NIFS 2 (1996) 3, 5-14

SIMILARITY MEASURES IN IFSs

Author : Ranjit Biswas
Department of Mathematics
Indian Institute of Technology
Kharagpur - 721302, West Bengal
INDIA

E-mail : ranjit@maths.iitkgp.ernet.in

Abstract :

In this paper a measure of similarity between two intuitionistic fuzzy sets (IFSs) as well as between two elements of IFSs are studied.

Keywords :

Fuzzy set, intuitionistic fuzzy set (IFS), similarity measure.

1. INTRODUCTION

The notion of fuzzy sets [11] was generalised by Atanassov [6] by introducing intuitionistic fuzzy sets (IFSs). Atanassov [6] stated how fuzzy sets can be regarded as IFSs but not conversely. He [6-10] stated various interesting operations on IFSs. In the present paper we study how to have a similarily measure between two IFSs and two elements of IFSs. Several authors [1-5, 12, 14, 15] suggested several measures of similarity between fuzzy sets. Out of those, the measures given in [4], [12] and [15] are interesting; and we shall follow mainly the measure suggested in [4] by Kwang, Song and Lee. Instead of geometric similarity and Hausdorff similarity, they [4] suggested some other type of similarity measures which are more useful to behaviour analysis in an organization.

2. PRELIMINARIES

We give below some basic preliminaries.

2.1 Similarity Between Fuzzy Sets

Similarity measure S(A,B) between two fuzzy sets A and B is as follows :

$$S(A,B) = \max_{x \in X} \min (\mu_A(x), \mu_B(x))$$

Some properties of this measure are (as studied in [4]) :

(1) $S(A,B)$ is the maximum membership degree in the intersection $A \cap B$.

(2) The similarity degree is bounded i.e. $0 \leq S(A,B) \leq 1$.

(3) If A and B are normalized, and $A = B$ then $S(A,B) = 1$.

If $A \cap B = \phi$, then $S(A,B) = 0$.

(4) The measure is commutative i.e. $S(A,B) = S(B,A)$.

(5) When the set A and B are crisp sets,

$$S = 0, \quad \text{if} \quad A \cap B = \phi$$

$$S = 1, \quad \text{if} \quad A \cap B \neq \phi.$$

2.2 Similarity Between Elements

The similarity measure between two elements $x, y \in X$ in fuzzy sets A_i of X, $i = 1,\ldots,n$, is defined as follows :

$$S_e(x,y) = \max_i \min\left(\mu_{A_i}(x), \ \mu_{A_i}(y) \right).$$

This measure satisfies the following properties :

(1) $0 \leq S_e(x,y) \leq 1$.

(2) If $x = y$, $S_e(x,y) = 1$.

(3) $S_e(x,y) = S_e(y,x)$.

Definition 2.3

Let a set E be fixed. An IFS A in E is an object having the form

$$A^{*} = \{ < x, \mu_A(x), \gamma_A(x) > | x \in E \}$$

where the functions $\mu_A : E \longrightarrow [0,1]$ and $\gamma_A : E \longrightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ to the set A, which is a subset of E, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1 ,$$

Definition 2.4

If A and B are two IFSs, then

$A \subset B$ iff $(\forall x \in E) (\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x))$

$A \supset B$ iff $B \subset A$.

$A = B$ iff $(\forall x \in E) (\mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x))$

$\overline{A} = \{ < x, \gamma_A(x), \mu_A(x) > | x \in E \}$

$A \cap B = \{ < x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x)) > | x \in E \}$

$A \cup B = \{ < x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x)) > | x \in E \}$

$A + B = \{ < x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \gamma_A(x).\gamma_B(x) > | x \in E \}$

$A.B = \{ < x, \mu_A(x).\mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x).\gamma_B(x) > | x \in E \}$

$\Box A = \{ < x, \mu_A(x), 1 - \mu_A(x) > | x \in E \}$

$\Diamond A = \{ < x, 1 - \gamma_A(x), \gamma_A(x) > | x \in E \}$.

Obviously every fuzzy set has the form $\{<x, \mu_A(x), \mu_A c(x)> : x \in E\}$.

In [6], Atanassov gave an example of an IFS which is not a fuzzy set. From now onwards in this paper, by an IFS A we shall mean the IFS (A, μ_A, ν_A), where the meaning is obvious.

3. SIMILARITY MEASURE

3.1 Similarity Between Two IFSs

Consider two IFSs A and B of E. The similarity measure between A and B is defined by an interval $I = I(A,B)$ given by

$$I(A,B) = [\, s_1 \wedge s_2,\ s_1 \vee s_2 \,]\text{-}$$

where $s_1 = S(\mu_A, \mu_B)$, $s_2 = S(\nu_A, \nu_B)$, and $\vee$, $\wedge$ denotes "sup" and "inf" respectively. If $I(A,B) = [\, i_1, i_2 \,]$, the i_1 gives the minimum simiarity between the two IFSs A and B, where i_2 gives the maximum similarity.

Clearly $I(A,B) \subseteq [0,1]$ assuming $[c,c] \subseteq [0,1]$ where $0 \le c \le 1$ (definition 2.1 of [13]).

Also, $i_1 \le \max\limits_x \mu_{A \cap B}(x)$ and $i_2 \le \max\limits_x \nu_{A \cup B}(x)$ and from definition, it is clear that $I(A,B) = I(B,A)$.

When A and B are crisp sets, then

$$I(A,B) = [0,0], \quad \text{if } A \cap B = \phi$$

$$= [k,1], \quad \text{if } A \cap B \ne \phi$$

where $K \in \{0,1\}$.

