# NEW WAYS FOR ALTERING THE INTUITIONISTIC FUZZY EXPERTS' ESTIMATIONS

**Krassimir T. Atanassov**

Centre of Biomedical Engineering, Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 105, Sofia-1113, Bulgaria
E-mail: *krat@bas.bg*

**Abstract**
Three new ways for altering the intuitionistic fuzzy experts' estimations are discussed for the case, when some expert gives incorrect estimations. An open problem is formulated.

## 1 INTRODUCTION

In [1, 2] the author described 5 different ways for altering incorrect intuitionistic fuzzy experts' estimations. Here, we will discuss other ways that are based on a property of the intuitionistic fuzzy interpretation triangle (for it, see [2]). In [3] the following problem related to the Intuitionistic Fuzzy Sets (IFSs; see [2]) is formulated and two solutions are given there:

**Problem:** *To find a continuous bijective transformation that transforms the unit square ABCD (from Figure 1 and Figure 2) to the IFS interpretation triangle ABD (from the same figures).*

The solution to this question gives the possibility:
• to prove that each bi-lattice (see e.g., [4]) can be interpreted by an IFS;
• to prove that each intuitionistic $L$-fuzzy set (see [2]) can be interpreted by an IFS.

In [3] it is particularly noted that the solution to the above problem will be used *"to construct a new algorithm for modifying incorrect expert estimations"*. There, the following two theorems are formulated and

proved.

**Theorem 1.** Transformation

$$F(x,y) = \begin{cases} \langle \frac{x^2}{x+y}, \frac{xy}{x+y} \rangle & \text{if } x,y \in [0,1] \text{ and } x \geq y \\ \\ \langle \frac{xy}{x+y}, \frac{y^2}{x+y} \rangle & \text{if } x,y \in [0,1] \text{ and } x \leq y \end{cases}$$

satisfies the conditions of the Problem.

**Theorem 2.** Transformation

$$G(x,y) = \begin{cases} \langle x - \frac{y}{2}, \frac{y}{2} \rangle & \text{if } x,y \in [0,1] \text{ and } x \geq y \\ \\ \langle \frac{x}{2}, y - \frac{x}{2} \rangle & \text{if } x,y \in [0,1] \text{ and } x \leq y \end{cases}$$

satisfies the conditions of the Problem (see Figure 2).

The forms of the two transformations are given on Figure 1 and 2.

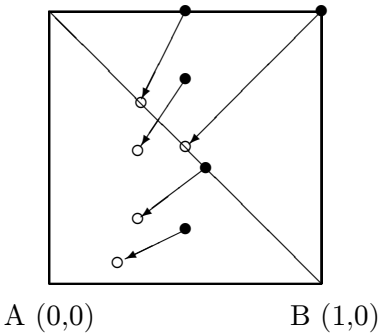


Figure 1

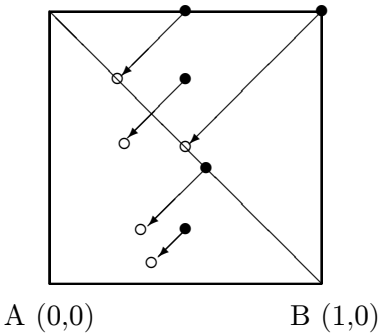


Figure 2

## 2 THREE WAYS FOR ALTERING THE INTUITIONISTIC FUZZY EXPERT'S ESTIMATIONS

In some cases $\mu$- and $\nu$-function values are calculated on the basis of expert knowledge. In this case some problems may arise in relation to

the correctness of the expert estimations. No such problems arise when we deal with ordinary fuzzy sets. In [1] we have discussed five ways for altering the experts' estimations of given events (or objects; for brevity, below we will use only the word "event") in case that the experts have made inacceptable errors in their Intuitionistic Fuzzy Estimations (IFEs).

While in an everyday conversation the expression "I am 1000% sure" is allowed, this is not the case when considering expert extimations. Clearly, it does not matter much whether the estimations will be expressed in percentage or in numbers from the $[0, 1]$ range.

Following the terminology of the fuzzy sets theory, we will assume that for a given estimated event or object the experts' estimations are expressed as numbers belonging to the $[0, 1]$ range, producing simultaneously two degrees (of truth and falsity, of membership and non-membership, etc.).

If we use fuzzy estimations and if an expert states that he is 101% convinced in something, then we could just assume that he is, for instance, joking or exaggerating, and feel free to adjust his estimation to 100%.

The situation that arises when working with IFS is essentially different than that. Now, it is not only a matter of consideration whether the expert has been joking or not, but rather whether his estimations are precise enough to be accepted.

