# Arc analysis in the intuitionistic fuzzy graph and its applications 

V. Nivethana ${ }^{1}$ and A. Parvathi ${ }^{2}$<br>${ }^{1}$ Department of Mathematics, Sri Venkateswara Institute of Science and Technology Thiruvallur, India<br>e-mail: vnivethana@yahoo.com<br>${ }^{2}$ Department of Mathematics, Avinashilingam University Coimbatore, India<br>e-mails: aparvathi.s@gmail.com

Received: 1 July 2015
Revised: 15 February 2016
Accepted: 21 February 2016


#### Abstract

In this paper, a two dimensional approach on arcs of an intuitionistic fuzzy graph is made and the arcs are classified into three types: Sturdy arc, Feeble arc and $\delta^{*}$ weak arc. A new concept of firm paths and infirm paths has been introduced and their application in a decision making problem has been shown. IF-bridges and IF-cutnodes are defined with a new notion and their properties are analyzed. We present with a necessary condition for an arc to be an IF-bridge.


Keywords: Intuitionistic fuzzy graph, Arcs in intuitionistic fuzzy graph, IF-bridges, IFcutnodes, Application in decision making.
AMS Classification: 03E72, 05C38.

## 1 Introduction

The idea of intuitionistic fuzzy set was first introduced by K. Atanassov in the year 1983. He introduced a new component called "degree of non-membership" to the definition of fuzzy sets. IF sets give both degree of membership and degree of non-membership, which are independent of each other to some extent, with the condition that their sum is not greater than one. In 1994, K. Atanassov and A. Shannon introduced the concept of intuitionistic fuzzy graph and afterwards more studies were conducted in this discipline (see e.g. [2, 3, 10], etc). As a major contribution, R. Parvathi, M. G. Karunambigai, and R. Buvaneswari in [5], have created a new insight on arcs, bridges and cutnodes of an intuitionistic fuzzy graph. In [1], M.

Akram, N. O. Alshehri introduced various types of intuitionistic fuzzy bridges, cutnodes in intuitionistic fuzzy graphs.

In this paper, arc analysis is carried out on a two dimensional approach for degree of membership ( $\mu$ ) and non-membership ( $v$ ) individually. Accordingly any arc of an IFG is categorized under Sturdy arc or Feeble arc or $\delta^{*}$ weak arc, and their properties are analyzed. Also the firm path and infirm path concepts are introduced and studied. In Section 4, we define intuitionistic fuzzy bridges and cutnodes and their characteristics are studied. We have provided the necessary condition for an arc to be an IF-bridge. We present an algorithm to find IF-bridge. In Section 5, we show an ideal application of intuitionistic fuzzy graph in a more familiar area of a decision making problem. An example of subject/subjects preferred by majority of students for higher studies is presented based on a survey conducted among 100 students of class $x$, who were randomly selected. The problem renders two important facts by using IFG as a tool. The first is the best combination of subjects opted by majority of students and the second is the least opted subject combination based on the interest rate of students.

## 2 Preliminaries and definitions

Definition 2.1. An intuitionistic fuzzy graph (IFG) is of the form $G:(V, E)$ where,
i. $\quad V$ is finite non-empty set of vertices such that $\mu_{A}: V \rightarrow[0,1]$ and $v_{A}: V \rightarrow[0,1]$ denotes the degree of membership and non-membership of the elements $x \in V$ respectively and $0 \leq \mu_{A}(x)+V_{A}(x) \leq 1$ for every $x \in V$.
ii. $\quad E \subset V \times V$ is a finite set of edges such that $\mu_{B}: V \times V \rightarrow[0,1]$ and $\nu_{B}: V \times V \rightarrow[0,1]$ are such that $\mu_{B}(x y) \leq \min \left\{\mu_{A}(x), \mu_{A}(y)\right\}$ and $\nu_{B}(x y) \leq \max \left\{v_{A}(x), \nu_{A}(y)\right\}$ and $0 \leq \mu_{B}(x y)+\nu_{B}(x y) \leq 1$ for every $(x, y) \in E$.

Note: Edge $(x, y)$ is represented hereafter by $(x y)$ whose membership function is $\mu_{B}(x y)$ and non-membership function is $v_{B}(x y)$.

Definition 2.2. An arc $(x, y)$ in IFG is strong if both $\mu_{B}(x y)=\min \left\{\mu_{A}(x), \mu_{A}(y)\right\}$ and $\nu_{B}(x y)=$ $\max \left\{v_{A}(x), v_{A}(y)\right\}$.

