1st Int. Workshop on IFSs, Mersin, 14 Nov. 2014
Notes on Intuitionistic Fuzzy Sets
ISSN 1310–4926
Vol. 20, 2014, No. 5, 50–56

Intuitionistic fuzzy radar chart interpretation for workload of the generalized net algorithms

Nora Angelova

Department of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Block 105, 1113 Sofia, Bulgaria
e-mail: nora.angelova@biomed.bas.bg

Abstract: A workload of the algorithm for transition functioning when merging of tokens is permitted and the general algorithm for the GN's functioning with respect to Intuitionistic fuzzy sets (IFS) and their geometrical interpretation based on radar chart is introduced. For this purpose we will use a Generalized Nets Model of a Wastewater Treatment Process.
Keywords: Generalized nets, Intuitionistic fuzzy sets, Genedit app, Geometrical interpretation, Radar chart.
AMS Classification: 03E72, 68Q85.

1 Intuitionistic fuzzy geometrical interpretation of the workload of the loop

The concept of Intuitionistic Fuzzy Set (IFS) and some geometrical interpretaqtions are defined in [1]. The concept of IFS include: a (crisp) set E witch is fixed and fixed set $A \subset E$.
An IFS A^* in E is an object of the following form

$$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ to the set A, respectively, and for every $x \in E$

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

If $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, then $\pi_A(x)$ is the degree of non-determinacy (uncertainty) of the membership of element $x \in E$ to set A. The IFS theory has different geometrical interpatations. One of them is a geometrical interpretation based on radar chart. This interpratation is proposed by Atanassova, [2]. In Figure 1, the innermost zone corresponds to the membership degree, the outermost zone to the non-membership degree and the region between both zones to the degree of uncertainty. This IFS-interpretation can be especially useful for data in time series.

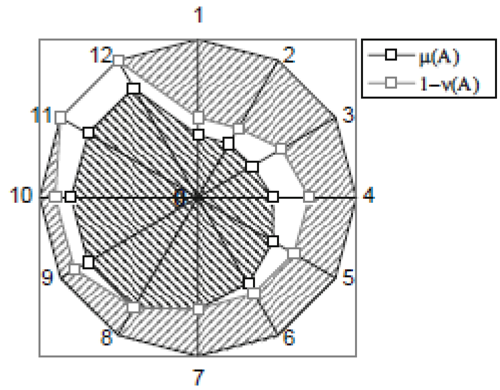

Figure 1: IFS geometrical interpretation based on radar chart

In computer programming, a loop is a sequence of instructions that is continually repeated until a certain condition is reached. A loop is a fundamental programming idea that is commonly used in writing programs. Some loop are using with data in time series with different size witch handled differently. Each pass through the loop is called an iteration. For example a loop can process data for the months of the year with 12 iterations etc.

Let us look an example for Wastewater Treatment Process. In this example, data for Wastewater Treatment is given by months. The data is for a years and it's processed with loop by months. The workload of processing for each month or loop iteration can be normalize in the range [0,1]. A formula is given below:

$$x_{new} = (x - x_{min})/(x_{max} - x_{min})$$

The results can be viewed in terms of the IFS where each result will corresponds to the membership degree and will be placed in geometrical interpretation based on radar chart. One complete cycle will be the data for a year. So far, this example can be interpret like ordinary fuzzy set where the degree of non-determinacy is zero, Figure 2.

Let us look the same example and same geometrical interpretation but with data for five years. In this case, for each month in the geometrical interperation will have five points. Now, if we connect the innermost points that reperesent the minumum workload for each month and the outermost points that represent the maximum workload for each month, the workload of the whole loop will be between these lines Figure 3.

So the workload will have a membership degree - the innermost zone, the non-membership degree - the outermost zone and the degree of uncertainty – the region between both zones.

