Intuitionistic fuzzy estimations of biological interactions

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Abstract

All living organisms sustain their life, reproduce and evolve through interaction with the biotic and abiotic components of their ecosystems. Therefore the mathematical formalism chosen to describe interactions generally predetermines the scope and properties of the ecological models it is part of. The goal of the present paper is to introduce Intuitionistic fuzzy estimations of the interaction between two living organisms capable of aggregating the positive and negative aspects of the interaction as well as the uncertainty with which they can be determined. Using the concept of intuitionistic fuzzy interaction new continuous formulation of the six basic types of interactions (neutralism, amensalism, commensalism, competition, mutualism, predation or parasitism) is presented. This formulation is capable of incorporating different sources of uncertainty, including the ambiguity in the discrimination of direct and environment mediated effects. In the special case when the uncertainty of interaction between two living organisms comes from unaccounted small changes in their relatively constant environment a method for estimation the interaction components based on observations of interacting objects is provided. A simple averaging algorithm is presented for generalizing the results of multiple observations. Future extensions and possible applications of the intuitionistic fuzzy estimations of biological interactions are also discussed.

Introduction

According to Ellis and Ramankutty (2008) more than 75% of the ice free Earth surface is altered as a result of human activities. The more extreme and faster these changes are the more off equilibrium the systems become. First to adapt to the new conditions are the most abundant and simplest short lifecycle life forms (like bacteria and microorganisms) which affect the food web and along with initiating factors the whole ecosystem. The key to understanding how organisms adapt or maintain their existence in a pristine natural or anthropogenic ecosystem is to understand how they interact with the environment and with other species. Such understanding will help us reconnect to the intricate web of life and benefit from the evolutionary gained knowledge of how to live together, utilise and recycle resources more efficiently, and become active self aware stability factors in our own ecosystems. Brief review some of the existing ecological theories directly involving the notion of interaction is provided below.

Replicator model is widely used for modelling the population dynamics and evolution of traits. Interactions in this model change the quantity of one species after encountering another. Populations are presumed to be well mixed and every species interact with every other species with fixed in time interaction strength. The model uses predefined interaction strengths in the replicator differential equation to determine species concentrations change. Different approaches are employed for assigning interaction strengths from random (Diederich and Opper 1989) trough presumption based i.e. Hebb rule (Poderoso, 2005) to directly measured (Fasham and Evans 1995; Berlow et al. 1999; Wootton and Emmerson 2005). In the classical replicator model only trophic (food web related) interactions are considered. Increasing number of studies (Cardinale et al. 2002, Snyder and Prasad 2010) suggest a significant role of non-trophic interactions which led some researches to introduce them into replicator model in the form of interaction modifications (Fontanari and de Olivera 2000;Goudard and Loreau 2008). Another extension of replicator model is the ability of the model to evolve interaction strength on a different time scale of the fast changes in the population dynamics (Poderoso and Fontanari 2007). Despite the recent developments the replicator model still possess some limitations emanated from the original assumptions that all species interact with fixed frequency in time and space. These two limitations are much better treated by another wide spread theory for modelling population dynamics cellular automata.

A cellular automata (Wolfram, 1986) is a grid of identical cells each of which can be in one of finite number of states. States are synchronously updated in function of cell's state and the state of neighbouring cells. Cells can represent individuals or local populations (Darwen and Green, 1996) and functions can represent individual or species interactions (Hoekstra et al. Ed 2010), fertility, mortality, migration and other. Comprehensive review of cellular automata applications is presented in Molofsky and Bever (2004). They separate the application of cellular automata in two broad categories: in illustrating theoretical consequences of different rules and empirical applications utilizing parameterized rule systems to model experimental observations. Cellular automata have proved to be very flexible and computationally efficient tool for investigation the various aspects of interspecies interactions, yet so far models have been build on a case by case basis with rules designed specifically for the considered set of species. Development of universal models is hampered by the definition of cellular automata in which each cell have to be identical and therefore the more species and possible interaction interactions are considered the more complex the definition of the cells become.