Definition 2.3. A path $v_{i}-v_{j}$ in an IFG is the sequence of distinct vertices $v_{1}, v_{2}, \ldots, v_{n}$ for all $(i, j=1,2, \ldots, n$.) such that either one of the following conditions is satisfied.
i. $\quad \mu_{B}\left(v_{i} v_{j}\right)>0$ and $v_{B}\left(v_{i} v_{j}\right)=0$ for some $i$ and $j$.
ii. $\quad \mu_{B}\left(v_{i} v_{j}\right)=0$ and $v_{B}\left(v_{i} v_{j}\right)>0$ for some $i$ and $j$.
iii. $\quad \mu_{B}\left(v_{i} v_{j}\right)>0$ and $v_{B}\left(v_{i} v_{j}\right)>0$ for some $i$ and $j$.

Definition 2.4. $\mu$-strength of a path, $s_{\mu}(x y)$ is defined as the least value of degree of membership of all the arcs in the path.

Definition 2.5. $v$-strength of a path, $s_{v}(x y)$ is defined as the maximum value of degree of nonmembership of all the arcs in the path.

Definition 2.6. $\mu$-strength of connectedness between two nodes $x$ and $y$ is defined as the maximum of $\mu$-strength of all the paths between $x$ and $y$ excluding the arc joining $x$ and $y$. It is denoted by ( $x y$ ).

Definition 2.7. $v$-strength of connectedness ((xy)) between two nodes $x$ and $y$ is defined as the minimum of $v$-strength of all the paths between $x$ and $y$ excluding the arc joining $x$ and $y$.

Definition 2.8. Total $\mu$-strength of connectedness denoted by $\operatorname{TCONN}_{\mu}(x y)$ is defined as the maximum of $\mu$-strength of all the paths between $x$ and $y$ including the arc joining $x$ and $y$.

Definition 2.9. Total $v$-strength of connectedness denoted by TCONN $(x y)$ is defined as the minimum of $v$-strength of all the paths between $x$ and $y$ including the arc joining $x$ and $y$.

Proposition 2.10. In a IFG, $(x y) \vee \mu_{\mathrm{B}}(x y)=\operatorname{TCONN}_{\mu}(x y)$ and $(x y) \wedge \nu_{\mathrm{B}}(x y)=\operatorname{TCONN}_{\nu}(x y)$. Proof: Proof of this proposition follows directly from Definitions 2.6. to 2.9.

Proposition 2.11. If $\operatorname{TCONN}_{\mu}(x y)=(x y)$ and $\operatorname{TCONN}_{\nu}(x y)=(x y)$ then either $\mu_{B}(x y)<(x y)$ and $V_{B}(x y)>(x y)$ or there is no arc joining the nodes $x$ and $y$.
Proof: Follows from Proposition 2.10.
With the above definitions, in the article [5], the authors have defined and classified arcs into three types $\alpha$-strong, $\beta$-strong and $\delta$-weak based on their strength of connectedness. In common it may be observed that if $\mu_{B}(x y)>(x y)$ then the non-membership value can be $\nu_{B}(x y) \geq(x y)$. So the corresponding arc (xy) cannot be classified as $\alpha$-strong or $\beta$-strong or $\delta$-weak arc as in [5]. Hence, there arises insufficiency in dividing the arcs based on their strength of connectedness. This gave rise to conduct further study on the arcs in IFGs. Here we try to define the types of arcs based on two dimensional view for degree of membership $(\mu)$ and non-membership ( $v$ ) separately in the proceeding section. We classify the arcs in IFG into three types: Sturdy arc, Feeble arc and $\delta^{*}$ weak arc. Based on it we introduce the firm path and infirm path and their properties are studied.

## 3 Types of arcs in IFGs

Definition 3.1. An $\operatorname{arc}(x, y)$ in $G$ with membership $\mu_{B}(x y)$ and non-membership $\gamma_{B}(x y)$ is called:

| i. | $\alpha-\mu$ strong arc if $\mu_{B}(x y)>(x y)$. | ii. | $\alpha-v$ strong arc if $v_{B}(x y)<(x y)$. |
| ---: | :--- | :--- | :--- |
| iii. | $\beta-\mu$ strong arc if $\mu_{B}(x y)=(x y)$. | iv. $\quad \beta$ - strong arc if $v_{B}(x y)=(x y)$. |  |
| v. | $\delta-\mu$ weak arc if $\mu_{B}(x y)<(x y)$. | vi. $\delta-v$ weak arc if $v_{B}(x y)>(x y)$. |  |