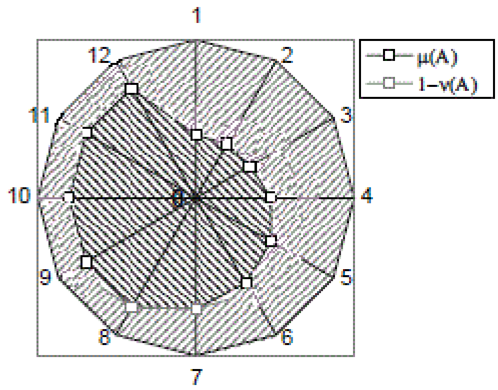

Figure 2: A workload of the Wastewater Treatment Process for a year

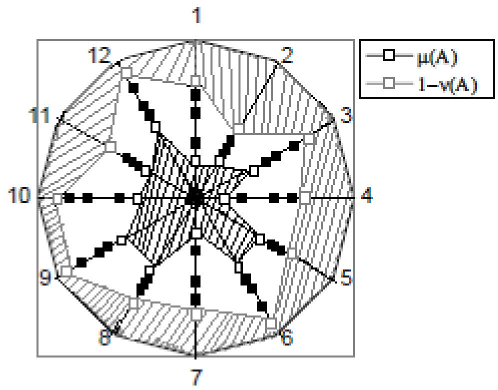

Figure 3: A workload of the Wastewater Treatment Process for 5 year

2 Intuitionistic fuzzy geometrical interpretation of the workload of the generalized nets transition

Generalized Nets (GNs) are extensions of Petri Nets [6]. They are defined in 1991 from Krasimir Atanasov [4]. They are defined in a way that is principally different from the ways of defining the other types of Petri nets. They are a tool for modelling of parallel processes.

In this section, we look at the example above but realized with Generalized Nets (GNs). The example itself and all results will be desrcibed in separate paper.

In the present paper we will discuss some possibilities to use IFS and their geometrical interpretation for assessment of the workload of the Generalizes Nets transition.

The main part of Generalized Net is called *transition* .

Formally, every transition is described by a seven-tuple:

$$Z = \langle L', L'', t_1, t_2, r, M, \Box \rangle,$$

where:

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively);

(b) t_1 is the current time-moment of the transition's firing;

(c) t_2 is the current value of the duration of its active state;

(d) r is the transition's *condition* determining which tokens will transfer from the transition's inputs to its outputs. Parameter r has the form of an IM:

$$r = \begin{array}{c|c} & l''_1 \ldots l''_j \ldots l''_n \\ \hline l'_1 & \\ \vdots & r_{i,j} \\ l'_i & (r_{i,j} \; - \; \text{predicate}) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array} ;$$

where $r_{i,j}$ is the predicate which expresses the condition for transfer from the i-th input place to the j-th output place. When $r_{i,j}$ has truth-value "$true$", then a token from the i-th input place can be transferred to the j-th output place; otherwise, this is impossible;

(e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|c} & l''_1 \ldots l''_j \ldots l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \ge 0 - \; \text{natural number or } \infty) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array} ;$$

(f) $\Box$ is called transition type and it is an object having a form similar to a Boolean expression. It may contain as variables the symbols that serve as labels for transition's input places, and it is an expression constructed of variables and the Boolean connectives $\wedge$ and $\vee$ determining the following conditions:

$\wedge(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ – every place $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$ must contain at least one token,

$\vee(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ – there must be at least one token in the set of places $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$, where $\{l_{i_1}, l_{i_2}, \ldots, l_{i_u}\} \subset L'$.

When the value of a type (calculated as a Boolean expression) is "$true$", the transition can become active, otherwise it cannot.

The GNs algorithm for transition functioning when merging of tokens is permitted and the general algorithm for the GN's functioning are described respectively in [3, 5].

Both algorithms above are cyclical and they are realized in Generalized Nets Integrated Development Environment (GN IDE). The GN IDE is a software tool, developed as a client to the GN simulation server, GNTicker.

Let us first look at the GNs algorithm for transition functioning when merging of tokens is permitted. For each iteration of the cycle sorted places are passed sequentially by their priority, starting with the place having the highest priority, which has at least one token, check if the output place is full, check if the selected token can be merged, evaluate some predicates, change the capacities of some arcs, places, the values of the characteristic function for the corresponding output place are assigned as a next token characteristic, check for ”termination of the transition functioning”.

The workload of the transition for each iteration will depend on the number of tokens witch are in the input places, characteristic functions and etc. The Wastewater Treatment Process can be simulate with an GN model where the token will have charachteristics with quantity and pollution. Each transition in the GN model will be a different treatment. The data can be pass by month, by day or hours. For this discussion the data will be treated by months.

In the example with a Wastewater Treatment Process an iteration of the transition algorithm will be different because the pollution will be different for each month and treatment will also be respectively different.

If the workload data of the transition will be normalize in the range [0,1], the result can be present with IFS geometrical interpretation. So, the workload of the each transition respectively of each treatment for five years can be present with IFS geometrical interpretation based on radar chart, where the membership degree is the innermost zone, the non-membership degree is the outermost zone and the degree of uncertainty is the region between both zones as in the Figure 3.