Another important property of natural interactions is their perceived uncertainty and variability. While both of these properties could be just the result of our limited knowledge of all contributing factors they still need to be considered when building models of biological interactions. Variability – different behaviour under seemingly preserved conditions is modelled by probabilities. In cellular automata models this is expressed as probabilistic rules (Wootton 2001) and in replicator model as random interactions. There are also entirely probabilistic models for predicting the population dynamics (Holland et al. 2009). Uncertainty in generally modelled as fuzziness. In cellular automata models is introduced in terms of fuzzy rules and states (Reiter 2002; Bone et al. 2005;). In niches theory, which models species interactions implicitly, fuzziness is also used to describe the degree of overlapping between niches (Cao 1995; Yimin and Hua, 2008).

All of the methods above are designed to model forward problems in ecology i.e. finding the state of ecosystem, when species and their interactions are known and understood. The inverse problem (Bellman et al. 1965) - quantify the interactions when observations of the ecosystem state are provided, proved to be even more challenging since discrete experimental observations has to be incorporated in often continuous theoretical models. A systematic overview of the origin of the major current methods is offered by Laska and Wootton (1998). The four basic theoretical models described in the overview are: community matrix, Jacobian matrix, Inverted Jacobian matrix and removal matrix. Community matrix represents the direct effect an individual from one species has on individual of other species. It can be derived from generalized Lotka-Voltera equation (Levin, 1968; Bender et al. 1984). This approach has been extensively studied (Mobley 1973; Kokkoris and Jansen 2003; Dambacher et al. 2003) and also applied to interacting populations instead of individuals. The elements of Jacobain matrix represent the direct effect on an individual of one species on the total population of another species at or near equilibrium (Bender et al. 1984; Neuhauser and Fargione, 2004). While community and Jacobian matrices account only for direct effects on one species to another the inverted Jacobian matrix also accounts for indirect interactions (Moon and Moon 2011). This matrix describes the total direct and indirect effects of one species to a constant rate of removal or addition of another species (Pine, 1992; Wootton, 1994). The removal matrix measures the interaction strength as a result of complete species removal. Each element in the matrix represents the difference in equilibrial abundance between a community with all species present, and the same community with one species removed. Various nontrivial extensions of those four major methods has been developed (Gleria et al. 2005; Dimakis and Müller-Hoissen 2002) as well as some principally different approaches (Dowd 2003) has been introduced. A common assumption in all of these methods is that the structure of the interaction model is known and only the interaction coefficients have to be determined. While this could be the case in some well studied simple ecosystems in general it is a limiting factor for the complex natural and anthropogenic systems. This outlines the need of a general model which can define the individual interaction strengths with no previous assumptions based on observations only.

The goal of the present paper is to introduce a formal description of the interaction between two living organisms capable of aggregating the mixture of positive and negative effects rendered from the one organism to the other and the ambiguity in the discrimination of direct and environment dependent components of the interaction.

Intuitionistic fuzzy estimations of biological interaction

Let $\Omega_{S+x}^{x} \in [0,1]$ be the health status of a living organism x in the presence of the set of objects, $S = \{z|z \in \aleph\}$, where 0 corresponds to dead organism and 1 to organism in perfect health, and S^{+x} denotes the set of objects S combined with the object x, or in general case

$$S^{+x_1,x_2,\dots,x_m} = \{z|z \in \aleph\} \cup \{x_1, x_2, \dots, x_m\}.$$

Then the ***effect*** of the object $y \in \aleph$ over the living organism $x \in \aleph$ in the presence of the objects in S could be described as the intuitionistic fuzzy number

$$\alpha_{S\langle x,y\rangle} = \langle \mu_S(\langle x,y\rangle), \vartheta_S(\langle x,y\rangle)\rangle,$$

where: $\langle x,y\rangle$ is the ordered tuple of the two interacting objects, $\mu_s: \aleph^2 \to [0,1]$ is the positive effect of y over x in the presence of S, $\vartheta_s: \aleph^2 \to [0,1]$ is the negative effect of y over x in the presence of S, and $0 \le \mu_s(\langle x,y\rangle) + \vartheta_s(\langle x,y\rangle) \le 1$. Level of uncertainty $\pi: \aleph^2 \to [0,1]$ can be defined as $\pi_s(\langle x,y\rangle) = 1 - \mu_s(\langle x,y\rangle) - \vartheta_s(\langle x,y\rangle)$.