Example 3.1

Consider two IFSs A and B given by :

x	μ_A , ν_A	μ_B , ν_B
x_1	.71, .24	.42, .31
x_2	.80, .20	.78, .13
x_3	.63, .31	.84, .03
x_4	.70, .28	.64, .21

Clearly s_1 = .78, s_2 = .24

$\therefore$ I(A,B) = [.24, .78]

Thus, minimum amount of similarity = .24, and

maximum amount of similarity = .78

3.2 Similarity Between Two Elements

Consider the IFSs A_i of X, $i = 1,2,3,\ldots,n$. Take any two elements $x, y \in X$. Similarity measure between x and y is defined by an interval $I_e = I_e(x,y)$ given by

$$I_e(x,y) = [\, e_1 \wedge e_2,\ e_1 \vee e_2 \,]$$

where, $e_1 = S_e(x,y)$ for membership functions μ_{A_i} and $e_2 = S_e(x,y)$ for non-membership functions ν_{A_i}.

Here $e_1 \wedge e_2$ gives minimum similarity and $e_1 \vee e_2$ gives the maximum similarity between the two elements x and y.

Clearly, $I_e(x,y) \subseteq [0,1]$, (using definition 2.1 of [13]). Also, $I_e(x,y) = I_e(y,x)$.

Example 3.2

Consider the IFSs A and B as given in Example 3.1. Let us find $I_e(x_1,x_4)$. Here, $e_1 = .70$, $e_2 = .24$

$\therefore$ $I_e(x_1, x_4) = [\ .24,\ .70\ ]$.

NOTE : In 2.2, the author [4] defined similarity measure between elements by $S_e(x,y)$. They said that if x = y, then $S_e(x,y) = 1$. But this is not true. The following example will support it.

Example 3.3

Consider the Table-3 of [4] which is

x	A_1	A_2
x_1	.2	.5
x_2	1	1
x_3	0	0
x_4	.5	1

Here, $I_e(x_1,x_1) = \max(\ .2,\ .5\ ) = .5$.

4. ANOTHER METHOD OF SIMILARITY MEASURE BETWEEN TWO IFSs

Chen [15] computed similarity between two fuzzy rules, using the dot product between the vectors representing the rules which is interesting. Thus, if A and B are two fuzzy sets and $\vec{A}$, $\vec{B}$ be the corresponding vectors (assuming membership values as elements of the vectors), then the similarity between A and B is defined by

$$\bar{S}(A,B) = \frac{\vec{A} \bullet \vec{B}}{\vec{A}^{2} \vee \vec{B}^{2}}$$

Now, we can define similarity between two IFSs A and B of X by the interval

$$\bar{I}(A,B) = [\ s_1 \wedge s_2,\quad s_1 \vee s_2\ ]$$

where, $s_1 = \bar{S}\ (\ \mu_A,\ \mu_B\ )$, $s_2 = \bar{S}\ (\ \nu_A,\ \nu_B\ )$.

Clearly, $\bar{I}\ (A,B) \subseteq [0,1]$, (using definition 2.1 of [13]),

$I(A,B) = I(B,A)$. and $I(A,A) = [1,1]$.

REFERENCES

[1] A. Kaufmann, Introduction to Fuzzy Arithmetic (Van Nostrand Reinhold, New York, 1985).

[2] C. Murthy, S.K. Pal and D. Dutta Majumder, Correlation between two fuzzy membership functions, Fuzzy Sets and Systems 17 (1985) 23-38.

[3] G.C. Oden, Integration of fuzzy linguistic information in language comprehension, Fuzzy Sets and Systems, 14 (1984) 29-91.

[4] Hyung Lee-Kwang, Yoon-Seon Song and Keon-Myung Lee, Similarity measures between fuzzy sets and between elements, Fuzzy Sets and Systems, 62 (1994) 291-292.

[5] K.S. Leung and W. Lam, Fuzzy concepts in Expert Systems, IEE Computer (Sept. 1988) 43-56.

[6] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1) (1986) 87-96.

[7] K. Atanassov, Two operators on intuitionistic fuzzy sets, Comptes Rendus de l'Academic Bulgare des Science 41(5) (1988) 35-38.

[8] K. Atanassov, More on intuitionistic fuzzy sets, Fuzzy Sets and Systems. 33(1) (1989) 37-46.

[9] K. Atanassov, A. universal operator over intuitionistic fuzzy sets, Comptes Rendus de l'Academic Bulgare des Sciences 46(1) (1993) 13-15.

[10] K. Ata n assov , New operations defined over the intuitionistic fuzzy sets, Fuzzy Sets and Systems 61 (1994) 137-142.

[11] L.A. Zadeh, Fuzzy Sets, Infor. and Control. 8 (1965) 338-353.

[12] L.B. Turksen and Zhao Zhoxy, An approxiate reasoning approach based on similarity measures, IEEE Trans. on Systems, Man and Cybernetics. SMC - 18 (6) (Nov/Dec. 1988), 1049-1055.

[13] Ranjit Biswas, Resenfeld's fuzzy subgroups with interval-valued membership functions, Fuzzy Sets and Systems, 63 (1994) 87-90.

[14] R. Zwick, E. Caristein and D. Budescu, Measures of similarity among fuzzy sets : a comparative analysis, Int. Jour. Approx. Reasoning, 1 (1987) 221-242.

[15] Shyi-Ming Chen, A new approach to handling fuzzy decision making problems, IEEE Trans. on Systems. Man and Cybernetics, SMC-18 (6) (Nov./Dec. 1988).