Issues in IFSs and GNs, Vol. 12, 2015/2016, 114–128

# Generalized Net Model for the Diagnosis of Asymptomatic Osteoporosis

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**Abstract:** In a previous paper (see [10]), Generalized net model of the process of diagnosing asymptomatic osteoporosis has been proposed. On the basis of this model, here we propose a model in which Generalized net with characteristics of the places is used. The two models are compared. **Keywords and phrases:** Generalized nets, Asymptomatic osteoporosis. **2000 Mathematics Subject Classification:** 68Q85.

# **1** Introduction

Generalized Nets (GNs; see [8]) have been applied to healthcare delivery systems, general and internal medicine. Many GN models are built which represent various types of organizational and patient workflows, diseases, symptoms and treatments, organs or states of human body. In fact, there is a proof that every dynamical system and every collection of dynamical systems can be described by a GN (see [5]). As it is shown in [6, 7, 11, 14, 15, 16], the GN models in medicine can be used for:

• simulation of real processes with educational aims;

- control of the corresponding hospital processes in real time;
- prognosis of the actual processes in hospital for the purposes of the hospital administration.

These models can also help:

- specialists in studying the logic of the processes related to diagnoses;
- medical students and new specialists in acquiring knowledge and diagnostic skills;
- lecturers in medical students examinations with real-time simulations;
- administrative personnel in taking decisions related to planning, management, organization and allocation of the available resources (materials, specialized apparatuses, personnel) and scheduling of the medical specialists.

Examples of GN models for medical diagnoses can be found in [12, 13]. A GN model of asymptomatic osteoporosis diagnosing is proposed in [10]. It has 8 transitions, 27 places and 8 types of tokens. The graphic representation of the net is shown in Figure 1.



Figure 1. GN model of asymptomatic osteoporosis diagnosing

In the present paper we propose an alternative model for the diagnosis of asymptomatic osteoporosis. In it, a recently introduced extension of the standard GNs is used.

# 2 A Short Note on Generalized Nets with Characteristics of the Places

During the years many extensions of the standard GNs have been defined [8]. In one of the most recent extensions – Generalized Nets with Characteristics of the Places (GNCP), defined in [2], some places can obtain characteristics during the functioning of the net. For more about the concept of GNCP refer to [1, 4].

A GNCP E is the ordered four-tuple

 $E = \langle \langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, t^0, t^* \rangle, \langle X, Y, \Phi, \Psi, b \rangle \rangle.$ 

All other components except the characteristic functions Y and  $\Psi$  are the same as in the standard GNs (see [8]). Here the characteristic function  $\Psi$  assigns characteristics to the places when tokens enter them and Y assigns initial characteristics to the places. These characteristics can be the number of tokens from each type in the place, the moments of time when they entered the place or other data that is relevant to the place.

In [2] it is proved that the class of all GNCP –  $\Sigma_{CP}$  – is a conservative extension of the class  $\Sigma$  of the ordinary GNs. Since the functioning and the results of the work of every GNCP can be described by an ordinary GN, it is important to know when it would be better to use a GNCP model instead of a GN model. The possibility of assigning characteristics to the places in GNCP can be used to construct nets with simpler graphic representation and less types of tokens. Also, the work of the places of the net in some sense based on their characteristics can be evaluated.

As a simple application of GNCP, consider a transition such that one or more of its places are both input and output. This is the case with place  $l_c$  for the transition Z in Figure 2.

In some models we have tokens that loop in such places and do not receive any characteristics. Instead their initial characteristic gives some criteria or other data which is used in the model. In other models we have tokens that loop in this place and receive new characteristics but they are not transferred to other output places for the transition. In such cases we can exclude the place  $l_c$  from the set of input places of the transition. When tokens enter the place the characteristics of the tokens in  $l_c$  will be assigned to the place in the sense of GNCP. Let the components of the transition be  $Z = \langle L', L'', t_1, t_2, r, M, \Box \rangle$ .



Figure 2. A sample transition with loop in place  $l_c$ .

The restriction that no tokens can be transferred from place  $l_c$  to other output places is given by the index matrix r of the transition's condition. In this case we have  $r = [L', L'', \{r_{l_i,l_j}\}]$ , where  $r_{l_i,l_j}$  is the predicate corresponding to the *i*-th input and *j*-th output place of the transition and  $(\forall l_j \in L'')(r_{l_c,l_j} = "false")$ . We can substitute every such transition Z with a new transition  $Z^*$  (see Figure 3) which has the same places as Z but  $l_c$  is not input place for  $Z^*$ .



Figure 3. A sample transition with place  $l_c$  not being an input.