Example 3.2. Consider the following graph $G:(V, E)$ in Fig. 1.
By repeated computation the values of strength of the paths, strength of connectedness and total strength of connectedness of the above graph are tabulated in Table 1 below and from the table it could be observed that the arc $(a, b)$ is $\alpha-\mu$ strong and $\alpha-v$ strong arc, $(a, d)$ is $\alpha-\mu$ strong and $\beta-\nu$ strong arc, $(a, c)$ is $\delta-\mu$ weak and $\alpha-\nu$ strong arc, $(b, c)$ is $\beta-\mu$ strong and $\delta-v$ weak arc and $(c, d)$ is $\beta-\mu$ strong and $\beta-v$ strong arc. Hence, it is obvious that if the arc is $\alpha-\mu$ strong, it need not be $\alpha-\nu$ strong also. Similarly, if it is $\delta \mu$ weak then there is no restriction that the arc should be $\delta v$ weak.

Based on the above Definition 3.1, we classify the arcs of any IFG into three types: Sturdy arc, Feeble arc and $\delta^{*}$ weak arc in the Definitions 3.5 to 3.7.


Figure 1. $G:(V, E)$

| End nodes | Paths | $s_{\mu}(x y)$ | $s_{\nu}(x y)$ | (xy) | (xy) | $\mathrm{TCONN}_{\mu}(x y)$ | $\mathrm{TCONN}_{\iota}(x y)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ab | $a-b$ | 0.2 | 0.3 | 0.15 | 0.55 | 0.2 | 0.3 |
|  | $a-c-b$ | 0.05 | 0.55 |  |  |  |  |
|  | $a-d-c-b$ | 0.15 | 0.55 |  |  |  |  |
| ac | $a-c$ | 0.05 | 0.45 | 0.15 | 0.5 | 0.15 | 0.45 |
|  | $a-b-c$ | 0.15 | 0.55 |  |  |  |  |
|  | $a-d-c$ | 0.15 | 0.5 |  |  |  |  |
| ad | $a-d$ | 0.3 | 0.5 | 0.15 | 0.5 | 0.3 | 0.5 |
|  | $a-c-d$ | 0.05 | 0.5 |  |  |  |  |
|  | $a-b-c-d$ | 0.15 | 0.55 |  |  |  |  |
| bc | $b-c$ | 0.15 | 0.55 | 0.15 | 0.45 | 0.15 | 0.45 |
|  | $b-a-c$ | 0.05 | 0.45 |  |  |  |  |
|  | $b-a-d-c$ | 0.15 | 0.5 |  |  |  |  |
| bd | $b-a-d$ | 0.2 | 0.5 | 0.2 | 0.5 | 0.2 | 0.5 |
|  | $b-c-d$ | 0.15 | 0.55 |  |  |  |  |
|  | $b-a-c-d$ | 0.05 | 0.5 |  |  |  |  |
|  | $b-c-a-d$ | 0.05 | 0.55 |  |  |  |  |
| cd | $c-d$ | 0.15 | 0.5 | 0.15 | 0.5 | 0.15 | 0.5 |
|  | $c-a-d$ | 0.05 | 0.5 |  |  |  |  |
|  | $c-b-a-d$ | 0.15 | 0.55 |  |  |  |  |

Table 1

Definition 3.3. An arc $\left(v_{i}, v_{j}\right)$ is called as a $\mu$-strong arc if it is $\alpha-\mu$ strong or $\beta-\mu$ strong.
Definition 3.4. An arc $\left(v_{i}, v_{j}\right)$ is called as a $v$-strong arc if it is $\alpha-v$ strong or $\beta$ - $v$ strong.
Definition 3.5. An arc is called a sturdy arc if it is both $\mu$-strong and $v$-strong arc.

Definition 3.6. An $\operatorname{arc}\left(v_{i}, v_{j}\right)$ is called as a feeble arc if it is either $\delta \mu$ weak or $\delta v$ weak.
Definition 3.7. An arc $\left(v_{i}, v_{j}\right)$ is called as a $\delta^{*}$ weak arc if it is both $\delta \mu$ weak and $\delta$ - $v$ weak.
Definition 3.8. A path $P$ is firm path if it contains only the sturdy arc.
Definition 3.9. A path $P$ is infirm path if it contains only the $\delta^{*}$ weak arc.
Definition 3.10. A path $P: x \rightarrow y$ is called a strong path if its strength equals $\operatorname{TCONN}_{\mu}(x y)$ and $\operatorname{TCONN}_{\nu}(x y)$, i.e., $s_{\mu}(x y)=\operatorname{TCONN}_{\mu}(x y)$ and $s_{\nu}(x y)=\operatorname{TCONN}_{\nu}(x y)$.

