.50
Expert2	**P1**	0.50	0.50	0.50	0.50	0.60	0.55	0.40	0.45	0.70	0.65	0.30	0.35	0.80	0.75	0.20	0.25
	P2	0.40	0.45	0.60	0.55	0.50	0.50	0.50	0.50	0.80	0.75	0.20	0.25	0.90	0.85	0.10	0.15
	P3	0.30	0.35	0.70	0.65	0.20	0.25	0.80	0.75	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35
	P4	0.20	0.25	0.80	0.75	0.10	0.15	0.90	0.85	0.30	0.35	0.70	0.65	0.50	0.50	0.50	0.50
Expert3	**P1**	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35	0.80	0.75	0.20	0.25
	P2	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35	0.70	0.65	0.30	0.35
	P3	0.30	0.35	0.70	0.65	0.30	0.35	0.70	0.65	0.50	0.50	0.50	0.50	0.70	0.65	0.30	0.35
	P4	0.20	0.25	0.80	0.75	0.30	0.35	0.70	0.65	0.30	0.35	0.70	0.65	0.50	0.50	0.50	0.50

Step 2. The consistency analysis, as defined in Step 2, was conducted to validate the internal coherence of the pairwise comparison matrices elicited from the experts. This process involved two stages: first, the transformation of expert judgments into precise numerical values via established scoring functions; and second, the subsequent calculation of the consistency ratio for each resulting matrix. The crisp values for the expert preferences and the consistency values for each expert preference are shown in Table 11.

Table 11. Consistency ratio values for expert assessments
(GM: Geometric mean, W: Crisp weights, CI: Consistency index).

		C1	**C2**	**C3**	**C4**	**GM**	**W**	**CI**		**P1**	**P2**	**P3**	**P4**	**GM**	**W**	**CI**
Expert1	**C1**	1.00	2.50	6.10	11.30	3.623	0.601	0.103	**P1**	1.00	1.00	2.00	2.50	1.495	0.317	0.032
	C2	0.40	1.00	2.50	6.10	1.572	0.260		**P2**	1.00	1.00	2.50	6.10	1.976	0.419	
	C3	0.16	0.40	1.00	0.50	0.426	0.071		**P3**	0.50	0.40	1.00	2.50	0.840	0.178	
	C4	0.09	0.16	2.00	1.00	0.413	0.068		**P4**	0.40	0.16	0.40	1.00	0.402	0.085	
Expert2	**C1**	1.00	6.10	6.10	11.30	4.528	0.705	0.001	**P1**	1.00	2.00	2.50	6.10	2.350	0.413	0.103
	C2	0.16	1.00	1.00	2.00	0.757	0.118		**P2**	0.50	1.00	6.10	11.30	2.423	0.426	
	C3	0.16	1.00	1.00	2.00	0.757	0.118		**P3**	0.40	0.16	1.00	2.50	0.636	0.112	
	C4	0.09	0.50	0.50	1.00	0.386	0.060		**P4**	0.16	0.09	0.40	1.00	0.276	0.049	
Expert3	**C1**	1.00	2.50	6.10	11.30	3.623	0.629	0.082	**P1**	1.00	1.00	2.50	6.10	1.976	0.416	0.038
	C2	0.40	1.00	0.50	2.50	0.841	0.146		**P2**	1.00	1.00	2.50	2.50	1.581	0.333	
	C3	0.16	2.00	1.00	2.50	0.952	0.165		**P3**	0.40	0.40	1.00	2.50	0.795	0.167	
	C4	0.09	0.40	0.40	1.00	0.345	0.060		**P4**	0.16	0.40	0.40	1.00	0.402	0.085	

Step 3. The pairwise comparison matrices filled out by the experts are combined into a co-decision matrix using the CIFAAWA operator provided in Step 3. When obtaining the co-decision matrix, the expert weights were determined as 0.25, 0.45, and 0.30 based on the experts' experience, respectively. Table 12 and Table 13 present the CIFAAWA values.