Since  $l_c$  is not input place for  $Z^*$  there is no arc going out of place  $l_c$ . The characteristics that are assigned to the tokens in place  $l_c$  of the original transition Z are assigned to the place  $l_c$  in the sense of GNCP. The two concentric circles representing place  $l_c$  are used to denote that the place can receive characteristics. The case in which there are more places of the same kind as  $l_c$  is analogous.

There are other situations, as we shall see in the next section, where through the use of GNCP the graphical representation of the net can be significantly simplified. For other applications of GNCP which are not relevant to this article the reader can refer to [3].

# **3** GNCP Model for the Diagnosis of Asymptomatic Osteoporosis

In this section, we use the GN model proposed in [10] to construct a GNCP model for the diagnosis of asymptomatic osteoporosis. The graphical representation of the net is in Figure 4.

The GNCP has 8 transitions which represent different stages of the diagnosing process.

- $Z_1$  represents the personal data of the patient.
- $Z_2$  represents the results from the OST (osteoporosis self-assessment tool) and questionnaire for risk factors.
- $Z_3$  represents the results from history and physical examination.
- $Z_4$  represents the set of BMD (bone mineral density) measurement tools.
- $Z_5$  represents the results from DXA(dual X-ray absorptiometry).
- $Z_6$  represents the results from QUS(quantitative ultrasound) or/and X-ray.
- $Z_7$  represents the final diagnosis.
- $Z_8$  represents the possible differential diagnosis and underlying causes of disease.



Figure 4. GNCP model of asymptomatic osteoporosis diagnosing

Token  $\alpha$  representing a patient enters the net in place  $l_1$  with initial characteristic:

"asymptomatic patient over 50 years of age".

Places  $l_3$ ,  $l_6$ ,  $l_{10}$ ,  $l_{13}$ ,  $l_{17}$  and  $l_{20}$  obtain characteristics during the functioning of the net as follows:

- Place *l*<sub>3</sub> collects the overall information obtained from the diagnostics steps in the personal record (personal data).
- Place  $l_6$  collects information about the OST and questionnaire.
- Place  $l_{10}$  collects information about the current functional status of the patient obtained from the history and physical examination.
- Place  $l_{13}$  collects information about the possible BMD measurement tools.
- Place  $l_{17}$  collects information about the results from DXA.
- Place  $l_{20}$  collects information about the results from QUS and/or X-ray.
- Place  $l_{22}$  collects information about the interpretation of the results from the imaging tests.

Transition  $Z_1$  has the following form:

$$Z_1 = \langle \{l_1, l_{10}, l_{22}\}, \{l_2, l_3\}, r_1, \Box_1 \rangle$$

where

$$r_1 = \frac{\begin{array}{c|cccc} l_2 & l_3 \\ \hline l_1 & true & true \\ l_{10} & false & true \\ l_{22} & false & true \end{array}}$$

and

$$\Box_1 = \lor (l_1, l_{10}, l_{22}).$$

The token  $\alpha$  in place  $l_1$ , representing the current patient, splits into two identical tokens which enter places  $l_2$  and  $l_3$ . In place  $l_2$  the token obtains characteristic:

"OST and questionnaire for risk factors are necessary".

When a token from at least one of the input places of the transition enters place  $l_3$  the place obtains the characteristic:

"name of the current patient, personal record (personal data)".

Transition  $Z_2$  has the following form:

$$Z_2 = \langle \{l_2\}, \{l_4, l_5, l_6\}, r_2 \rangle,$$

where

$$r_2 = \frac{l_4 \quad l_5 \quad l_6}{l_2 \quad W_{2,4} \quad W_{2,5} \quad true}$$

and

- $W_{2,4}$  = "the score from OST is 10 or greater and the score from questionnaire is between 0-6 points";
- $W_{2,5}$  = "the score from OST is less than 10 and the score from questionnaire is above 6 points".

When the truth value of the predicate  $W_{2,4}$  is "true" the token in place  $l_2$  splits into two identical tokens one of which enters place  $l_6$  and the other one enters place  $l_4$  where it obtains the characteristic:

# "there is a low risk of osteoporosis and no further immediate action is needed".

When the truth value of the predicate  $W_{2,5}$  is "true" the token in place  $l_2$  splits into two identical tokens one of which enters place  $l_6$  and the other one enters place  $l_5$  where it obtains the characteristic:

"there is an increased risk of osteoporosis and detailed history, physical examination and diagnosing tests are necessary".

The token entering place  $l_6$  does not obtain characteristic. Place  $l_6$  obtains the characteristic:

"information about the OST and questionannaire".