Note: From Example 3.2. and the table it can be observed that the path $b-a-d$ is a strong path. Also the arcs $(a, b),(c, d)$ and $(a, d)$ are sturdy arcs and the arcs $(a, c)$ and $(b, c)$ are feeble arcs.

Proposition 3.11. An $\operatorname{arc}(x, y)$ is sturdy iff $\mu_{B}(x y)=\operatorname{TCONN}_{\mu}(x y)$ and $\nu_{B}(x y)=\operatorname{TCONN}_{\iota}(x y)$.
Proof: Let the $\operatorname{arc}(x, y)$ be the sturdy arc then $\mu_{B}(x y) \geq(x y)$ and $v_{B}(x y) \leq(x y)$. By Proposition 2.10, $\operatorname{TCONN}_{\mu}(x y)=\mu_{B}(x y)$ and $\operatorname{TCONN}_{k}(x y)=\nu_{B}(x y)$.
Conversely, if $\mu_{B}(x y)=\operatorname{TCONN}_{\mu}(x y)$ then again by Proposition $2.10,(x y) \leq \mu_{B}(x y)$. Hence, the arc $(x, y)$ must be either $\alpha-\mu$ strong or $\beta$ - $\mu$ strong arc. Hence, $(x, y)$ is a $\mu$-strong arc. Similarly, the argument can be repeated for $v$, and it can be shown that the $\operatorname{arc}(x, y)$ is a $v$-strong arc. Therefore $(x, y)$ must be a sturdy arc.

Proposition 3.12. A strong path has only sturdy arcs.
Proof: Let $P: v_{1}, v_{2}, \ldots, v_{n}$ be the strong path. Consider the $\operatorname{arc}\left(v_{1}, v_{2}\right)$ in the path $P$. Let $\mu_{B}\left(v_{1} v_{2}\right)$ has the least membership value in the path $P$. Hence, $s_{\mu}\left(v_{1} v_{n}\right)=\mu_{B}\left(v_{1} v_{2}\right)$. Hence, for all arcs in that path $P$,

$$
\begin{equation*}
\mu_{B}\left(v_{i} v_{j}\right) \geq s_{\mu}\left(v_{1} v_{n}\right)=\mu_{B}\left(v_{1} v_{2}\right), \tag{1}
\end{equation*}
$$

for all $i, j=1,2,3, \ldots, n$.
Since $P$ is strong $s_{\mu}\left(v_{1} v_{n}\right)=\operatorname{TCONN}_{\mu}\left(v_{1} v_{n}\right)$. By Proposition 2.10, $s_{\mu}\left(v_{1} v_{n}\right) \geq\left(v_{1} v_{n}\right)$. Hence, from (1), $\mu_{B}\left(v_{i} v_{j}\right) \geq\left(v_{1} v_{n}\right)$ for all $i, j=1,2,3, \ldots, n$. Therefore by Definition 3.1, every arc in the path $P$ must be $\alpha-\mu$ strong or $\beta-\mu$ strong. Similarly repeating the argument for $v$-values, it can be shown that all the arcs in path $P$ must be $\alpha-v$ strong or $\beta-v$ strong. Hence, $P$ has only $\mu$-strong and $v$-strong arcs.

Corollary 3.13. A strong path is a firm path but not conversely.
Proof: From Proposition 3.12., one way of the proof is obviously true. Conversely from Example 3.2., the path $a-d-c$ is a firm path but it is not strong path.