The ***interaction*** between two objects $x, y \in \aleph$ in the presence of the objects in S is defined by the tuple

$$I_S(x,y) = \left(\alpha_{S\langle x,y\rangle}, \alpha_{S\langle y,x\rangle}\right).$$

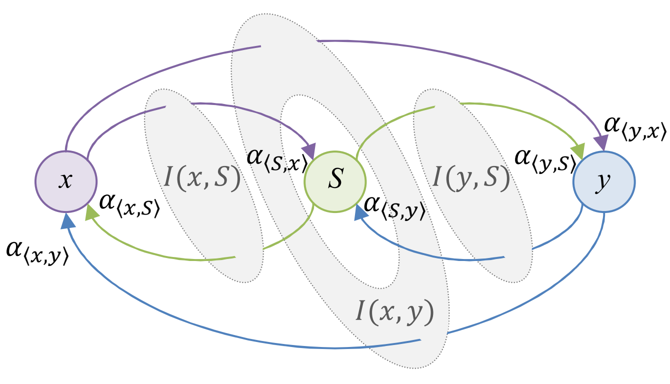
The ***pure*** or ***direct*** effect of y over x can be defined as the effect observable in the constant environment $\bar{S}$:

$$\alpha_{\bar{S}\langle x,y\rangle} = \alpha_{\langle x,y\rangle} = \langle \mu(\langle x,y\rangle), \vartheta(\langle x,y\rangle)\rangle,$$

The ***constant environment*** is $\bar{S}$ such an environment which is not affected by the presence of x and y, or formally:

$$\forall(z \in S), \Omega_{S+x,y}^{z} = \Omega_{S+y}^{z} = \Omega_{S+x}^{z} = \Omega_{S}^{z}, \text{ and}$$

$$\forall(z_1, z_2 \in S), I_S(z_1,z_2) = I_{S^{+x}}(z_1,z_2) = I_{S^{+y}}(z_1,z_2) = I_{S^{+x,y}}(z_1,z_2).$$



The interaction $I_S(x,y)$ is the cumulative effect of all direct ineractions in time and space between *x, y* and *S* (see fig.1). In general case this could be very comlicated and mutually dependent relationship which can not be easily divided into inependent components. In the special case when *S* could be treated as relatively unchanged set of objects as a result of presnece of *x* and *y* then the interaction between *x* and *y* in the enviroment of *S* is close to direct ineraction. In this case we can assume that all uncertainty π_s for the direct interaction $I(x,y)$ comes from the unaccounted small changes in S. This case provides a frameowrk with a great practical significance where pure interactions can be studied by comparing the health indeces Ω_{S+x}^{x} and Ω_{S+y}^{y} of x and y living by themselves in the enviroment S with thir health indices $\Omega_{S+x,y}^{x}$ and $\Omega_{S+x,y}^{y}$ wnen living together. One possible way to estimate the direct effect $\alpha_{\bar{S}\langle x,y\rangle}$ is

$$\mu_{\tilde{S}\langle x,y\rangle} = \left(\frac{1}{2} + \frac{\Delta\Omega_{\tilde{S}}^{x}}{2 * \max\left(\Omega_{\tilde{S}+x}^{x}, 1 - \Omega_{\tilde{S}+x}^{x}\right)}\right)\left(1 - \pi_{\tilde{S}}(\langle x, y\rangle)\right)$$

$$\vartheta_{\tilde{S}\langle x,y\rangle} = \left(\frac{1}{2} - \frac{\Delta\Omega_{\tilde{S}}^{x}}{2 * \max\left(\Omega_{\tilde{S}+x}^{x}, 1 - \Omega_{\tilde{S}+x}^{x}\right)}\right)\left(1 - \pi_{\tilde{S}}(\langle x, y\rangle)\right)$$

where

$$\Delta\Omega_{\tilde{S}}^{x} = \Omega_{\tilde{S}+x,y}^{x} - \Omega_{\tilde{S}+x}^{x}$$

