Notes on Intuitionistic Fuzzy Sets
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Identification of a sufficient number of the best attributes in the intuitionistic fuzzy models

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Abstract: Dimension reduction of the models, i.e., pointing out only the necessary number of input variables (attributes, features) is an important task enabling the efficient performance of different algorithms. This paper is a continuation of our previous works on a new method of selection of the attributes in the models making use of Atanassov's intuitionistic fuzzy sets. We consider classification problems trying to point out the reduced number of the attributes and still obtain satisfactory results. We investigate the previously proposed method in more details comparing its performance with a well-known method of extraction parameters, namely Principal Component Analysis (PCA), and with a well-known method of selecting the attributes in which the so-called Gain Ratio is used. We illustrate our considerations using benchmark data from UCI Machine Learning Repository.

Keywords: Selection of attributes, Classification, Intuitionistic fuzzy sets, Principal Component Analysis, Gain Ratio.
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1 Introduction

Dimensionality reduction of the models is an important task that refers to reducing the number of input variables (attributes, features) in a dataset. Reduced but still sufficient number of the input variables makes a model more transparent and simpler from the point of view of calculations. The problem is challenging from both theoretical and practical points of view. The existing methods have their pros and cons but there is not one "best method". There are two approaches to model reduction, namely, feature (attribute) extraction and feature (attributes) selection. Feature extraction uses a combination of features (attributes), deriving some new ones. The results of feature extraction can be difficult to interpret. The second method of model reduction, namely, feature (attributes) selection boils down to the selection of the most relevant features. In this paper, we will examine attribute selection for the sets of data expressed by the Atanassov's intuitionistic fuzzy sets (IFSs, for short).

Atanassov's intuitionistic fuzzy sets (Atanassov [2–4]) are a generalization of the fuzzy sets (Zadeh [48]). The IFSs can be viewed as a tool that may help better model the systems in the presence of a lack of knowledge. An advantage of the IFSs is an inherent possibility to take a lack of knowledge into account by using the so-called hesitation margin or intuitionistic fuzzy index.

Certainly, the problem of too many variables occurs for the IFSs models as for other types of models. The counterpart of the well-known Principal Component Analysis (PCA) (Jackson [9], Jolliffe [10], Marida et al. [12]) for the IFSs (cf. Szmidt and Kacprzyk [37]), Szmidt [15]) gives correct results but, again, it is complicated from the point of view of calculations, and the final result is not transparent enough for some users.

Here we analyze a simple method of feature selection for the data sets which are expressed by intuitionistic fuzzy sets (IFSs). We make use of the three term representation of IFSs enabling us to construct a convincing, simple and efficient, transparent, and easy from the point of view of calculations method of feature selection. Moreover, the considered here approach makes it possible to rank the attributes (not all methods enable it).

The discussed method is tested on well-known benchmark data from the UCI Machine Learning Repository (`https://archive.ics.uci.edu/ml/datasets`). We deal with classification tasks trying to reduce the number of input attributes and still obtain satisfactory results. The results of our approach are compared to Principal Component Analysis (cf. Jackson [9], Jolliffe [10], Marida et al. [12]) and with the method using the well known Gain Ratio (Quinlan [13]). Additionally, we propose to reduce the number of calculations by using a graphical representation of the proposed method.

2 A brief introduction to IFSs

One of the possible generalizations of a fuzzy set in X (Zadeh [48]) given by

$$A^{'} = \{\langle x, \mu_{A'}(x)\rangle | x \in X\} \tag{1}$$

where $\mu_{A'}(x) \in [0, 1]$ is the membership function of the fuzzy set $A^{'}$, is an intuitionistic fuzzy set (Atanassov [2–4]) A is given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\} \tag{2}$$

where: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x), \nu_A(x) \in [0,1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively. (See Szmidt and Baldwin [16] for assigning memberships and non-memberships for IFSs from data.)

Obviously, each fuzzy set may be represented by the following IFS:

$$A = \{\langle x, \mu_{A'}(x), 1 - \mu_{A'}(x)\rangle \mid x \in X\}.$$

An additional concept for each IFS in X, that is not only an obvious result of (2) and (3) but which is also relevant for applications, we will call (Atanassov [3])

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{4}$$

a *hesitation margin* of $x \in A$, which expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [3]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The hesitation margin turns out to be important while considering the distances (Szmidt and Kacprzyk [17, 18, 21, 28, 30], entropy (Szmidt and Kacprzyk [22, 32]), similarity (Szmidt and Kacprzyk [33, 44]) for the IFSs, etc. i.e., the measures that play a crucial role in virtually all information processing tasks (Szmidt [15]).

