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# Intercriteria Decision Making:
# A New Approach for Multicriteria Decision Making, Based on Index Matrices and Intuitionistic Fuzzy Sets

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**Abstract:** A new approach for multicriteria decision making is introduced in the paper. It is called "Intercriteria decision making". It is based on the apparatusa of the index matrices and the intuitionistic fuzzy sets and can be applied for decision making in different areas of science and practice.
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## 1 Introduction

A novel method for decision making, based on Index Matrices (IMs; see [1, 2, 3]) and Intuitionistic Fuzzy Sets (IFSs, see [4]) is introduced.

The IMs are esentially new and not widely known mathematical objects, that are extensions of the ordinary matrices. They are discussed in Section 3. In the paper we use also the concept of an Intuitionistic Fuzzy Pair (IFP, see [5]), that will be described in Section 2.

The new approach for multicriteria decision making gives possibility to compare some criteria or estimated by them objects. By this reason it is called an *intercriteria decision making method.* It is discussed in Section 4. A possible application is discussed in Section 5. Formulas for evaluation of the predicted values are discussed in Section 6.

## 2 Short remarks on intuitionistic fuzzy pairs

Initially, we give some remarks on Intuitionistic Fuzzy Pairs (IFPs; see [5]). The IFP is an object in the form of an ordered pair $\langle a, b\rangle$, where $a, b \in [0, 1]$ and $a + b \leq 1$, that is used as an evaluation of some object or process, and which components ($a$ and $b$) are interpreted, respectively, as degrees of membership and non-membership to a given set, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc.

Let us have two IFPs $x = \langle a, b\rangle$ and $y = \langle c, d\rangle$.

In [5], we defined the relations

$$\begin{array}{lll} x < y & \text{iff} & a < c \text{ and } b > d \\ x \leq y & \text{iff} & a \leq c \text{ and } b \geq d \\ x = y & \text{iff} & a = c \text{ and } b = d \\ x \geq y & \text{iff} & a \geq c \text{ and } b \leq d \\ x > y & \text{iff} & a > c \text{ and } b < d \end{array}$$

## 3 Short remarks on index matrices

The concept of Index Matrix (IM) was introduced in [1] and discusssed in more details in [2, 3]. Here, following [2], the basic definitions and properties related to IMs are given.

Let $I$ be a fixed set of indices and $\mathcal{R}$ be the set of all real numbers. By IM with index sets $K$ and $L$ $(K, L \subset I)$, we mean the object,

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \ldots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \ldots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \ldots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \ldots & a_{k_m,l_n} \end{array},$$

where $K = \{k_1, k_2, ..., k_m\},\ L = \{l_1, l_2, ..., l_n\}$, and for $1 \leq i \leq m$, and $1 \leq j \leq n : a_{k_i,l_j} \in \mathcal{R}$.

On the basis of the above definition, in [3] the new object – the Intuitionistic Fuzzy IM (IFIM) – was introduced in the form

$$[K, L, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}]$$

$$\equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1} \rangle & \langle \mu_{k_1,l_2}, \nu_{k_1,l_2} \rangle & \dots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n} \rangle \\ k_2 & \langle \mu_{k_2,l_1}, \nu_{k_2,l_1} \rangle & \langle \mu_{k_2,l_2}, \nu_{k_2,l_2} \rangle & \dots & \langle \mu_{k_2,l_n}, \nu_{k_2,l_n} \rangle \\ \vdots & & & & \\ k_m & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1} \rangle & \langle \mu_{k_m,l_2}, \nu_{k_m,l_2} \rangle & \dots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n} \rangle \end{array},$$

where for every $1 \leq i \leq m, 1 \leq j \leq n$: $0 \leq \mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} \leq 1$, i.e., $\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle$ is an IFP.

# 4 The proposed InterCriteria decision making method

Let us have an IM

$$A = \begin{array}{c|ccccccc} & O_1 & \dots & O_k & \dots & O_l & \dots & O_n \\ \hline C_1 & a_{C_1,O_1} & \dots & a_{C_1,O_k} & \dots & a_{C_1,O_l} & \dots & a_{C_1,O_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_i & a_{C_i,O_1} & \dots & a_{C_i,O_k} & \dots & a_{C_i,O_l} & \dots & a_{C_i,O_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_j & a_{C_j,O_1} & \dots & a_{C_j,O_k} & \dots & a_{C_j,O_l} & \dots & a_{C_j,O_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_m & a_{C_m,O_1} & \dots & a_{C_m,O_k} & \dots & a_{C_m,O_l} & \dots & a_{C_m,O_n} \end{array},$$

where for every $p, q,\ (1 \leq p \leq m, 1 \leq q \leq n)$:

- $C_p$ is a criterion, taking part in the evaluation,
- $O_q$ is an object, being evaluated.
- $a_{C_p,O_q}$ is a real number or another object, that is comparable about relation $R$ with the other $a$-objects, so that for each $i, j, k$: $R(a_{C_k,O_i}, a_{C_k,O_j})$ is defined. Let $\overline{R}$ be the dual relation of $R$ in the sense that if $R$ is satisfied, then $\overline{R}$ is not satisfied and vice versa. For example, if "$R$" is the relation "$<$", then $\overline{R}$ is the relation "$>$", and vice versa.