Transition  $Z_3$  has the following form:

$$Z_3 = \langle \{l_5\}, \{l_7, l_8, l_9, l_{10}\}, r_3 \rangle,$$

where

$$r_3 = \frac{l_7 \quad l_8 \quad l_9 \quad l_{10}}{l_5 \quad W_{5,7} \quad W_{5,8} \quad W_{5,9} \quad true}$$

and

- $W_{5,7}$  = "there are key risk factors for osteoporosis";
- $W_{5,8}$  = "there is a history of prior low-impact fragility fracture";
- $W_{5,9}$  = "the history and physical examination do not show critical signs of osteoporosis, no further diagnosing tests are necessary".

When the truth value of the predicate  $W_{5,7}$  is "true" the token in place  $l_5$  splits into two identical tokens one of which enters place  $l_{10}$  and the other one enters place  $l_7$  where it obtains the characteristic:

"consider BMD measurements".

When the truth value of the predicate  $W_{5,8}$  is "true" the token in place  $l_5$  splits into two identical tokens one of which enters place  $l_{10}$  and the other one enters place  $l_8$  where it obtains the characteristic:

When the truth value of the predicate  $W_{5,9}$  is "true" the token in place  $l_5$  splits into two identical tokens one of which enters place  $l_{10}$  and the other one enters place  $l_9$  where it obtains the characteristic:

"inform patient and give leaflet on bone health".

Upon entering place  $l_{10}$  the token coming from place  $l_5$  obtains the characteristic:

"results from physical examination and diagnostic tests".

Place  $l_{10}$  obtains the characteristics:

*"information about the current functional status of the patient obtained from the history and physical examination".* 

Transition  $Z_4$  has the following form:

$$Z_4 = \langle \{l_7, l_8\}, \{l_{11}, l_{12}, l_{13}\}, r_4, \Box_4 \rangle,$$

where

$$r_4 = \frac{\begin{array}{c|cccc} l_{11} & l_{12} & l_{13} \\ \hline l_7 & W_{7,11} & W_{7,12} & true \\ \hline l_8 & W_{8,11} & W_{8,12} & true \end{array}}$$

and

- $W_{7,11} = W_{8,11} =$  "the patient is suitable and agreed for DXA testing";
- $W_{7,12} = W_{8,12} = "DXA is unavailable".$

When the truth value of the predicate  $W_{7,11}$  is "true" the token from place  $l_7$  (or  $l_8$ ) splits into two tokens one of which enters place  $l_{13}$  and the other one enters place  $l_{11}$ . In place  $l_{11}$  the token obtains the characteristic:

"perform a DXA".

When the truth value of the predicate  $W_{7,12}$  is "true" the token from place  $l_7$  (or  $l_8$ ) splits into two tokens one of which enters place  $l_{13}$  and the other one enters place  $l_{12}$ . In place  $l_{12}$  the token obtains the characteristic:

"perform a QUS and/or X-ray".

The tokens entering place  $l_{13}$  obtain the characteristic:

#### "results from BMD".

Place  $l_{13}$  obtains the characteristic:

"information about possible BMD tools".

The transition's type is:

$$\Box_4 = \lor (l_7, l_8).$$

Transition  $Z_5$  has the following form:

$$Z_5 = \langle \{l_{11}\}, \{l_{14}, l_{15}, l_{16}, l_{17}\}, r_5 \rangle,$$

where

$$r_5 = \frac{l_{14} \quad l_{15} \quad l_{16} \quad l_{17}}{l_{11} \quad W_{11,14} \quad W_{11,15} \quad W_{11,16} \quad W_{11,17}}$$

and

- $W_{11,14} =$  "the result from DXA is  $T score \ge -1$ ";
- $W_{11,15} =$  "the result from DXA is  $T score \leq -2.5$ ";
- $W_{11,16} =$  "the result from DXA is -2.5 < T score < -1".

When the truth of value of the predicate  $W_{11,14}$  is "true" the token in place  $l_{11}$  splits into two tokens one of which enters place  $l_{17}$  and the other one enters place  $l_{14}$ . In place  $l_{14}$  the token obtains the characteristic:

#### "exclude osteoporosis".

When the truth of value of the predicate  $W_{11,15}$  is "true" the token in place  $l_{11}$  splits into two tokens one of which enters place  $l_{17}$  and the other one enters place  $l_{15}$ . In place  $l_{15}$  the token obtains the characteristic:

"consider osteoporosis".