The hesitation margin turns out to be relevant for applications – in image processing (cf. Bustince et al. [7]), the classification of imbalanced and overlapping classes (cf. Szmidt and Kukier [45–47]), the classification applying intuitionistic fuzzy trees (cf. Bujnowski [6]), attribute selection [41, 42], ranking of alternatives [43], multiagent decisions, negotiations, voting, group decision making, etc. (cf. [5,11,19,20,23–25,28,29,31,34]), genetic algorithms [14]. Sometimes the concept of the hesitation margin is just indispensable, for example, for a proper definition of the Hausdorff distance [36] and seeing IFSs like different ones from interval- valued fuzzy sets [40].

3 Three term representation of the IFSs as a basis for attribute selection

In this paper we use the three term representation of the IFSs, i.e., take into account membership values μ, non-membership values ν, and hesitation margins π. The tree term representation is very useful especially from practical points of view (cf. Szmidt [15], Szmidt and Kacprzyk [21, 22, 26, 27, 34–36, 38, 39]).

We also used an algorithm [16] of how to derive IFS parameters of a model from relative frequency distributions (histograms) but in further consideration it is assumed that the parameters are known.

Having in mind the interpretation of the three terms we can indicate the most relevant attributes. As the values of each attribute A_k, $k = 1, \ldots, K$ for different instances are different, an attribute can be described by average values of memberships (5), non-memberships (6), and hesitation margins (7), that are obtained by the weight operator W (cf. [4]), i.e.:

$$\overline{\mu}_{A_k} = \frac{1}{n}\sum_{i=1}^{n} \mu_{A_k}(x_i) \tag{5}$$

$$\overline{\nu}_{A_k} = \frac{1}{n}\sum_{i=1}^{n} \nu_{A_k}(x_i) \tag{6}$$

$$\overline{\pi}_{A_k} = \frac{1}{n}\sum_{i=1}^{n} \pi_{A_k}(x_i) \tag{7}$$

where n is a number of instances.

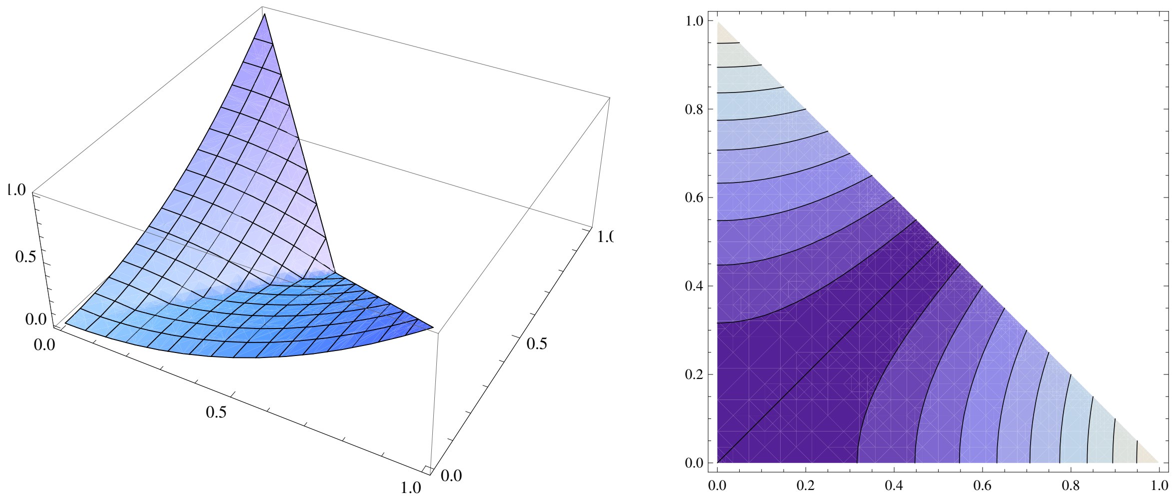
Description of the attributes by (5)–(7) makes it possible to indicate the most discriminative attributes. An intuitionistic fuzzy attribute A_k is most discriminative if its average intuitionistic fuzzy index (7) is as small as possible, and the difference between average membership value and average non-membership value $|\overline{\mu}_{A_k} - \overline{\nu}_{A_k}|$ is as big as possible. The simplest function which makes it possible to find out the most relevant attributes, i.e., the one fulfilling conditions for π and $|\overline{\mu}_{A_k} - \overline{\nu}_{A_k}|$ is:

$$f(A_k) = [(1 - \overline{\pi}_{A_k})(|\overline{\mu}_{A_k} - \overline{\nu}_{A_k}|)] \tag{8}$$

Function $f(A_k)$ (8) has the following properties

1. $0 \leq f(A_k) \leq 1$.
2. $f(A_k) = (f(A_k)^C)$
3. If a value of $|\overline{\mu_k} - \overline{\nu_k}|$ is fixed, $f(A_k)$ increases while π decreases.
4. If a value of π is fixed, $f(A_k)$ behaves dually to a very simple sort of entropy measure $|\overline{\mu_k} - \overline{\nu_k}|$ (i.e., as $1 - (|\overline{\mu_k} - \overline{\nu_k}|)$).