Let $S^{\mu}_{k,l}$ be the number of cases is which $R(a_{C_k,O_i}, a_{C_k,O_j})$ and $R(a_{C_l,O_i}, a_{C_l,O_j})$ are simultaneously satisfied. Let $S^{\nu}_{k,l}$ be the number of cases is which $R(a_{C_k,O_i}, a_{C_k,O_j})$ and $\overline{R}(a_{C_l,O_i}, a_{C_l,O_j})$ are simultaneously satisfied.

Obviously,

$$S^{\mu}_{k,l} + S^{\nu}_{k,l} \leq \frac{n(n-1)}{2}.$$

Now, for every $k, l$, such that $1 \leq k < l \leq m$ and for $n \geq 2$, we define

$$\mu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{n(n-1)}, \quad \nu_{C_k,C_l} = 2\frac{S^{\nu}_{k,l}}{n(n-1)}.$$

Therefore, $\langle \mu_{C_k,C_l}, \nu_{C_k,C_l} \rangle$ is an IFP. Now, we can construct the IM

$$\begin{array}{c|ccc} & C_1 & \dots & C_m \\ \hline C_1 & \langle \mu_{C_1,C_1}, \nu_{C_1,C_1} \rangle & \dots & \langle \mu_{C_1,C_m}, \nu_{C_1,C_m} \rangle \\ \vdots & \vdots & \vdots & \\ C_m & \langle \mu_{C_m,C_1}, \nu_{C_m,C_1} \rangle & \dots & \langle \mu_{C_m,C_m}, \nu_{C_m,C_m} \rangle \end{array},$$

that determine the degrees of correspondence between criteria $C_1, ..., C_m$.

Let $\alpha, \beta \in [0, 1]$ be given, so that $\alpha + \beta \leq 1$. We say that criteria $C_k$ and $C_l$ are in

- $(\alpha, \beta)$-positive consonance, if

$$\mu_{C_k,C_l} > \alpha \text{ and } \nu_{C_k,C_l} < \beta;$$

- $(\alpha, \beta)$-negative consonance, if

$$\mu_{C_k,C_l} < \beta \text{ and } \mu_{C_k,C_l} > \alpha;$$

- $(\alpha, \beta)$-dissonance, otherwise.

# 5 An application of the method for prediction

Let the IM $A$ be given and let criterion $D$ (e.g., one of the criteria $C_1, ..., C_m$) be fixed. Let us reduce IM $A$ to the IM $B$, omitting, if necessary, some rows, so that all criteria corresponding to the rows of $B$, be in $(\alpha, \beta)$-positive or $(\alpha, \beta)$-negative consonance with $D$.

$$B = \begin{array}{c|ccccccc} & O_1 & \dots & O_k & \dots & O_l & \dots & O_n \\ \hline C_1 & a_{C_1,O_1} & \dots & a_{C_1,O_k} & \dots & a_{C_1,O_l} & \dots & a_{C_1,O_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_i & a_{C_i,O_1} & \dots & a_{C_i,O_k} & \dots & a_{C_i,O_l} & \dots & a_{C_i,O_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_p & a_{C_p,O_1} & \dots & a_{C_p,O_k} & \dots & a_{C_p,O_l} & \dots & a_{C_p,O_n} \\ D & b_{D,O_1} & \dots & b_{D,O_k} & \dots & b_{D,O_l} & \dots & b_{D,O_n} \end{array}$$

For brevity, we say that these criteria are in consonance.

The important particularity in this case is that elements $b_{D,O_1}, ..., b_{D,O_n}$ are evaluated hardlier than the rest $a$-elements of $B$.

Let us have a new object $X$ with estimations $x_1, ..., x_p$ w.r.t. the criteria $C_1, ..., C_p$. Then we can solve the following problem: "Predict the value $y$ of object $X$ w.r.t. criterion $D$".

To solve the problem, we can use one of the following two algorithms.

## 5.1 First algorithm

We realize the following steps for each $i, 1 \leq i \leq p$:

1.1. Determine the values $a_{C_i,O_j}$ and $a_{C_i,O_k}$ so that $a_{C_i,O_j} < a_{C_i,O_k}$ and $a_{C_i,O_j} \leq x_i \leq a_{C_i,O_k}$ and $a_{C_i,O_j}$ is the highest $a_{C_i,O_r}$ with this property and $a_{C_i,O_k}$ is the lowest $a_{C_i,O_s}$ with this property (for $1 \leq r, s \leq p$).

