4th IFSCOM, 3–7 May 2017, Mersin, Turkey
Notes on Intuitionistic Fuzzy Sets
Print ISSN 1310–4926, Online ISSN 2367–8283
Vol. 23, 2017, No. 1, 100–103

Errata, or a new form of the uniformly expanding intuitionistic fuzzy operator

Krassimir T. Atanassov

Dept. of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering
Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., Sofia - 1113, Bulgaria
and
Intelligent Systems Laboratory
Prof. Asen Zlatarov University
1 Prof. Yakimov Blvd., Bourgas - 8010, Bulgaria
e-mail: `krat@bas.bg`

Abstract: A correction in the definition of the intuitionistic fuzzy topological operator U, introduced in [3], is given. Some of the basic properties of operator U in its new form are studied.
Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy topological operator.
AMS Classification: 03E72.

1 Introduction

In [3], I introduced the uniformly expanding intuitionistic fuzzy operator U, but writing the book [4], I saw that in its definition there is a mistake and by this reason, I had not included in the book the text about this operator. In a recent research of theirs, my colleagues O. Roeva and P. Vassilev, using operator U, also observed the mistake in the definition and shared with me their discovery. This motivated to write the present note. In it, a new form of operator U is given.

Let E be a fixed universe and let $A \subset E$ be a fixed set. The object

$$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

is called an Intuitionistic Fuzzy Set (IFS, see, e.g., [1, 2]), where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the *degree of membership* and the *degree of non-membership* of the element $x \in E$ to the set A, respectively, and for every $x \in E$

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Below, we use only the IFS A^* and by this reason, for the sake of brevity, we omit the asterisk, writing A instead of A^*. Let

$$E^* = \{\langle x, 1, 0\rangle | x \in E\},$$

$$O^* = \{\langle x, 0, 1\rangle | x \in E\}.$$

The topological operators C and I are defined for every IFS A, by

$$C(A) = \{\langle x, \sup_{y\in E} \mu_A(y), \inf_{y\in E} \nu_A(y)\rangle | x \in E\},$$

$$I(A) = \{\langle x, \inf_{y\in E} \mu_A(y), \sup_{y\in E} \nu_A(y)\rangle | x \in E\}.$$

The modal operators $\Box$ and $\Diamond$ are defined for every IFS A, by

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\},$$

$$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\}.$$

2 New form of the uniformly expanding intuitionistic fuzzy operator U

In [3], the operator U over the IFS A, such that

$$\sup_y \mu_A(y) > \inf_y \mu_A(y),$$

$$\sup_y \nu_A(y) > \inf_y \nu_A(y),$$

is defined by:

$$U(A) = \left\{\left\langle x, \frac{\mu_A(x) - \inf_y \mu_A(y)}{\sup_y \mu_A(y) - \inf_y \mu_A(y)}, \frac{\nu_A(x) - \inf_y \nu_A(y)}{\sup_y \nu_A(y) - \inf_y \nu_A(y)}\right\rangle | x \in E\right\}.$$

In the proof of the correctness of the so-defined operator, there is a mistake: there are elements of universe E that operator U transforms to points that are already not intuitionistic fuzzy pairs.

By this reason, here we introduce the following modification of the above operator, that can be defined over an IFS A such that

$$\Delta(A) \equiv \inf_{y\in E} \mu_A(y) + \inf_{y\in E} \nu_A(y) + \inf_{y\in E} \pi_A(y) < 1. \quad (1)$$

Its form is:

$$U(A) = \left\{ \left\langle x, \frac{\mu_A(x) - \inf\limits_y \mu_A(y)}{1 - \Delta(A)}, \frac{\nu_A(x) - \inf\limits_y \nu_A(y)}{1 - \Delta(A)} \right\rangle | x \in E \right\}.$$

Immediately, we can see that if we denote

$$U(A, x) \equiv U(A, \langle \mu_A(x), \nu_A(x) \rangle)$$

$$= \left\langle \frac{\mu_A(x) - \inf\limits_y \mu_A(y)}{1 - \Delta(A)}, \frac{\nu_A(x) - \inf\limits_y \nu_A(y)}{1 - \Delta(A)} \right\rangle,$$

where for $x \in E$, $\langle x, \mu_A(x), \nu_A(x) \rangle \in A$, then we obtain the degrees of membership and non-membership of element $x \in E$ after applying operator U over IFS A.

Now, we must prove that the definition is correct if inequality (1) is valid, i.e.,

$$X \equiv \frac{\mu_A(x) - \inf\limits_y \mu_A(y)}{1 - \Delta(A)} + \frac{\nu_A(x) - \inf\limits_y \nu_A(y)}{1 - \Delta(A)} \leq 1. \tag{2}$$

Really,

$$X = \frac{\mu_A(x) - \inf\limits_y \mu_A(y) + \nu_A(x) - \inf\limits_y \nu_A(y)}{1 - \Delta(A)} \leq 1,$$

because

$$1 - (\inf_{y \in E} \mu_A(y) + \inf_{y \in E} \nu_A(y) + \inf_{y \in E} \pi_A(y)) - (\mu_A(x) - \inf_y \mu_A(y) + \nu_A(x) - \inf_y \nu_A(y))$$

$$= 1 - \inf_{y \in E} \pi_A(y) - \mu_A(x) - \nu_A(x) = \pi_A(x) - \inf_{y \in E} \pi_A(y) \geq 0.$$

Therefore, the definition of the new operator U is correct.

The following assertions are proven analogously.

Theorem 1. For every IFS A satisfying (1):

(a) $U(C(A)) = C(U(A))$,

(b) $U(I(A)) = I(U(A))$,

(c) $\neg U(\neg A) = U(A)$.

Theorem 2. For every IFS A satisfying (1):

(a) $U(\Box A) \supseteq \Box U(A)$,

(b) $U(\Diamond A) \subseteq \Diamond U(A)$.

Theorem 3. For every IFS A satisfying (1):

$$U(U(A)) = U(A).$$

3 Conclusion

The so introduced operator can be a basis for defining a new quantifier in intuitionistic fuzzy predicate logic, that will be a theme of the next author's research.

The operator U can be used for the aims of InterCriteria analysis (see, e.g. [5]) for modifying the regions in the intuitionistic fuzzy interpretational triangle, when the results of this analysis are collected in the central parts of the triangle. So, we can increase the region of the these results.

Acknowledgements

The author is grateful for the support provided by the project DFNI-I-02-05 "InterCriteria Analysis: A New Approach to Decision Making" funded by the National Science Fund, Bulgarian Ministry of Education and Science.

References

[1] Atanassov, K. (1999). *Intuitionistic Fuzzy Sets,* Springer, Heidelberg.

[2] Atanassov, K. (2012). *On Intuitionistic Fuzzy Sets Theory,* Springer, Berlin.

[3] Atanassov, K. (2016). Uniformly expanding intuitionistic fuzzy operator. *Notes on Intuitionistic Fuzzy Sets*, 22(2), 48–52.

[4] Atanassov, K. (2017). *Intuitionistic Fuzzy Logics,* Springer, Cham.

[5] Atanassov K., Mavrov, D., & Atanassova, V. (2014). Intercriteria Decision making: a new approach for multicriteria decision making, based on index matrices and intuitionistic fuzzy sets. *Issues in Intuitionistic Fuzzy Sets and Generalized Nets*, 11, 1–8.