Table 12. CIFAAWA values for main criteria

	C				P			
	ω	β	τ	δ	ω	β	τ	δ
C	0.09	0.09	0.09	0.09	0.17	0.14	0.04	0.06
P	0.04	0.06	0.17	0.14	0.09	0.09	0.09	0.09

Table 13. CIFAAWA values for sub-criteria

	C1				C2				C3				C4			
	ω	β	τ	δ	ω	β	τ	δ	ω	B	τ	δ	ω	β	τ	δ
C1	0.088	0.088	0.088	0.088	0.256	0.209	0.023	0.033	0.306	0.250	0.013	0.020	0.479	0.379	0.003	0.007
C2	0.023	0.033	0.256	0.209	0.088	0.088	0.088	0.088	0.140	0.119	0.068	0.073	0.212	0.172	0.040	0.052
C3	0.013	0.020	0.306	0.250	0.068	0.073	0.140	0.119	0.088	0.088	0.088	0.088	0.160	0.130	0.047	0.062
C4	0.003	0.007	0.479	0.379	0.040	0.052	0.212	0.172	0.047	0.062	0.160	0.130	0.088	0.088	0.088	0.088
	P1				**P2**				**P3**				**P4**			
	ω	β	τ	δ	ω	β	τ	δ	ω	β	τ	δ	ω	β	τ	δ
P1	0.088	0.088	0.088	0.088	0.113	0.099	0.075	0.080	0.190	0.155	0.037	0.049	0.285	0.232	0.018	0.026
P2	0.075	0.080	0.113	0.099	0.088	0.088	0.088	0.088	0.256	0.209	0.023	0.033	0.378	0.300	0.017	0.025
P3	0.037	0.049	0.190	0.155	0.023	0.033	0.256	0.209	0.088	0.088	0.088	0.088	0.205	0.167	0.029	0.040
P4	0.018	0.026	0.285	0.232	0.017	0.025	0.378	0.300	0.029	0.040	0.205	0.167	0.088	0.088	0.088	0.088

Step 4. The CIF values in the co-decision matrix given above are defuzzified using the equation given in Step 4 in Section 4. In the defuzzified matrix, calculations are performed by taking the inverse of numbers less than 1 above the diagonal and greater than 1 below the diagonal if any. The defuzzified values are given in Table 14, columns **C** and **P**, and in Table 15, columns **C1–C4** and **P1–P4**.

Table 14. The defuzzified values for main criteria

	C	P	GM	W
C	1.00	1.50	1.23	0.60
P	0.67	1.00	0.82	0.40

Table 15. The defuzzified values for sub-criteria

	C1	C2	C3	C4	GM	W		P1	P2	P3	P4	GM	W
C1	1.00	2.92	3.73	6.04	2.85	0.56	**P1**	1.00	2.37	1.86	3.37	1.96	0.42
C2	0.34	1.00	1.16	2.10	0.96	0.19	**P2**	0.42	1.00	2.92	4.54	1.54	0.33
C3	0.27	0.86	1.00	1.32	0.74	0.15	**P3**	0.54	0.34	1.00	2.17	0.79	0.17
C4	0.17	0.48	0.76	1.00	0.49	0.10	**P4**	0.30	0.22	0.46	1.00	0.42	0.09

Step 5. The relative weights of the criteria are obtained by AHP method. The weights for main criteria and sub-criteria are given in Table 14 and Table 15, columns **GM** and **W**.

Step 6. The final weights of the criteria are given in Table 16. According to the table, he most decisive criterion in the selection of electric vehicles is the purchasing cost (**C1**) with 34%. This is followed by driving range (P1) with 17% and charging duration (**P2**) with 13%. Battery cost (**C2**, 11%) and operating cost (**C3**, 9%) hold moderate importance, while brand reliability (P3, 7%), maintenance cost (**C4**, 6%), and battery life (**P4**, 4%) are relatively less significant. These results indicate that decision-makers primarily focus on cost, while among performance criteria, driving range and charging duration are given greater importance.

Table 16. Final weights

Criteria	**C1**	**C2**	**C3**	**C4**	**P1**	**P2**	**P3**	**P4**
Weights	0.34	0.11	0.09	0.06	0.17	0.13	0.07	0.04

6 Conclusion

This study introduces a novel Complex Intuitionistic Fuzzy Analytic Hierarchy Process (CIF-AHP) method for solving multi-criteria decision-making problems. By integrating complex intuitionistic fuzzy sets into the classical AHP framework, the proposed method effectively captures not only membership and non-membership degrees and provide a more effective representation of expert judgments.

To validate the applicability of the CIF-AHP approach, a real-world case study was conducted in the context of electric vehicle (EV) selection. The results shows that purchasing cost, driving range, and charging duration were the most influential factors in EV selection. These outcomes are consistent with consumer preferences observed in sustainability-focused markets. When compared to traditional Intuitionistic Fuzzy AHP, the CIF-AHP method demonstrated enhanced capability in handling hesitation, vague evaluations, and the inherent complexity of subjective decision-making.

In future studies, the proposed CIF-AHP can be integrated with other MCDM methods such as VIKOR or ELECTRE. The method can also be used in other decision making problems where uncertainty, ambiguity, and expert subjectivity play a critical role,

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