Proposition 3.14. An arc to the end vertex is a sturdy arc iff its non-membership value is zero. Proof: Let $v_{n}$ be an end vertex and so the only arc connecting $v_{n}$ be ( $v_{m}, v_{n}$ ). Hence, the strength of connectedness $\left(v_{m} v_{n}\right)=0$ and $\left(v_{m} v_{n}\right)=0$, since $v_{n}$ is end vertex, there is no other path from $v_{m}$ to $v_{n} . \therefore \mu_{B}\left(v_{m} v_{n}\right) \geq\left(v_{m} v_{n}\right)=0$ and $v_{B}\left(v_{m} v_{n}\right) \geq\left(v_{m} v_{n}\right)=0$.
$\therefore$ The arc is $\alpha-\mu$ strong arc.
Case (i): If $v_{B}\left(v_{m} v_{n}\right)=0$, then $v_{B}\left(v_{m} v_{n}\right)=\left(v_{m} v_{n}\right)$, i.e., the arc $v_{m} v_{n}$ is $\beta$ - $v$ strong arc. Hence, from (1) and above statement the arc $\mathrm{v}_{\mathrm{m}} \mathrm{v}_{\mathrm{n}}$ is a sturdy arc.

Case (ii): If $\nu_{B}\left(v_{\mathrm{m}} \mathrm{v}_{\mathrm{n}}\right) \neq 0$, then $v_{B}\left(v_{m} v_{n}\right)>\left(v_{m} v_{n}\right)$. By definition the arc $v_{m} v_{n}$ is $\delta \boldsymbol{\delta} v$ weak arc. $\therefore$ The arc is a feeble arc.

Proposition 3.15. If there is more than one strong path between a pair of vertices $v_{i}$ and $v_{j}$, then all the paths are of equal strength.
Proof: Let $P_{1}$ and $P_{2}$ be two strong paths between vertices $v_{i}$ and $v_{j}$.
If not, let the strength of path $P_{1}<$ the strength of the path $P_{2}$. Since both $P_{1}$ and $P_{2}$ are strong, for $P_{1} \rightarrow \mathrm{~s}_{\mu}\left(v_{i} v_{j}\right)=\operatorname{TCONN}_{\mu}\left(v_{i} v_{j}\right)$ and for $P_{2} \rightarrow s_{\mu}\left(v_{i} v_{j}\right)=\operatorname{TCONN}_{\mu}\left(v_{i} v_{j}\right)$.

Comparing $P_{1}$ and $P_{2}, \operatorname{TCONN}_{\mu}\left(v_{i} v_{j}\right)<\operatorname{TCONN}_{\mu}\left(v_{i} v_{j}\right)$ is meaningless. Hence, we arrive at a contradiction. $\therefore$ The strength of paths $P_{1}$ and $P_{2}$ are equal.

## Remark: 3.16.

a) If vertices $v_{i}$ and $v_{j}$ are not adjacent, then $\left(v_{i} v_{j}\right)=\operatorname{TCONN}_{\mu}\left(v_{i} v_{j}\right)$ and $\left(v_{i} v_{j}\right)=\operatorname{TCONN}_{\iota}\left(v_{i} v_{j}\right)$.
Proof: Since vertices $v_{i}$ and $v_{j}$ are not adjacent, $\operatorname{TCONN}_{\mu}\left(v_{i} v_{j}\right)$ and $\left(v_{i} v_{j}\right)$ are equal since there is no arc joining $v_{i}$ and $v_{j}$. Similarly, $\left(v_{i} v_{j}\right)=\operatorname{TCONN}_{\nu}\left(v_{i} v_{j}\right)$.
b) In a connected graph, the strength of connectedness for a path $v_{i}-v_{j}$ is zero, then $v_{i}$ and $v_{j}$ are adjacent vertices and either $v_{i}$ or $v_{j}$ must be an end vertex.
Proof: Since the strength of connectedness is zero, there is no other path from $v_{i}$ to $v_{j}$ other than the arc $\left(v_{i}, v_{j}\right)$. Hence, $v_{i}$ and $v_{j}$ must be adjacent and either $v_{i}$ or $v_{j}$ must be an end vertex.

## 4 Intuitionistic fuzzy bridges and intuitionistic fuzzy cutnodes

Definition 4.1. An arc $\left(v_{i}, v_{j}\right)$ is said to be a IF-bridge in $G$ if the deletion of the $\operatorname{arc}\left(v_{i}, v_{j}\right)$ reduces the total $\mu$-strength of connectedness and increases the total $\nu$-strength of connectedness between some pair of vertices at the same time.

Example: In Fig. 1, the arc $(a, b)$ is an IF-bridge since the removal of the $\operatorname{arc}(a, b)$ reduces $\operatorname{TCONN}_{\mu}(a b)$ and increases $\operatorname{TCONN}_{\nu}(a b)$ at the same time between the nodes $a$ and $b$. Also the arc $(a, c)$ is not a bridge since removal of $(a, c)$ does not reduce TCONN $_{\mu}(a c)$ between nodes a and cor elsewhere.