1.2. If criteria $C_i$ and $D$ are in positive consonance, then calculate the value

$$y_i = b_{D,O_j} + (x_i - a_{C_i,O_j}).\frac{b_{D,O_k} - b_{D,O_j}}{a_{C_i,O_k} - a_{C_i,O_j}}$$

and if criteria $C_i$ and $D$ are in negative consonance, then calculate the value

$$y_i = b_{D,O_j} + (x_i - a_{C_i,O_j}).\frac{b_{D,O_j} - b_{D,O_k}}{a_{C_i,O_k} - a_{C_i,O_j}}.$$

1.3. Determine the values

$$y_{min} = \min_{1 \leq i \leq p} y_i,$$

$$y_{ave} = \frac{1}{p} \sum_{1 \leq i \leq p} y_i,$$

$$y_{max} = \max_{1 \le i \le p} y_i.$$

Now, the value of $y$ can be $y_{ave}$ or some other number in interval $[y_{min}, y_{max}]$.

If there is no number $a_{C_i,O_j}$ such that $a_{C_i,O_j} \le x_i$, or $a_{C_i,O_k}$ such that $x_i \le a_{C_i,O_k}$, then Step 1.2 is omitted and in Step 1.3, the denominator is $p - s$, where $s$ is the number of omitted cases (if they are smaller than $p$). If in Step 1.1, $a_{C_i,O_j} = x_i = a_{C_i,O_k}$ and $b_{D,O_j} < b_{D,O_k}$, then

$$y_i = \frac{1}{2}(b_{D,O_k} - b_{D,O_j})$$

for the case of positive consonance between criteria $C_i$ and $D$ and

$$y_i = \frac{1}{2}(b_{D,O_j} - b_{D,O_k})$$

for the case of negative consonance between these criteria.

### 5.2 Second algorithm

2.1. Determine those objects $O_j$, for which for each $i$ ($1 \le i \le p$): $a_{C_i,O_j} \le x_i$ and those objects $O_k$, for which for each $i$ ($1 \le i \le p$): $a_{C_i,O_k} \ge x_i$.

2.2. Determine object $O_r$, so that $a_{C_i,O_r}$ is the highest $a$-element from the determined in Step 2.1 and $a_{C_i,O_r} \le x_i$.

2.3. Determine object $O_s$, so that $a_{C_i,O_s}$ is the lowest $a$-element from the determined in Step 2.1 and $a_{C_i,O_s} \ge x_i$.

2.4. Determine

$$y = \begin{cases} b_{D,O_r} + \frac{b_{D,O_s} - b_{D,O_r}}{p} . \sum_{i=1}^{p} \frac{x_i - a_{C_i,O_r}}{a_{C_i,O_s} - a_{C_i,O_r}}, & \text{if } b_{D,O_s} \ge b_{D,O_r} \\ \\ b_{D,O_s} + \frac{b_{D,O_r} - b_{D,O_s}}{p} . \sum_{i=1}^{p} \frac{x_i - a_{C_i,O_r}}{a_{C_i,O_s} - a_{C_i,O_r}}, & \text{otherwise} \end{cases} .$$

## 6 Formulas for evaluation of the predicted values

Now we discuss two (standard) formulas for evaluation of the $y$-values. Let the IM $B$ be given:

$$B = \begin{array}{c|ccccc} & O_1 & \ldots & O_k & \ldots & O_n \\ \hline C_1 & a_{C_1,O_1} & \ldots & a_{C_1,O_k} & \ldots & a_{C_1,O_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_i & a_{C_i,O_1} & \ldots & a_{C_i,O_k} & \ldots & a_{C_i,O_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_p & a_{C_p,O_1} & \ldots & a_{C_p,O_k} & \ldots & a_{C_p,O_n} \\ D & b_{D,O_1} & \ldots & b_{D,O_k} & \ldots & b_{D,O_n} \end{array}$$

1. For every $k$ $(1 \leq k \leq n)$ we construct the IM

$$B_k = B_{(\perp,O_k)}.$$

2. For every $i$ $(1 \leq i \leq p)$ we put $x_i = a_{C_i,O_k}$.

3. Using the two above described methods (for the fixed number $k$), for $B_k$ and $x_1, ..., x_p$, we determine $y$-values $y_{k,1}, y_{k,2}$.

4. For $s$ $(s = 1, 2)$, we determine numbers

$$z_{k,s} = |y_{k,s} - b_{D,O_k}|.$$

5. Evaluate the standard deviation by:

$$\sigma'_s = \frac{1}{n(B_2 - B_1)} \sum_{k=1}^{n} z_{k,s},$$

$$\sigma''_s = \frac{1}{B_2 - B_1} \sqrt{\frac{1}{n} \sum_{k=1}^{n} z_{k,s}^2}.$$

# 7 Conclusion

In future, the new method can be applied to different areas. For example, in medicine, it can shows some intercriteria dependencies, related to criteria for decision making about the status of a patient from medical experts. The method can be used for searching of the values of objects, for which we have only partial information, and others.

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