The shape of (8), and its contour plot are in Figure 1.



a) *b)*

Figure 1. a) Shape of (8); b) Contourplot of (8)

Making use of the characteristic of each attribute $f(A_k)$ (8) we find "the best" attribute

$$\arg\max_{A_k}[(1-\overline{\pi}_{A_k})(|\overline{\mu}_{A_k}-\overline{\nu}_{A_k}|)] \tag{9}$$

where A_k is the k-th attribute , $k = 1, \ldots, K$.

We can rank all K attributes from the most to the least discriminative by repeating (9) $K - 1$ times.

4 Results

We tested the selection method (8)–(9) using the Diabetic Retinopathy dataset available in UCI Repository: `https://archive.ics.uci.edu/ml/datasets/Diabetic+Retinopathy+Debrecen+Data+Set`.

The Diabetic Retinopathy dataset contains features extracted from the Messidor image set [1]. The Diabetic Retinopathy dataset has 20 attributes. The last (20th) attribute is the classification of whether an image contains signs of diabetic retinopathy or not. There are 1151 instances.

The order of the first 10 best attributes and respective values of measure (8) are in Table 1. In Figure 2 there are all the attributes evaluated by (8) and presented in descended order from the best to the worst one.

Table 1. "Diabetic Retinopathy" – first ten attributes selected by $f(A_k)$ (8)

	1	2	3	4	5
Attribute No	3	4	5	6	19
Measure $f(A_k)$	0.020	0.016	0.011	0.006	0.006
	6	7	8	9	10
Attribute No	8	7	10	11	12
Measure $f(A_k)$	0.005	0.004	0.004	0.003	0.003

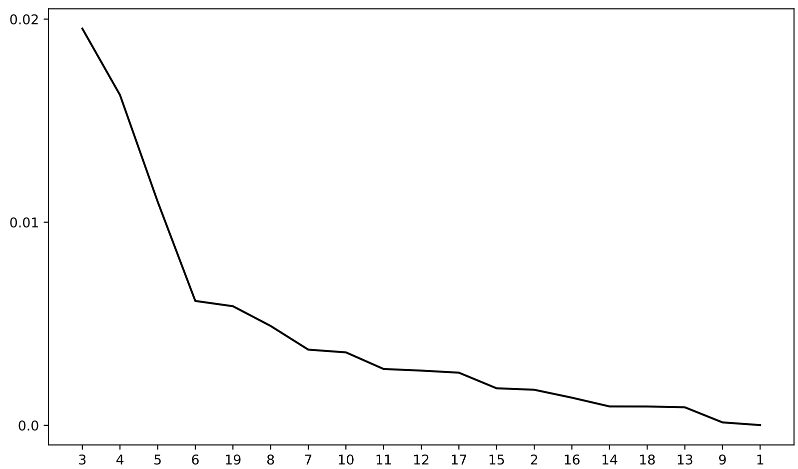


Figure 2. The values of (8) for all the Diabetic Retinopathy attributes ranked from the best to the worst

Next, using WEKA (`http://www.cs.waikato.ac.\-nz/ml/weka/`) we evaluated the accuracy of different 12 classifiers using all 19 attributes (without selection). A simple cross-validation method was applied with 10 experiments of the 10-fold cross-validation. The best results of the classification were obtained for the algorithms:

- function Logistic;
- trees LMT;
- Multilayer Perceptron;
- Random Forest.

Besides the classification accuracy (total proper identification of the instances belonging to the classes considered), we have also paid attention to the area under ROC curve [8]. The results are in Table 2.

The accuracy by the best algorithms and all the attributes (Table 2) is equal to 74.62% obtained by a function Logistic. Accuracy of the other algorithms with best results, namely, tree LMT, Multilayer Perceptron, and Random Forest is equal to: 71.95%, 71.16%, 68.66%, respectively.