Definition 4.2. A node (vertex) is an intuitionistic fuzzy cutnode of an IFG if the removal of it reduces the total $\mu$-strength of connectedness and increases the total $v$-strength of connectedness at the same time between some other pair of nodes.

Example: In Example 3.2., the node ' $a$ ' is the IF-cutnode since if ' $a$ ' is removed, $\operatorname{TCONN}_{\mu}(b d)$ $=0.15<0.2$ and $\operatorname{TCONN}_{\nu}(b d)=0.55>0.5$.

Proposition 4.3. In a IFG the $\operatorname{arc}(a, b)$ is an IF-bridge then $\mu_{B}(a b)=\operatorname{TCONN}_{\mu}(a b)$ and $\nu_{B}(a b)$ $=\operatorname{TCONN}_{r}(a b)$.
Proof: By the definition of IF-bridge, $(a b)<\mu_{\mathrm{B}}(a b)$.
$\therefore(a b) \vee \mu_{B}(a b)=\mu_{B}(a b)$.
$\therefore$ From Proposition 2.10., $\operatorname{TCONN}_{\mu}(a b)=\mu_{B}(a b)$. Similarly we can prove that $v_{B}(a b)=\operatorname{TCONN}_{\downarrow}(a b)$.

Corollary 4.4. In an IFG, the arc $(a, b)$ is an IF-bridge then $\mu_{B}(a b) \geq(a b)$ and $\nu_{B}(a b) \leq(a b)$. Proof: Follows from the above proposition.

Proposition 4.5. Every bridge is sturdy arc, but a sturdy arc need not be a bridge.
Proof: From Corollary 4.3., if $(a, b)$ is a bridge, $\mu_{B}(a b) \geq(a b)$ and $v_{B}(a b) \leq(a b)$. From the definition of $\alpha-\mu$ strong, $\beta-\mu$ strong, $\alpha-v$ strong and $\beta-v$ strong arc, the arc must be both $\mu$-strong and $v$-strong arc and Hence, the sturdy arc. But the sturdy arc need not be a bridge is obvious from Example 3.2., i.e., the arc $(c, d)$ is a sturdy arc but it is not a bridge.

## Remark 4.6.

1. It should be observed that an IF-bridge between nodes $x$ and $y$ is not necessarily the arc joining $x$ and $y$. It may be any arc in some path between $x$ and $y$.
2. In a unique strong path every arc is an IF-bridge is not true according to our definition of IF-bridge.
Example: In Fig. 1., the path b-a-d is strong path. Deletion of arc $(a, d)$ reduces $\mathrm{TCONN}_{\mu}(b d)$ but does not increase $\mathrm{TCONN}_{\mathrm{v}}(b d)$. Hence, $(a, d)$ is not an IF-bridge. But arc $(a, b)$ is an IF-bridge.

Proposition 4.7. In an IFG, the necessary condition for arcs $\left(x_{i} y_{j}\right)$ to be an IF-bridges is,
a. The arcs $\left(x_{i} y_{j}\right)$ lie in a unique path between $v_{i}$ and $v_{j}$.
b. In the unique path, there may be any number of IF-bridges.

Proof: (a) It may be observed that two IF-bridges between some pair of vertices indicate that both must have the same membership and non-membership values. If suppose both IF-bridges lie in two distinct paths between $v_{i}$ and $v_{j}$. Let the arcs $(P Q)$ and $(R S)$ be two IF-bridges between vertex pair $v_{i}$ and $v_{j}$ and let $(P Q)$ and $(R S)$ lie in two distinct paths, then the removal of (PQ) does not reduce the total $\mu$-strength of connectedness and increase the total $v$-strength of connectedness because there exist another path connecting $v_{i}$ and $v_{j}$, via $(R S)$. Hence, $(P Q)$ is not an IF-bridge which is a contradiction to our assumption. Hence, $(P Q)$ and $(R S)$ cannot lie in two distinct paths. Hence, the path containing IF-bridge is unique.
(b) Also the arcs $(P Q)$ and $(R S)$ may lie in the same path where the removal of any one of it disconnects the path. Hence, both $(P Q)$ and $(R S)$ are IF-bridges which lie in the unique path. Hence, the unique path may contain any number of IF-bridges.