When the truth of value of the predicate  $W_{11,16}$  is "true" the token in place  $l_{11}$  splits into two tokens one of which enters place  $l_{17}$  and the other one enters place  $l_{16}$ . In place  $l_{16}$  the token obtains the characteristic:

#### "consider osteopenia".

Upon entering place  $l_{17}$  the tokens do not obtain any characteristic. Place  $l_{17}$  obtains the characteristic:

Transition  $Z_6$  has the following form:

$$Z_6 = \langle \{l_{12}\}, \{l_{18}, l_{19}, l_{20}\}, r_6 \rangle,$$

where

$$r_6 = \frac{\begin{array}{c|cccc} l_{18} & l_{19} & l_{20} \end{array}}{\begin{array}{c|ccccc} l_{12} & W_{12,18} & W_{12,19} & true \end{array}}$$

and

- $W_{12,18} =$  "the results from QUS and/or X-ray are normal";
- $W_{12,19} = "QUS and/or X-ray show bone stiffness and reduced bone mass".$

When the truth of value of the predicate  $W_{12,18}$  is "true" the token in place  $l_{12}$  splits into two tokens one of which enters place  $l_{20}$  and the other one enters place  $l_{18}$ . In place  $l_{18}$  the token obtains the characteristic:

"follow-up observation is necessary".

When the truth of value of the predicate  $W_{12,19}$  is "true" the token in place  $l_{12}$  splits into two tokens one of which enters place  $l_{20}$  and the other one enters place  $l_{19}$ . In place  $l_{19}$  the token obtains the characteristic:

"consider osteopenia and further testing".

Upon entering place  $l_{20}$  the tokens do not obtain any characteristic. Place  $l_{20}$  obtains the characteristic:

"results from QUS and/or X-ray".

Transition  $Z_7$  has the following form:

$$Z_7 = \langle \{l_{15}, l_{16}, l_{19}\}, \{l_{21}, l_{22}\}, r_7, \Box_7 \rangle,$$

where

$$r_{7} = \frac{\begin{array}{c|cccc} l_{21} & l_{22} \\ \hline l_{15} & W_{15,21} & true \\ l_{16} & W_{16,21} & true \\ \hline l_{19} & W_{19,21} & true \end{array}}$$

and

•  $W_{15,21} = W_{16,21} = W_{19,21} =$  "the asymptomatic osteoporosis is confirmed".

When the truth of value of the predicate  $W_{15,21}$  is "true" the token in place  $l_{15}$  (or  $l_{16}$ , or  $l_{19}$ ) splits into two tokens one of which enters place  $l_{22}$  and the other one enters place  $l_{21}$ . In place  $l_{21}$  the token obtains the characteristic:

"laboratory tests are necessary".

Upon entering place  $l_{22}$  the token obtains the characteristic:

"set of results from the imaging tests (the data for personal record of the patient)".

Place  $l_{22}$  obtains the characteristic:

"information about the interpretation of the results from the imaging tests".

The transition's type is:

$$\Box_7 = \lor (l_{15}, l_{16}, l_{19}).$$

Transition  $Z_8$  has the following form:

$$Z_8 = \langle \{l_{21}\}, \{l_{23}, l_{24}, l_{25}\}, r_8 \rangle,$$

where

$$r_8 = \frac{l_{23} \quad l_{24} \quad l_{25}}{l_{21} \quad W_{21,23} \quad W_{21,24} \quad W_{21,25}}$$

and

- $W_{21,23} =$  "vitamin D test shows: 25-hydroxyvitamin D levels of ; 20 nanograms/mL";
- $W_{21,24} =$  "serum PTH levels are elevated and/or high levels of parathyroid hormone and/or abnormal levels of thyroid hormone";
- $W_{21,25} =$  "there is osteomalacia".

When the truth of value of the predicate  $W_{21,23}$  is "true" the token in place  $l_{21}$  enters place  $l_{23}$  with characteristic:

"consider vitamin D deficiency".

When the truth of value of the predicate  $W_{21,24}$  is "true" the token in place  $l_{21}$  enters place  $l_{24}$  where it obtains the characteristic:

"consider hyperthyroidism or hyperparathyroidism".

When the truth of value of the predicate  $W_{21,25}$  is "true" the token in place  $l_{21}$  enters place  $l_{25}$  where it obtains the characteristic:

"bone biopsy is necessary".

## 4 Conclusions

The GNCP model for diagnosis of asymptomatic osteoporosis proposed here has 8 transitions, 25 places and all tokens are of one type. In the GN model from [10], there are 8 transitions, 27 places and 8 different types of tokens. In every transition of the GN (See Figure 1), except  $Z_8$ , there is a place which is both input and output for the respective transition. In the GNCP in Figure 4 there are no places which are both input and output for the transitions. As a result, the number of arcs in the graphic representation of the net is reduced. The use of characteristics of the places allowed places  $l_5$  and  $l_6$  of transition  $Z_2$  and places  $l_{23}$  and  $l_{24}$  of  $Z_7$  to be replaced by just one output place for the transitions in the GNCP model. Thus transitions  $Z_3$  and  $Z_7$  of the GNCP have one output place less than their corresponding transitions in the GN.

## Acknowledgements

The work presented here is partially supported by the "Program for career development of young scientists, BAS", Grant number DFNP-142/2016.

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