Notes on Intuitionistic Fuzzy Sets
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Three de-intuitionistic fuzzification procedures over circular intuitionistic fuzzy sets

Lilija C. Atanassova

Institute of Information and Communication Technologies,
Bulgarian Academy of Sciences
Acad. Georgi Bonchev Str., Bl. 2, Sofia-1113, Bulgaria
e-mail: l.c.atanassova@gmail.com

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Abstract: Circular Intuitionistic Fuzzy Sets (CIFS) are one of the newest extensions of the intuitionistic fuzzy sets. In the present paper, three procedures for de-intuitionistic fuzzification of CIFS are discussed.
Keywords: Circular intuitionistic fuzzy sets, De-i-fuzzification procedure.
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1 Introuction

The idea for de-intuitionistic fuzzification of Intuitionistic Fuzzy Sets (IFSs, see, e.g., [1]) was discussed for the first time in [7] by Adrian Ban, Janusz Kacprzyk and Krassimir Atanassov. Later, in [6] Vassia Atanassova and Sotir Sotirov introduced a new version of this procedure. In [5], Piotr Dworniczak and the author gave another interpretation of the same procedure. Meantime, in [8] R. Parvathi and C. Radhamani introduced similar procedure for the case of Temporal IFSs (TIFSs, see [1]).

In [2], Atanassov introduced the concept of Circular IFSs (CIFSs). In the present paper, we introduce some procedures for de-intuitionistic fuzzification of CIFSs.

2 Preliminaries

All necessary definitions for IFSs are from [1]. Here, following [2], we give only the basic definition of CIFS as a main object of investigation in the present paper.

Let us have a fixed universe E and its subset A. The set

$$A_r = \{\langle x, \mu_A(x), \nu_A(x); r\rangle \mid x \in E\}$$

is called *Circular IFS (CIFS)*, where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ with $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, represent the *degree of membership (validity, etc.)* and *non-membership (non-validity, etc.)* of an element $x \in E$ to the fixed set $A \subseteq E$, and $r \in [0,1]$ is the radius of a circle constructed around the graphical interpretation of each element $x \in E$ onto the intuitionistic fuzzy interpretational triangle.

In addition, we can define also the function $\pi_A : E \to [0,1]$ as the complement of the sum of membership and non-membership to 1: $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, thus corresponding to the *degree of indeterminacy (uncertainty, etc.).*

3 Three procedures for de-intuitionistic fuzzification of CIFSs

3.1 First procedure

First, we discuss the procedure that is an analogous of the procedures from [5–7].

Let the CIFS A_r be given. Then we can juxtapose to each its element with a circular form living in the Intuitionistic Fuzzy Interpretation Triangle (IFIT, see [1]), its center (see Fig. 1). In a result we obtain a standard IFS A.

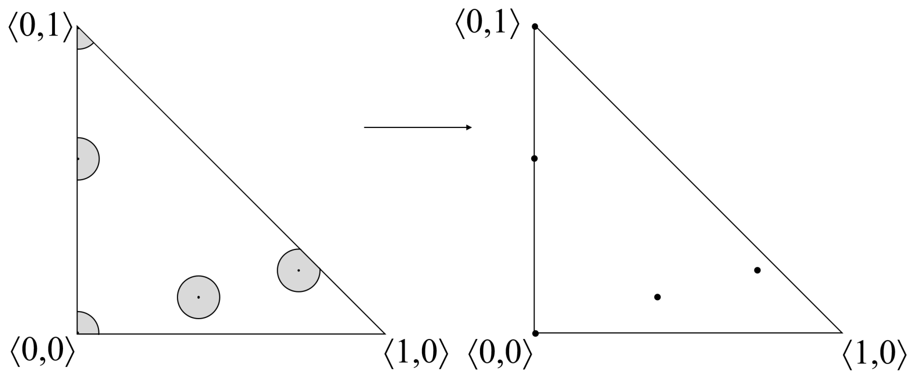

Figure 1. A geometrical interpretation of elements of a CIFS.

Having in mind that operator D_α (see [1]) is defined for each IFS B by:

$$D_\alpha(B) = \{\langle x, \mu_B(x) + \alpha\pi_B(x), \nu_B(x) + (1-\alpha)\pi_B(x)\rangle \mid x \in E\},$$

we can apply it over the obtained standard IFS A. Therefore, the IFS A is transformed to a fuzzy set (see [9]).

On the other hand, following [6], we can juxtapose to each element of E with degrees $\langle \mu_A(x), \nu_A(x)\rangle$ the intuitionistic fuzzy pair $\langle \frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)}, \frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)}\rangle$ and as it is shown in [6], for it is valid that

$$\frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)} + \frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)} = 1,$$

i.e., this pair belong to the hypotenuse of the IFIT.

This result can be obtained also if we apply the operator Δ (see, [4,5]), defined over the IFS A.

3.2 Second procedure

Let us define for the element $x \in E$ the set:

$$\overline{x} = \{\langle y, \mu_A(y), \nu_A(y)\rangle | y \in E \;\&\; \sqrt{(\mu_A(x)-\mu_A(y))^2 + (\nu_A(x)-\nu_A(y))^2} \leq r\}.$$

Obviously,

$$\overline{x} \sqsubset A_r,$$

where relation "$\sqsubset$" is the set-theoretical relation "inclusion"; and $\overline{x}$ is an IFS over the set:

$$R(x) = \{y \mid \langle y, \mu_A(y), \nu_A(y)\rangle \in \overline{x}\}$$

that is a subset of E.