## An Algorithm to find IF-bridge

Step 0: [Initialize the ' $r$ ' distinct paths between every pair of vertices say $v_{i}$ and $v_{j}$ ]
For $n=1$ to $r$
$p[n]=n \leftarrow v_{i}-v_{i 1}-v_{i 2}-\ldots-v_{i m}-v_{j}$.
Step 1: For $P[n]=1$ to $r$
$1.1 s_{\mu}(x y)[n] \leftarrow$ minimum of $\mu_{B}(x y)$ of all the arcs say $\left(v_{i a} v_{i b}\right)$ in the path $P[n]$ and initialize the $\operatorname{arc}\left(v_{i a} v_{i b}\right)$ as $K$.
$1.2 s_{\vee}(x y)[n] \leftarrow$ maximum of $v_{B}(x y)$ of all the arcs say $\left(v_{i c} v_{i d}\right)$ in the path $P[n]$ and initialize the $\operatorname{arc}\left(v_{i c} v_{i d}\right)$ as $L$.
Step 2: 2.1 $\operatorname{TCONN}_{\mu}(x y) \leftarrow$ maximum of $s_{\mu}(x y)[n]$ and assume $n=n$.
2.2 $\operatorname{TCONN}_{\downarrow}(x y) \leftarrow$ minimum of $s_{\nu}(x y)[n]$ and assume $N^{*}=n$.

Step 3: If $n \neq N^{*}$
Output "There is no IF-bridge between the vertex pair $v_{i}$ and $v_{j}$."
Goto Step 0: with a different pair of vertices.
Else $n=N^{*}$
If $K=L$, output "The arc $\left(v_{i a} v_{i b}\right)$ is the IF-bridge"
Goto Step 0: with a different pair of vertices.
Else $K \neq L$, output "There is no IF-bridge between the vertex pair $v_{i}$ and $v_{j}$." Goto Step 0: with a different pair of vertices.

## 5 Application of intuitionistic fuzzy graph (IFG) in decision making problem

Today's students are blessed with vast range of career options. In addition to a few courses with high level of demand, all the options available become best choice unless the individual student is enhanced with adequate planning and is driven with interest to the subject/career. Preparation with interest alone helps to reach proficiency in any field we opt for. Students at the end of secondary education are in need to make their first choice in their career determination. At this stage, providing adequate information to students for proper career choice parallel to their interest must be emphasized. In this section, based on the survey conducted among random sample of 100 students of class $x$, the percentage of students with interest/disinterest towards a particular subject and pair of subjects that they have studied till class $x$ is calculated and tabulated below. Depending on the data, we use intuitionistic fuzzy graph as a tool since it incorporates the degree of membership (interest of percentage of students to a subject or pair of subjects) and the degree of non-membership (disinterest of percentage of students to a subject or pair of subjects). Using this IFG we may analyze the best combination of subjects, i.e., the group containing the subjects which might be fruitful to a large number of students and will probably result in best academic performance of more students.

Let $\mathbf{S}=\{\operatorname{English}(E), \operatorname{Language}(L), \operatorname{Maths}(M)$, Science $(S)$, Social Science $(S S)\}$ be the set of vertices. The following table illustrates the percentage of students with interest/disinterest towards a subject and Pair of subjects.

| Subject/Subject <br> combination | Interest $\%$ |  |
| :---: | :---: | :---: |
|  | Disinterest $\%$ <br> etc. |  |
| $E$ | 0.7 | 0 |
| $L$ | 0.38 | 0.45 |
| $M$ | 0.79 | 0.07 |
| $S$ | 0.75 | 0.21 |
| $S S$ | 0.36 | 0.63 |
| $E-M$ | 0.45 | 0 |
| $E-L$ | 0.33 | 0.4 |


| Sub/Sub combination | Interest \% | Disinterest \% |
| :---: | :---: | :---: |
| $E-S$ | 0.28 | 0.1 |
| $E-S S$ | 0.15 | 0.5 |
| $L-M$ | 0 | 0 |
| $L-S$ | 0.25 | 0.27 |
| $L-S S$ | 0.12 | 0.63 |
| $M-S$ | 0.73 | 0.1 |
| $M-S S$ | 0.12 | 0.47 |
| $S-S S$ | 0.12 | 0.5 |

Table 2.

Based on the above table we generate an IFG as follows (Fig. 2).