Table 2. "Diabetic Retinopathy" – comparison of the classification accuracy by different classifiers with all 19 attributes

	Classification accuracy ($\bar{x} \pm \sigma$) in %	
Algorithm (no selection)	**Accuracy of both classes**	**AUC ROC**
Function Logistic	74.62 ± 3.38	0.83 ± 0.03
Trees LMT	71.95 ± 3.75	0.79 ± 0.04
Multilayer Perceptron	71.16 ± 4.60	0.80 ± 0.04
Random Forest	68.66 ± 3.74	0.76 ± 0.04

We wished to see how many attributes are redundant, i.e., for how many attributes after selection we will still have high accuracy. We started the calculation from only one, the best attribute, and in the next steps we were adding one by one the next "the best" attribute verifying accuracy obtained. The procedure of adding the attributes was continued until obtaining satisfactory accuracy. For data set "Diabetic Retinopathy", taking into account only 4 "best" attributes (Table 3) we obtained accuracy 74.07% for function Logistic, 72.19% for tree LMT, 72.02% for Multilayer Perceptron, and 67.82% for the fourth algorithm (Random Forest).

In the same (Table 3) we have results obtained by PCA, and by the Gain Ratio. The PCA results are worse for the three first algorithms than the results obtained by the measure (8).

The results obtained by the Gain Ratio (Table 3) are considerably worse and equal: 59%, 59%, 55%, 60% in comparison with 74%, 72%, 72%, 68% obtained by the measure (8).

Summing up, the selecting algorithm (8)–(9) meets our expectations for Diabetic Retinopathy data set.

Table 3. “Diabetic Retinopathy” – comparison of the classification accuracy with 4 attributes pointed out by: $f(A_k)$ (9), PCA, and the Gain Ratio

Algorithm (4 attributes)	**Classification accuracy ($\bar{x} \pm \sigma$) in %**		
	$f(A_k)$ (9)	**PCA**	**Gain Ratio**
Function Logistic	74.07 ± 3.71	68.06 ± 3.67	59.03 ± 3.52
Trees LMT	72.19 ± 3.13	69.28 ± 3.99	58.84 ± 3.72
Multilayer Perceptron	72.02 ± 4.41	70.40 ± 4.01	55.03 ± 3.13
Random Forest	67.82 ± 3.91	71.08 ± 3.96	59.51 ± 3.81

The advantage of selecting the attributes by the measure (8) in comparison with selecting the attributes by the Gain Ration is illustrated in Figures 2 and 3.

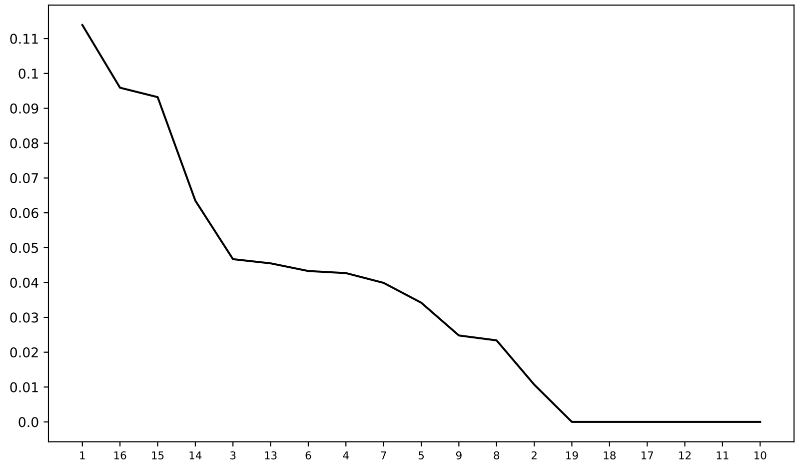


Figure 3. The values of the Gain Ratio for all the Diabetic Retinopathy attributes ranked from the best to the worst

First, we can see that the order of the attributes obtained by the measure (8) in Figure 2, and the order of the attributes obtained by the Gain Ratio in Figure 3 are different. The best attributes by (8) are: 3, 4, 5, 6 whereas by the Gain Ratio are quite different: 1, 16, 15, 14. We have calculated cumulative percentage participation of the four attributes in both measures (cf. Table 4 and Table 5). It turns out that:

- cumulative percentage participation of the four attributes (3, 4, 5, 6) in the measure (8) is equal to 60.9%,
- the cumulative percentage participation of the four attributes (1, 16, 15, 14) in the Gain Ratio is equal to 54.1%.

In other words, the four attributes selected by the measure (8) “cover” more area (60.9%) under the curve in Figure 2 than the four attributes selected by the Gain Ratio (cover less area – 54.1% under the curve in Figure 3). This result explains why the accuracy obtained by the four attributes selected by the measure (8) is better than the accuracy obtained by the four attributes selected by the Gain Ratio (cf. Table 3).

