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Short remark on two covering topological operators over intuitionistic fuzzy sets

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Abstract: Here we construct intuitionistic fuzzy sets with integral form of their degrees of membership and non-membership, that cover interval type-2 fuzzy sets.
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1 Introduction

In [5], we compared the concepts of interval type-2 fuzzy sets [3, 4, 6] and intuitionistic fuzzy sets (IFSs; [1, 2]). A type-2 fuzzy set $\overline{A}$, is defined by:

$$\overline{A} = \{\langle (x, u), \mu_{\overline{A}}(x, u)\rangle | x \in X, u \in J_x \subseteq [0, 1]\}, \tag{1}$$

where X is a fixed universe, that is a closed interval and $\mu_{\overline{A}}(x,u) \in [0,1]$.

As we showed in [5], the IFS A, that represents the type-2 fuzzy set $\overline{A}$, is defined by:

$$A = \{\langle (x,u), \mu_A(x,u), \nu_A(x,u)\rangle | x \in X, u \in J_x \subseteq [0,1]\}, \tag{2}$$

where $\mu_A(x,u), \nu_A(x,u) \in [0,1]$ and $\mu_A(x,u) + \nu_A(x,u) \leq 1$. Therefore, this IFS is from the type of so called temporal IFSs (see [1, 2]). Here, for this type of IFSs we introduce two operators from topological type. They are essentially different from the existing topological operators over IFSs (see [2]). Here, we study some of their properties, but in general, this will be our aim for the near future.

2 Main results

Here, we introduce two new topological operators over intuitionistic fuzzy sets having the form of (2). The proposed operators are $acl(A)$ and $vcl(A)$, whose names are derived from the concept of closure (operator cl), and the concepts of area and volume, hence the prefixes a and v.

2.1. For the IFS A, given by (2) we introduce the operator acl as follows:

$$acl(A) = \left\{ \left\langle x, \int\limits_{J_x} \mu_A(x,u)du, \int\limits_{J_x} \nu_A(x,u)du \right\rangle | x \in X, u \in J_x \subseteq [0,1] \right\}, \tag{3}$$

where μ_A and ν_A are continuous functions in $X \times J_x$, $J_x = [\inf J_x, \sup J_x]$ is a proper subinterval of $[0,1]$ and

$$\sup J_x - \inf J_x \leq 1. \tag{4}$$

Proposition 1. For every IFS A given by (2), $acl(A)$ is an IFS.
Proof: For every $x \in X$ and for (4) it follows that

$$\int\limits_{J_x} \mu_A(x,u)du + \int\limits_{J_x} \nu_A(x,u)du = \int\limits_{J_x} (\mu_A(x,u) + \nu_A(x,u))du$$

$$\leq \sup J_x - \inf J_x \leq 1.$$

For two IFSs A and B (see [1, 2]):

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x \in X\},$$

$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle | x \in X\},$$

$$\neg A = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}.$$

This completes the proof. □

Theorem 1. For every two IFSs A and B, having the form of (2):

(a) $acl(A) \cap acl(B) \supseteq acl(A \cap B)$,

(b) $acl(A) \cup acl(B) \subseteq acl(A \cup B)$,

(c) $\neg acl(\neg A) = acl(A)$.

Proof: Let A and B be two IFSs. For (a) we obtain

$$acl(A) \cap acl(B)$$

$$= \left\{ \left\langle x, \int\limits_{J_x} \mu_A(x,u)du, \int\limits_{J_x} \nu_A(x,u)du \right\rangle | x \in X, u \in J_x \subseteq [0,1] \right\}$$

$$\cap \left\{ \left\langle x, \int\limits_{J_x} \mu_B(x,u)du, \int\limits_{J_x} \nu_B(x,u)du \right\rangle | x \in X, u \in J_x \subseteq [0,1] \right\}$$

$$= \left\{ \left\langle x, \min\left(\int\limits_{J_x} \mu_A(x,u)du, \int\limits_{J_x} \mu_B(x,u)du \right), \right. \right.$$

$$\left. \left. \max\left(\int\limits_{J_x} \nu_A(x,u)du, \int\limits_{J_x} \nu_B(x,u)du \right) \right\rangle | x \in X, u \in J_x \subseteq [0,1] \right\}$$

$$\supseteq \left\{ \left\langle x, \int\limits_{J_x} \min(\mu_A(x,u), \mu_B(x,u))du, \right. \right.$$