Figure 2. $G_{1}(V, E)$

In our graph for all vertices the degree of membership represents percentage of students with passion for a particular subject and degree of non-membership is the percentage of students with no interest in the subject from a random sample of 100 students of class $x$ selected for survey. Also membership/non-membership of edges of the graph indicates the likes/dislikes of the students to study the combination of any two subjects at the higher secondary level. From the above graph, the edge ( $L, S S$ ) having high degree of non-membership indicates most of the students do not wish to study the combination of Language and Social Science and the edge; $(M, S)$ having high degree of membership indicates many of the students have passion to study the combination of Maths and Science. Also there is no like/dislike to study the combination of Tamil and Maths shows the subject need not be combined. Hence, a high (low) degree of membership of any edge indicates the high (low) weightage for the combination of the subjects at higher studies.

Using the definitions in Section 2 and Section 3 for the above graph by repeated computation we observe that the arcs $(E, M)$ and $(M, S)$ are sturdy arcs. Hence, the path $E-M-S$ is the firm path. The $\operatorname{arcs}(E, S),(E, S S),(E, L),(L, S)$ and $(M, S S)$ are feeble arcs. Also the arcs $(L, S S)$ and $(S, S S)$ are $\delta^{*}$ weak arcs. Hence, the path $L-S S-S$ is the infirm path. Hence, it may be concluded that the combination of subjects English, Maths and Science (firm path) derives high interest among students and the subject combination of Language, Science and Social Science (infirm path) is not liked by majority of students for their higher studies.

This simple analysis indicates that IFG may be used in decision making situations for all real life and day today problems. More applications in artificial intelligence and decision making situations can be discussed. For example, in medical analysis the effect of the drug "Septilin" in various septic conditions such as sinusitis, tonsillitis, otorrhoea, furunculosis and improving body's defence mechanism on a group of people under continuous medical
examination can be formulated as an IFG and can be analyzed for best results. This helps us to identify the right way for the right usage of the drug.

## Conclusion

The application of intuitionistic fuzzy graphs provides us with a novel and ideal results that looks to be greatly significant because it gives accurate and Proper guidance in various situations of decision making problems. This can also be extended to artificial intelligence, networking etc. Also when the number of vertices and edges on an IFG increases, the manual calculation of $(x y),(x y), \operatorname{TCONN}_{\mu}(x y)$ and $\operatorname{TCONN}_{\mathrm{v}}(x y)$ becomes very tedious as the number of paths between any two pair of vertices may be more. So in our next paper we will try to provide a "C program" to find the type of each arc, firm paths, infirm paths and strong paths for any IFG, based on the algorithm provided in this article.

## References

[1] Akram, M. \& Alshehri, N. O. (2014) Intuitionistic fuzzy cycles and intuitionistic fuzzy trees. The Scientific World Journal, 2014, Article ID 305836, DOI: 10.1155/2014/ 305836.
[2] Shannon, A., \& Atanassov, K. (2006) On a generalization of intuitionistic fuzzy graphs, Notes on Intuitionistic Fuzzy Sets, 12(1), 24-29.
[3] Atanassov, K. (1995) On intuitionistic fuzzy graphs and intuitionistic fuzzy relations. Proceedings of the $6^{\text {th }}$ IFSA World Congress. Sao Paulo, Brazil, July 1995, 551-554.
[4] Bhutani, K. R., \& Rosenfeld, A. (2003) Strong arcs in fuzzy graphs. Information Sciences, 152, 319-322.
[5] Karunambigai, M. G., Parvathi, R. \& Buvaneswari, R. (2012) Arcs in intuitionistic fuzzy graphs. Notes on Intuitionistic Fuzzy Sets, 18(4), 48-58.
[6] Mathew, S., \& Sunitha, M.S. (2009) Types of arcs in a fuzzy graph. Information Sciences, 179, 1760-1768.
[7] Tom, M., \& Sunitha, M. S. (2013) On strongest paths, delta arcs and blocks in fuzzy graphs. World Applied Sciences Journal, 22, 10-17.
[8] Tom, M., Sunitha, M. S., \& Mathew, S. (2014) Notes on types of arcs in fuzzy graphs. Journal of Uncertainty in Mathematical Sciences, 2014, Article ID jums-00004, doi:10.5899/2014/jums-00004.
[9] Nivethana, V., \& Parvathi, A. (2015) On Complement of Intuitionistic Fuzzy Graphs, International Journal of Computational and Applied Mathematics, 10(1), 17-26.
[10] Parvathi, R., Shannon, A., Chountas, P. \& Atanassov, K. (2011) On intuitionistic fuzzy tree-interpretations by index matrices, Notes on Intuitionistic Fuzzy Sets, 17(2), 17-24.

