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ON INTUITIONISTIC FUZZY VERSIONS OF L. ZADEH'S EXTENSION PRINCIPLE

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1 Introduction and basic concepts

K. Atanassov introduced the concept of the Intuitionistic Fuzzy Sets (IFS) in [1]. The book [2] contains some of the most important results of this theory by 1999.

Here we shall discuss some IFS-versions of the extension principle introduced by L. Zadeh in [6] and discussed in a lot of publications, e.g., [3, 4].

Let us have a fixed universe E and its subset A. The set

$$\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in E\},$$

where

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{1}$$

is called IFS and functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ represent *degree of membership (validity, etc.)* and *non-membership (non-validity, etc.).*

For every two IFSs A and B a lot of operations, relations and operators are defined (see, e.g. [2]), two of them are related to the present research:

$$\begin{array}{rcl} C(A) & = & \{\langle x, K, L\rangle | x \in E\}, \\ I(A) & = & \{\langle x, k, l\rangle | x \in E\}, \end{array}$$

where

$$K = \underset{y\in E}{sup}\ \mu_A(y), \quad L = \underset{y\in E}{inf}\ \nu_A(y)$$

and

$$k = \underset{y\in E}{inf}\ \mu_A(y), \quad l = \underset{y\in E}{sup}\ \nu_A(y).$$

Let E_1 and E_2 be two universes and let

$$\begin{array}{rcl} A & = & \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E_1\}, \\ B & = & \{\langle x, \mu_B(y), \nu_B(y)\rangle | y \in E_2\}, \end{array}$$

be two IFSs; A – over E_1, and B – over E_2.

Following [2] we will define:

$$A \times_1 B = \{\langle\langle x,y\rangle, \mu_A(x).\mu_B(y), \nu_A(x).\nu_B(y)\rangle | x \in E_1 \& y \in E_2\},$$

$$A \times_2 B = \{\langle\langle x,y\rangle, \mu_A(x)+\mu_B(y)-\mu_A(x).\mu_B(y), \nu_A(x).\nu_B(y)\rangle | x \in E_1 \& y \in E_2\},$$

$$A \times_3 B = \{\langle\langle x,y\rangle, \mu_A(x).\mu_B(y), \nu_A(x)+\nu_B(y)-\nu_A(x).\nu_B(y)\rangle | x \in E_1 \& y \in E_2\},$$

$$A \times_4 B = \{\langle\langle x,y\rangle, \min(\mu_A(x),\mu_B(y)), \max(\nu_A(x),\nu_B(y))\rangle | x \in E_1 \& y \in E_2\}$$

$$A \times_5 B = \{\langle\langle x,y\rangle, \max(\mu_A(x),\mu_B(y)), \min(\nu_A(x),\nu_B(y))\rangle | x \in E_1 \& y \in E_2\}.$$

2 Main results

Now, we shall formulate some IFS-versions of the extension principle introduced by L. Zadeh.

Let X and Y be fixed universes and let $f : X \to Y$. Let A and B be IFSs over X. Then we can construct the sets

$$D_i = A \times_i B,$$

where $i = 1, 2, ..., 5$ and can obtain the sets

$$F_i = f(D_i).$$

Having in mind the idea of G. Pasi, K. Atanassov and R. Yager for intuitionistic fuzzy optimistic and pessimistic estimations (see [5]), the extension principle will have the following 10 forms.

Optimistic forms of the extension principle are:

$$F_1^{opt} = \{\langle z, \sup_{z=f(x,y)}(\mu_A(x).\mu_B(y)), \inf_{z=f(x,y)}(\nu_A(x).\nu_B(y))\rangle | x \in E_1 \& y \in E_2\},$$

$$F_2^{opt} = \{\langle z, \sup_{z=f(x,y)}(\mu_A(x)+\mu_B(y)-\mu_A(x).\mu_B(y)), \inf_{z=f(x,y)}(\nu_A(x).\nu_B(y))\rangle | x \in E_1 \ y \in E_2\},$$

$$F_3^{opt} = \{\langle z, \sup_{z=f(x,y)}(\mu_A(x).\mu_B(y)), \inf_{z=f(x,y)}((\nu_A(x)+\nu_B(y)-\nu_A(x).\nu_B(y))\rangle | x \in E_1 \ y \in E_2\},$$

$$F_4^{opt} = \{\langle z, \sup_{z=f(x,y)}(\min(\mu_A(x),\mu_B(y)), \inf_{z=f(x,y)}(\max(\nu_A(x),\nu_B(y))\rangle | x \in E_1 \ y \in E_2\},$$

$$F_5^{opt} = \{\langle z, \sup_{z=f(x,y)}(\max(\mu_A(x)),\mu_B(y)), \inf_{z=f(x,y)}((\min(\nu_A(x),\nu_B(y))\rangle | x \in E_1 \ y \in E_2\}.$$