$$\left. \left. \int\limits_{J_x} \max(\nu_A(x,u), \nu_B(x,u))du \right\rangle | x \in X, u \in J_x \subseteq [0,1] \right\}$$

$$= acl(A \cap B).$$

For (b) we obtain

$$acl(A) \cup acl(B)$$

$$= \left\{ \left\langle x, \int\limits_{J_x} \mu_A(x,u)du, \int\limits_{J_x} \nu_A(x,u)du \right\rangle | x \in X, u \in J_x \subseteq [0,1] \right\}$$

$$\cup \left\{ \left\langle x, \int\limits_{J_x} \mu_B(x,u)du, \int\limits_{J_x} \nu_B(x,u)du \right\rangle | x \in X, u \in J_x \subseteq [0,1] \right\}$$

$$= \left\{ \left\langle x, \max\left(\int\limits_{J_x} \mu_A(x,u)du, \int\limits_{J_x} \mu_B(x,u)du \right), \right. \right.$$

$$\left. \left. \min\left(\int\limits_{J_x} \nu_A(x,u)du, \int\limits_{J_x} \nu_B(x,u)du \right) \right\rangle | x \in X, u \in J_x \subseteq [0,1] \right\}$$

$$\subseteq \left\{ \left\langle x, \int\limits_{J_x} \max(\mu_A(x,u), \mu_B(x,u))du, \right.\right.$$

$$\left.\left. \int\limits_{J_x} \min(\nu_A(x,u), \nu_B(x,u))du \right\rangle |x \in X, u \in J_x \subseteq [0,1] \right\}$$

$$= acl(A \cup B).$$

For (c) we obtain

$$\neg acl(\neg A) = \neg acl(\{\langle (x,u), \nu_A(x,u), \mu_A(x,u)\rangle | x \in X, u \in J_x \subseteq [0,1]\})$$

$$= \neg \left\{ \left\langle x, \int\limits_{J_x} \nu_A(x,u)du, \int\limits_{J_x} \mu_A(x,u)du \right\rangle |x \in X, u \in J_x \subseteq [0,1] \right\}$$

$$= \left\{ \left\langle x, \int\limits_{J_x} \mu_A(x,u)du, \int\limits_{J_x} \nu_A(x,u)du \right\rangle |x \in X, u \in J_x \subseteq [0,1] \right\}.$$

This completes the proof. □

2.2. Let $X = [a,b]$, where $b > a \geq 0$. Now, for the IFS A, given by (2), we define the operator vcl as follows:

$$vcl(A) = \left\{ \left\langle x, \frac{1}{b-a} \int\limits_a^b \int\limits_{J_y} \mu_A(y,u)dudy, \frac{1}{b-a} \int\limits_a^b \int\limits_{J_y} \nu_A(y,u)dudy \right\rangle |y \in X, u \in J_y \subseteq [0,1] \right\},$$

where μ_A and ν_A are continuous functions in $X \times J_x$, $J_x = [\inf J_x, \sup J_x]$ is a proper subinterval of $[0,1]$ and (4) is valid.

Proposition 2. For every IFS A given by (2), $vcl(A)$ is an IFS.
Proof: For every $x \in X$ and for (4) it follows that

$$\frac{1}{b-a} \left(\int\limits_a^b \int\limits_{J_y} \mu_A(y,u)dudy + \int\limits_a^b \int\limits_{J_y} \nu_A(y,u)dudy \right)$$

$$= \frac{1}{b-a} \int\limits_a^b \int\limits_{J_y} (\mu_A(y,u) + \nu_A(y,u))dudy$$

$$\leq \frac{1}{b-a} \int\limits_a^b \int\limits_{J_y} 1.dudy$$

$$\leq \frac{1}{b-a} \int\limits_a^b \int\limits_0^1 1.dudy$$

$$= \frac{1}{b-a}(b-a) = 1.$$

This completes the proof. □

Theorem 2. For every two IFSs A and B, given by (2):

(a) $vcl(A) \cap vcl(B) \supseteq vcl(A \cap B)$,

(b) $vcl(A) \cup vcl(B) \subseteq vcl(A \cup B)$,

(c) $\neg vcl(\neg A) = vcl(A)$.

Proof: The proof is similar to the proof of Theorem 1. □

3 Conclusion

In future, we will discuss the relations between the new operators and the different IFS operations and operators from modal, topological and level types.

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