and $\pi_{\tilde{S}}(\langle x, y\rangle) = \max\left(\left|\mu\left(\langle\tilde{S}, x\rangle\right) - \vartheta\left(\langle\tilde{S}, x\rangle\right)\right|, \left|\mu\left(\langle\tilde{S}, y\rangle\right) - \vartheta\left(\langle\tilde{S}, y\rangle\right)\right|\right)$ - one possible measure of the changes in $\tilde{S}$ as a result of presence of x and y. In the expressions above $\tilde{S}$ denotes ***relatively constant environment*** which formally can be defined as following:

$$\forall(z \in S), \Omega_{S+x,y}^{z} \cong \Omega_{S+y}^{z} \cong \Omega_{S+x}^{z} \cong \Omega_{S}^{z}, \text{ and}$$

$$\forall(z_1, z_2 \in S), I_S(z_1, z_2) \cong I_{S+x}(z_1, z_2) \cong I_{S+y}(z_1, z_2) \cong I_{S+x,y}(z_1, z_2).$$

The estimation method above presumes maximum activity and represents neutral effects ($\Delta\Omega_{\tilde{S}}^{x}$) with their equal maximal values $\mu_{\tilde{S}\langle x,y\rangle} = \vartheta_{\tilde{S}\langle x,y\rangle} = 0.5$, when $\pi_{\tilde{S}}(\langle x, y\rangle) = 0$.

The formal classification of biological interactions is given in table. 1. Naturally occurring interactions often does not exclusively fall in a single category but are rather located in the continuum between any of the six main categories. This fact is well accommodated by their intuitionistic fuzzy definition which also provides additional parameter π quantifying the uncertainty of the described relationship. It also must be noted that these relationships are defined as pure interactions in a constant environment $\bar{S}$ (the index in the definitions is omitted for better readability)

Interaction type	**Description**	**Effect on x**	**Effect on y**	**Intuitionistic Fuzzy Definition**	***Extreme crisp case*** $\pi(\langle x,y\rangle) = 0$, $\pi(\langle y,x\rangle) = 0$
Neutralism	Neutralism describes the relationship between two objects which interact but do not affect each other.	0	0	$\begin{cases}\mu(\langle x,y\rangle) = \vartheta(\langle x,y\rangle)\\ \mu(\langle y,x\rangle) = \vartheta(\langle y,x\rangle)\end{cases}$	$\begin{cases}\mu(\langle x,y\rangle) = 0.5\\ \vartheta(\langle x,y\rangle) = 0.5\\ \mu(\langle y,x\rangle) = 0.5\\ \vartheta(\langle y,x\rangle) = 0.5\end{cases}$
Amensalism	Amensalism between two objects x, y involves y impeding the success of x while the x has no effect on y	-	0	$\begin{cases}\mu(\langle x,y\rangle) < \vartheta(\langle x,y\rangle)\\ \mu(\langle y,x\rangle) = \vartheta(\langle y,x\rangle)\end{cases}$	$\begin{cases}\mu(\langle x,y\rangle) = 0\\ \vartheta(\langle x,y\rangle) = 1\\ \mu(\langle y,x\rangle) = 0.5\\ \vartheta(\langle y,x\rangle) = 0.5\end{cases}$
Commensalism	Commensalism between two objects x, y occurs when x benefits from y, while x has no effect on y	+	0	$\begin{cases}\mu(\langle x,y\rangle) > \vartheta(\langle x,y\rangle)\\ \mu(\langle y,x\rangle) = \vartheta(\langle y,x\rangle)\end{cases}$	$\begin{cases}\mu(\langle x,y\rangle) = 1\\ \vartheta(\langle x,y\rangle) = 0\\ \mu(\langle y,x\rangle) = 0.5\\ \vartheta(\langle y,x\rangle) = 0.5\end{cases}$