Let $|X|$ be the cardinality of an arbitrary set X. Then, obviously,

$$|\overline{x}| = |R(x)|.$$

Now, we use operator W (see [1]) that for each finite IFS B over E is defined by

$$W(B) = \left\{\left\langle y, \frac{1}{|E|}\sum_{z\in E}\mu_{R(x)}(z), \frac{1}{|E|}\sum_{z\in E}\nu_{R(x)}(y)\right\rangle \mid y \in E\right\}$$

as follows:

$$W(\overline{x}) = \left\{\left\langle y, \frac{1}{|R(x)|}\sum_{z\in R(x)}\mu_{R(x)}(z), \frac{1}{|R(x)|}\sum_{z\in R(x)}\nu_{R(x)}(y)\right\rangle \mid y \in R(x)\right\}.$$

Therefore, it juxtaposes to each element $x \in E$ with degrees $\langle \mu_A(x), \nu_A(x)\rangle$, the intuitionistic fuzzy pair

$$P(x) = \left\langle \frac{1}{|R(x)|}\sum_{z\in R(x)}\mu_{R(x)}(z), \frac{1}{|R(x)|}\sum_{z\in R(x)}\nu_{R(x)}(y)\right\rangle.$$

It corresponds to the weight-center of the circle around element x. This weight-center can coincide with the element x, but this is not obligatory. An example is shown on Fig. 2.

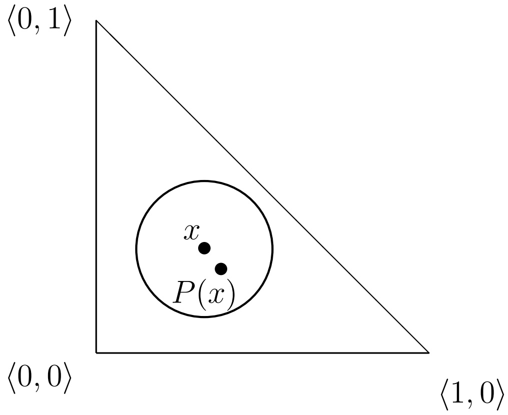

Figure 2. The geometrical interpretation of an elements $x \in E$ and $P(x)$.

The final step of this procedure is the same as in the first procedure, but now we use intuitionistic fuzzy pair $P(x)$ instead of the pair with the degrees of x.

3.3 Third procedure

The third procedure is similar to the first two, but now to each element $x \in E$ we juxtapose a point $X(x)$ over the hypotenuse of the IFIT with coordinates, i.e., intuitionistic fuzzy pair

$$\left\langle \mu_A(x) + \frac{\pi_A(x)}{2}, \nu_A(x) + \frac{\pi_A(x)}{2} \right\rangle = \left\langle \frac{1 + \mu_A(x) - \nu_A(x)}{2}, \frac{1 - \mu_A(x) + \nu_A(x)}{2} \right\rangle$$

(see Fig. 3).

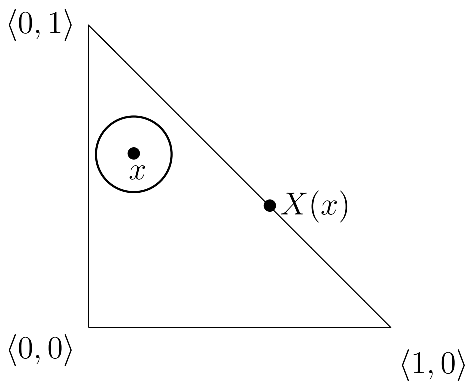

Figure 3. The geometrical interpretation of elements $x \in E$ and $X(x)$.

Hence, in each one of these three procedures, we obtain a point $X(x)$ on the hypotenuse of the IFIT. On the other hand, we know that the radius of the element x is r. So, over the hypotenuse we can construct two additional points with coordinates as follows (see Fig. 4):

- point $X_L(x)$ with coordinates

$$\left\langle \max\left(0, \frac{1 + \mu_A(x) - \nu_A(x) - \sqrt{2}r}{2}\right), \min\left(1, \frac{1 - \mu_A(x) + \nu_A(x) + \sqrt{2}r}{2}\right)\right\rangle$$

- point $X_R(x)$ with coordinates

$$\left\langle \min\left(1, \frac{1+\mu_A(x)-\nu_A(x)+\sqrt{2}r}{2}\right), \max\left(0, \frac{1-\mu_A(x)+\nu_A(x)-\sqrt{2}r}{2}\right)\right\rangle$$

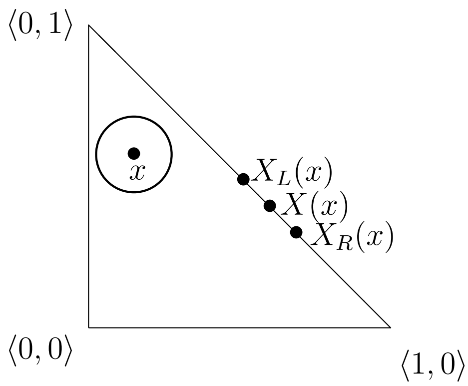

Figure 4. The geometrical interpretation of elements $x \in E$, $X(x)$, $X_L(x)$ and $X_R(x)$.

4 Conclusion

In the present research, we introduce procedures for a de-intuitionistic fuzzification of a given CIFS. In anet research, we will do the same for the case of the elliptic IFS (see [3]).

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