Table 4. "Diabetic Retinopathy" – the order of the attributes by $f(A_k)$ (8),
the values of the measure (8), and the cumulative value of (8) for the ordered attributes

	1	2	3	4	5	6	7
Attribute No	3	4	5	6	19	8	7
Measure $f(A_k)$ (8)	0.020	0.016	0.011	0.006	0.006	0.005	0.004
Cumulative $f(A_k)$ (8) [%]	22.47	41.18	53.86	60.90	67.65	73.28	77.57
	8	9	10	11	12	13	14
Attribute No	10	11	12	17	15	2	16
Measure $f(A_k)$ (8)	0.004	0.003	0.003	0.003	0.002	0.002	0.001
Cumulative $f(A_k)$ (8) [%]	81.70	84.89	87.99	90.98	93.08	95.09	96.66
	15	16	17	18	19		
Attribute No	14	18	13	9	1		
Measure $f(A_k)$ (8)	0.0009	0.0009	0.0009	0.0002	0.000		
Cumulative $f(A_k)$ (8) [%]	97.73	98.80	99.82	99.99	100		

Table 5. "Diabetic Retinopathy" – the order of the attributes by the Gain Ratio (GR),
the values of the measure Gain Ratio, and the cumulative values of the Gain Ratio [%]
for the ordered attributes

	1	2	3	4	5	6	7
Attribute No	1	16	15	14	3	13	6
Values of GR	0.114	0.096	0.093	0.064	0.047	0.046	0.043
Cumulative values of GR [%]	16.81	30.96	44.71	54.08	60.97	67.68	74.07
	8	9	10	11	12	13	14
Attribute No	4	7	5	9	8	2	19
Values of GR	0.043	0.040	0.034	0.025	0.023	0.011	0.00
Cumulative values of GR [%]	80.37	86.26	91.31	94.97	98.42	100	100
	15	16	17	18	19		
Attribute No	18	17	12	11	10		
Values of GR 0	0.000	0.000	0.000	0.000	0.000		
Cumulative values of GR [%]	100	100	100	100	100		

The presented method gave promising results also for other data sets (cf. [41]. However, to determine the satisfactory number of attributes selected, we were performing calculations using WEKA – for many algorithms and for every algorithm we have started from only one attribute adding in the next steps other attributes, one by one, in the order pointed out by the proposed function $f(A_k)$ (8). The approach enables to indicate a satisfactory number of the attributes. However, having the promising method (8)–(9) of the attributes selection, it is still profitable to simplify the calculations. We have observed that the number of the selected attributes giving satisfactory results can be found out using a figure showing the order of the attributes selected by $f(A_k)$ (8). For the data set Diabetic Retinopathy we use Figure 2. The function $f(A_k)$ (8) is a decreasing one. We can observe regions of decreasing followed by the regions of similar values $f(A_k)$ (8) for the ordered attributes. For example, for attributes: 3, 4, 5, 6 the function decreases, whereas for the next attributes: 6 and 19 is almost the same, next, the function decreases for attributes 19, 8, and 7, next, for attributes 7 and 10 is very similar, and next, decreases for attributes 10 and 11, to be almost the same for attributes 11, 12, 17, etc. The idea of simplifying the calculations is to verify the successive subsets of the attributes for which the function $f(A_k)$ (8) decreases instead of verifying the accuracy adding the attributes one by one. In Figure 2 we can see that the first subset of such attributes is {3, 4, 5, 6}, next subsets are: {19, 8, 7}, {10, 11}, {17, 15}, etc. As we have already verified, the first subset of the attributes {3, 4, 5, 6} is enough to obtain satisfactory accuracy of classification (74.1% (Table 3) instead of 74.6% for all the attributes (Table 2)). The method of finding out a satisfactory number of the attributes graphically does not work in the case of the Gain Ration (cf. Figure 3). We have observed similar dependencies testing other data sets.

5 Conclusions

We have tested and discussed in more details an earlier proposed method for feature selection for the sets of data represented by the intuitionistic fuzzy sets (IFSs). The IFSs were represented by the three terms, i.e., by taking into account the degree of membership, non-membership and hesitation margin. A transparent and easy-to-understand function evaluating the attributes and making it possible to order them was investigating. The ordered attributes are a basis to select them. We also used a graphical representation of the ordered attributes with the respective values of the evaluating function to simplify the calculations. The method is easy to explain and interpret. simple from the point of view of calculations and gives promising results.

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