Competition	Competition is an interaction between two objects that is mutually detrimental.	-	-	$\begin{cases}\mu(\langle x,y\rangle) < \vartheta(\langle x,y\rangle)\\ \mu(\langle y,x\rangle) < \vartheta(\langle y,x\rangle)\end{cases}$	$\begin{cases}\mu(\langle x,y\rangle) = 0\\ \vartheta(\langle x,y\rangle) = 1\\ \mu(\langle y,x\rangle) = 0\\ \vartheta(\langle y,x\rangle) = 1\end{cases}$
Mutualism	Mutualism is an interaction between two objects, which is mutually beneficial.	+	+	$\begin{cases}\mu(\langle x,y\rangle) > \vartheta(\langle x,y\rangle)\\ \mu(\langle y,x\rangle) > \vartheta(\langle y,x\rangle)\end{cases}$	$\begin{cases}\mu(\langle x,y\rangle) = 1\\ \vartheta(\langle x,y\rangle) = 0\\ \mu(\langle y,x\rangle) = 1\\ \vartheta(\langle y,x\rangle) = 0\end{cases}$
Predation or Parasitism	Predation or Parasitism between two x, y organisms is when x benefits at the expense of the y	+	-	$\begin{cases}\mu(\langle x,y\rangle) > \vartheta(\langle x,y\rangle)\\ \mu(\langle y,x\rangle) < \vartheta(\langle y,x\rangle)\end{cases}$	$\begin{cases}\mu(\langle x,y\rangle) = 1\\ \vartheta(\langle x,y\rangle) = 0\\ \mu(\langle y,x\rangle) = 0\\ \vartheta(\langle y,x\rangle) = 1\end{cases}$

The more complex interacting organisms are and richer their constant environment is, the more individual differences and the accuracy of observational health index estimations can influence the measure of the pure interaction. Therefore a simple algorithm can be applied to smooth those differences and combine them in an average estimate. This method can be used when all interactions take place in the same constant environment. For simplicity as in above definitions the index $\bar{S}$ will be omitted from the description below.

We can construct the following Indexed matrix

$$\begin{array}{c|cccc} & O_1 & O_2 & \cdots & O_m \\ \hline O_1 & \alpha_{\langle 1,1\rangle} & \alpha_{\langle 2,1\rangle} & \cdots & \alpha_{\langle m,1\rangle} \\ O_2 & \alpha_{\langle 1,2\rangle} & \alpha_{\langle 2,2\rangle} & \cdots & \alpha_{\langle m,2\rangle} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_m & \alpha_{\langle 1,m\rangle} & \alpha_{\langle 2,m\rangle} & \cdots & \alpha_{\langle m,m\rangle} \end{array}$$

where $O_1, O_2, \cdots O_m$ are the interacting objects and $\alpha_{\langle i,j\rangle}$ are the intuitionistic fuzzy estimations of the pure effect of object O_j over object O_i in the constant environment $\bar{S}$, where $i, j = 1,2, \dots m$. The direct interaction can be expressed as above:

$$I(O_i, O_j) = \left(\alpha_{\langle O_i,O_j\rangle}, \alpha_{\langle O_i,O_j\rangle}\right).$$

Initially all effects are unknown and $\pi_{\langle i,j\rangle} = \pi_{\langle j,i\rangle} = 1$,

$$\begin{array}{c|cccc} & O_1 & O_2 & \cdots & O_m \\ \hline O_1 & \langle 0,0\rangle & \langle 0,0\rangle & \cdots & \langle 0,0\rangle \\ O_2 & \langle 0,0\rangle & \langle 0,0\rangle & \cdots & \langle 0,0\rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_m & \langle 0,0\rangle & \langle 0,0\rangle & \cdots & \langle 0,0\rangle \end{array}$$

Let $E^k_{\langle i,j\rangle}$ be the k-th observation of the effect of object j on object i

$$E^k_{\langle i,j\rangle} = \langle \mu_{E^k}(\langle i,j\rangle), \vartheta_{E^k}(\langle i,j\rangle)\rangle,$$

where $i, j \in \aleph \;\&\; 0 \le \mu_{E^k}(\langle x,y\rangle) + \vartheta_{E^k}(\langle x,y\rangle) \le 1$

then the k-th intuitionistic fuzzy interaction estimation for the objects I, j can be obtained as a weighted combination of the $a_{\langle i,j\rangle}^{k-1}$ and the observation $E_{\langle i,j\rangle}^{k}$

$$a_{\langle i,j\rangle}^{k} = \langle \frac{(k-1)\mu_{a^{k-1}}(\langle i,j\rangle)+\mu_{E^k}(\langle i,j\rangle)}{k}, \frac{(k-1)\vartheta_{a^{k-1}}(\langle i,j\rangle)+\vartheta_{E^k}(\langle i,j\rangle)}{k} \rangle.$$