Pessimistic forms of the extension principle are:

$$F_1^{pes} = \{\langle z, \inf_{z=f(x,y)}(\mu_A(x).\mu_B(y)), \sup_{z=f(x,y)}(\nu_A(x).\nu_B(y))\rangle | x \in E_1 \& y \in E_2\},$$

$$F_2^{pes} = \{\langle z, \inf_{z=f(x,y)} (\mu_A(x) + \mu_B(y) - \mu_A(x).\mu_B(y)), \sup_{z=f(x,y)} (\nu_A(x).\nu_B(y))\rangle | x \in E_1 \; y \in E_2\},$$

$$F_3^{pes} = \{\langle z, \inf_{z=f(x,y)} (\mu_A(x).\mu_B(y)), \sup_{z=f(x,y)} ((\nu_A(x) + \nu_B(y) - \nu_A(x).\nu_B(y))\rangle | x \in E_1 \; y \in E_2\},$$

$$F_4^{pes} = \{\langle z, \inf_{z=f(x,y)} (\min(\mu_A(x), \mu_B(y)), \sup_{z=f(x,y)} (\max(\nu_A(x), \nu_B(y))\rangle | x \in E_1 \; y \in E_2\},$$

$$F_5^{pes} = \{\langle z, \inf_{z=f(x,y)} (\max(\mu_A(x)), \mu_B(y)), \sup_{z=f(x,y)} ((\min(\nu_A(x), \nu_B(y))\rangle | x \in E_1 \; y \in E_2\}.$$

Average forms of the extension principle are:

$$F_1^{ave} = \{\langle z, \frac{1}{card(\{z|z = f(x,y)\}} \sum_{z=f(x,y)} (\mu_A(x).\mu_B(y)),$$

$$\frac{1}{card(\{z|z = f(x,y)\}} \sum_{z=f(x,y)} (\nu_A(x).\nu_B(y))\rangle | x \in E_1 \& y \in E_2\},$$

$$F_2^{ave} = \{\langle z, \frac{1}{card(\{z|z = f(x,y)\}} \sum_{z=f(x,y)} (\mu_A(x) + \mu_B(y) - \mu_A(x).\mu_B(y)),$$

$$\frac{1}{card(\{z|z = f(x,y)\}} \sum_{z=f(x,y)} (\nu_A(x).\nu_B(y))\rangle \; | x \in E_1 \; y \in E_2\},$$

$$F_3^{ave} = \{\langle z, \frac{1}{card(\{z|z = f(x,y)\}} \sum_{z=f(x,y)} (\mu_A(x).\mu_B(y)),$$

$$\frac{1}{card(\{z|z = f(x,y)\}} \sum_{z=f(x,y)} ((\nu_A(x) + \nu_B(y) - \nu_A(x).\nu_B(y))\rangle \; | x \in E_1 \; y \in E_2\},$$

$$F_4^{ave} = \{\langle z, \frac{1}{card(\{z|z = f(x,y)\}} \sum_{z=f(x,y)} (\min(\mu_A(x), \mu_B(y)),$$

$$\frac{1}{card(\{z|z = f(x,y)\}} \sum_{z=f(x,y)} (\max(\nu_A(x), \nu_B(y))\rangle | x \in E_1 \; y \in E_2\},$$

$$F_5^{ave} = \{\langle z, \frac{1}{card(\{z|z = f(x,y)\}} \sum_{z=f(x,y)} (\max(\mu_A(x)), \mu_B(y)),$$

$$\frac{1}{card(\{z|z = f(x,y)\}} \sum_{z=f(x,y)} ((\min(\nu_A(x), \nu_B(y))\rangle | x \in E_1 \; y \in E_2\}.$$

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