3 Intuitionistic fuzzy geometrical interpretation of the workload of the generalized nets functioning

Similar to the GNs algorithm for transition functioning when merging of tokens is permitted and the general algorithm for the GN’s functioning is cyclic.

The Generalized Net is called the ordered four-tuple

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, b\rangle\rangle,$$

where

(a) A is a set of transitions (see above);

(b) π_A is a function giving the priorities of the transitions, i.e., $\pi_A : A \to \mathcal{N}$;

(c) π_L is a function giving the priorities of the places, i.e., $\pi_L : L \to \mathcal{N}$, where

$$L = pr_1 A \cup pr_2 A$$

and obviously, L is the set of all GN-places;

(d) c is a function giving the capacities of the places, i.e., $c : L \to \mathcal{N}$;

(e) f is a function that calculates the truth values of the predicates of the transition’s conditions;

(f) θ_1 is a function giving the next time-moment, for which a given transition Z can be activated, i.e., $\theta_1(t) = t'$, where $pr_3 Z = t, t' \in [T, T + t^*]$ and $t \leq t'$; the value of this function is calculated at the moment when the transition terminates its functioning. Here and below $pr_i X$ is the i-th projection of the n-dimensional set X.

(g) θ_2 is a function giving the duration of the active state of a given transition Z, i.e., $\theta_2(t) = t'$, where $pr_4 Z = t \in [T, T + t^*]$ and $t' \geq 0$; the value of this function is calculated at the moment when the transition starts functioning;

(h) K is the set of the GN's tokens. In some cases, it is convenient to consider this set in the form

$$K = \bigcup_{l \in Q^I} K_l,$$

where K_l is the set of tokens which enter the net from place l, and Q^I is the set of all input places of the net;

(i) $\pi_{K,T}$ In the standard GNs $\pi_K : K \rightarrow N$. In the GNPDT $\pi_{K,T}$ is a function giving the priorities of the tokens by the token and the current time, i.e. $\pi_{K,T} : K \times [T, T + t^*] \rightarrow N$;

(j) θ_K is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

(k) T is the time-moment when the GN starts functioning; this moment is determined with respect to a fixed (global) time-scale;

(l) t^0 is an elementary time-step, related to the fixed (global) time-scale;

(m) t^* is the duration of the GN functioning;

(n) X is a function which assigns initial characteristics to every token when it enters input place of the net;

(o) Φ is a characteristic function that assigns new characteristics to every token when it makes a transfer from an input to an output place of a given transition;

(p) b is a function giving the maximum number of characteristics witch a given token can receive, i.e., $b : K \rightarrow N$.

The general algorithm for the GN's functioning check the value of the current time, check all transitions for which the time-component is exactly equal to the current time-moment, check the transition's types of all transitions, add all transitions which the transition types are satisfied, apply algorithm from previous section over the transition, remove all transitions which are inactive at the current time moment and increase the current time.

Similarly to the previous section, here can be introduce IFS geometrical interpretation of the workload of the Generalized Nets functioning. The workload of the Generalized Nets functioning for a month will be the average from the workload of the Generalized Nets functioning during all tokens for a month does not enter and leave the GNs model.

In the example with the Wastewater Treatment Process an iteration of the transition algorithm will be different because the pollution will be different for each month and treatment will also be respectively different.

Again, if the workload data of the transition will be normalize in the range [0,1], the result can be present with IFS geometrical interpretation. So, the workload of the model or respectively of whole Wastewater Treatment Process for five years can be present with IFS geometrical interpretation based on radar chart, where the membership degree is the innermost zone, the non-membership degree is the outermost zone and the degree of uncertainty is the region between both zones as in the Figure 3.

References

[1] Atanasov, K., *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin, 2012.

[2] Atanassova, V., Representation of fuzzy and intuitionistic fuzzy data by Radar charts. *Notes on Intuitionistic Fuzzy Sets*, Vol. 16, 2010, No. 1, 21–26.

[3] Andonov V., N. Angelova, Modifications of the algorithms for transition functioning in GNs, GNCP, IFGNCP1 and IFGNCP3 when merging of tokens is permitted (in press).

[4] Atanassov, K., *Generalized Nets*. World Scientific, Singapore, 1991.

[5] Atanassov, K., *On Generalized Nets Theory*. "Prof. M. Drinov" Academic Publ. House, Sofia, 2007.

[6] Murata, T., Petri Nets: Properties, Analysis and Applications. *Proceedings of the IEEE*. Vol. 77, 1989, No. 4, 541–580.