Alternatively to prevent error accumulation the above expression can be written in the following form:

$$a_{\langle i,j\rangle}^{k} = \langle \frac{1}{k}\sum_{l=1}^{k}\mu_{E^l}(\langle i,j\rangle), \frac{1}{k}\sum_{l=1}^{k}\vartheta_{E^l}(\langle i,j\rangle)\rangle$$

Using simple averaging does not lead to reduction of uncertainty with increased number of observations, which could be a desired property when the algorithm is applied for interaction in relatively constant environment $\tilde{S}$ since the uncertainty there comes from unaccounted cumulative effects. In case of constant environment $\bar{S}$, other operators (Atanassov ???) reducing uncertainty can be used to aggregate the estimations form different observations.

Discussion

The formalism above focuses on the general case of interaction of two living organisms in given environment. These results can be applied in the case when one of the interacting objects is not a living organism but rather media or resource. In this case the health status Ω_{S+x}^{x} can be viewed as a measure of the availability of the resource within the range of: 0 – resource is not available, and 1 resource is available in infinite quantities. Another special case of general definition is the interaction without environment $S = \emptyset$. In this case the interaction is direct and does not have any third party mediated components. This is also the case which is considered in the majority of the existing models including those based on Lotka-Volterra equation and replicator dynamics.

Direct effect observed in one environment $\tilde{S}_1$ does not necessary coincide with the direct effect observed in other $\tilde{S}_2$. Further investigation is needed to revile what conditions need to be met in order direct effect observed in one environment can be adjusted to estimate the direct effect observed in another environment.

In the current definition only the “quality” (positive, neutral, negative) of the interaction is considered but not its magnitude / straight. This however is important factor when the quantitative estimation of the result of the interaction is needed.

In its present form the intuitionistic fuzzy estimation of the biological interaction could not explicitly accommodate important properties of natural interactions. Some of these properties are:

- Living organism state dependency. A simple example of such dependency is the interaction between the predator and prey – depending on the current need for food of the predator. Such dependencies can be modelled in the future by introduction of state machines models of the organisms;
- Temporal aspect. This aspect is tightly related to the state dependencies and introduces the time scale in which these state changes develop. A generalized measure of such temporal changes is the life stage of the living organism which clearly presents different needs and different interaction properties in the course of organism development, maturation and decline. Temporal aspect could be easily introduced utilizing the existing research in the field of Temporal Intuitionistic Fuzzy Sets (Atanassov ???);
- Spatial aspect. Interaction strength and polarity (positive neutral negative) generally are significantly altered by the distance between the interacting objects. For example a nitrogen fixing plant at optimal

distance could provide nitrogen for other plants without creating too much root or light competition, thus creating overall positive effect. The same plant if closer than optimal could have detrimental effect because of shading and diminishing effect if it is too far;

- Evolutionary aspect. Crossbreeding and mutations – the driving force of evolution also change the way living organisms interact;
- Multidimensional. Interactions can have multiple "dimensions" and can be beneficial in certain aspects while detrimental in others.

Considering all of these properties in a single model might look as an over-complication and unattractive for practical application but it could be the only plausible way to establish their relative importance while building a better model of arguably complex natural phenomena. More over when considering state machine represented individual interacting objects in space, time and physical environment utilizing game theory a simulation of such virtual ecosystem does not look unattainable. Such virtual models could chose not to describe the abundant diversity of life forms but rather utilize the existing taxonomy classifications and build general models which simulates the relationships of kingdoms, genus, classes or families of organisms investigating the evolutionary preserved properties of respective taxonomy ranks.

Conclusions

The introduction of intuitionistic fuzzy estimation of biological interactions, offers simple yet practically applicable framework for studying anthropogenic or natural ecosystems. It presents unified formalism which can be used for modelling of both: forward and inverse problems in ecology. A benefit over existing approaches is that intuitionistic fuzzy estimations provide explicit estimations of both positive and negative aspects of an interaction as well as its uncertainty. It is also important to emphasize that interactions in this framework are conditionally defined and depend on the environment in which they are occurring. Intuitionistic fuzzy interactions are also defined on individual level and can be used for encoding or predicting direct observations.

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