Let us call the expert $E_i$ ($i \in I$; $I$ is an index set, related to the experts) *unconscientious* if among his estimations $\{\langle \mu_{i,j}, \nu_{i,j} \rangle / j \in J_i\}$, where

$$J = \bigcup_{i \in I} J_i$$

is an index set, related to the events estimated by the experts indexed by the elements of the set $I$, there are ones for which $\mu_{i,j} \leq 1$, $\nu_{i,j} \leq 1$, but

$$\mu_{i,j} + \nu_{i,j} > 1.$$

In this case, different ways for a post-adjustment of his incorrect estimations can be introduced in order to convert them to a correct IF-form.

It is obvious that if the estimations are fuzzy, then if an expert happens to assert that a given event is 101% possible, we can simply assume his estimation is 1. However, in the case of an IF-estimation the procedure of post-adjustment is very complicated. Below, we will show several different

ways for post-adjustment of the estimations of the unconscientious (inaccurate) experts.

Let among the estimations $\{\langle\mu_{i,j},\nu_{i,j}\rangle/j \in J_i\}$ made by expert $E_i$ there be a subset $\{\langle\mu_{i,k},\nu_{i,k}\rangle/k \in K_i\}$, where $K_i \subset J_i$, for which $\mu_{i,k} \leq 1$, $\nu_{i,k} \leq 1$, and $\mu_{i,k} + \nu_{i,k} > 1$.

The first two ways for solving the problem with the adjustments of the incorrect estimations involve using the two transformation formulas above. Obviously, they will transform the expert's estimations to new correct ones. In both cases, we change the expert's estimations only on the basis of his estimations, i.e., without using any additional information, as it has been done for some of the five algorithms in [1].

These two new algorithms are simple from computational point of view. The third algorithm that we will discuss below decreases to a less extent the original expert's estimations.

Let us determine for the $i$-th expert the pair $\langle p_i, q_i\rangle$, so that $p_i = \mu_{i,m}, q = \nu_{i,m}$ and

$$\mu_{i,m}^2 + \nu_{i,m}^2 = \max_{k\in K_i}(\mu_{i,k}^2 + \nu_{i,k}^2).$$

The new transformation formula have the form

$$H(\mu_{i,k},\nu_{i,k}) = \langle\frac{\mu_{i,k}p_i}{p_i+q_i}, \frac{\nu_{i,k}q_i}{p_i+q_i}\rangle.$$

**Theorem 3** Transformation $H$ is a bijective one.

**Proof.** Let the expert's estimations have geometrical interpretation by points with coordinates $\langle\mu_{i,k},\nu_{i,k}\rangle$ and $\langle\mu_{i,j},\nu_{i,j}\rangle$ (for $k, j \in J_i$). Then, from

$$H(\mu_{i,k},\nu_{i,k}) = H(\mu_{i,j},\nu_{i,j})$$

it follows that

$$\langle\frac{\mu_{i,k}p_i}{p_i+q_i}, \frac{\nu_{i,k}q_i}{p_i+q_i}\rangle = \langle\frac{\mu_{i,j}p_i}{p_i+q_i}, \frac{\nu_{i,j}q_i}{p_i+q_i}\rangle,$$

i.e.

$$\frac{\mu_{i,k}p_i}{p_i+q_i} = \frac{\mu_{i,j}p_i}{p_i+q_i}$$

and

$$\frac{\nu_{i,k}q_i}{p_i+q_i} = \frac{\nu_{i,j}q_i}{p_i+q_i},$$

from where it directly follows that

$$\langle \mu_{i,k}, \nu_{i,k} \rangle = \langle \mu_{i,j}, \nu_{i,j} \rangle.$$

On the other hand, if

$$\langle \mu_{i,k}, \nu_{i,k} \rangle \neq \langle \mu_{i,j}, \nu_{i,j} \rangle,$$

then

$$\mu_{i,k} \neq \mu_{i,j} \text{ or } \nu_{i,k} \neq \nu_{i,j}.$$

Hence

$$\frac{\mu_{i,k} p_i}{p_i + q_i} \neq \frac{\mu_{i,j} p_i}{p_i + q_i} \text{ or } \frac{\nu_{i,k} q_i}{p_i + q_i} \neq \frac{\nu_{i,j} q_i}{p_i + q_i},$$

i.e., $H(\mu_{i,k}, \nu_{i,k}) \neq H(\mu_{i,j}, \nu_{i,j})$.

Therefore, $H$ is a bijective transformation. It can be easily seen that $H$ is a continuous transformation and that

$$\begin{cases} \lim\limits_{x \to 0} \dfrac{x p_i}{p_i + q_i} = 0 \\ \\ \lim\limits_{y \to 0} \dfrac{y q_i}{p_i + q_i} = 0 \end{cases}$$

**Open problem**: What other non-linear bijective transformations are possible? For example, if the values of the expert's estimation $\langle \mu_i, \nu_i \rangle$ satisfy inequalities

$$\mu_i + \nu_i \geq 1,\ 0 \leq \mu_i \leq 1,\ 0 \leq \nu_i \leq 1$$

can we find the smallest real number $n > 1$ such that

$$(\mu_i)^n + (\nu_i)^n \leq 1?$$

# 3 ACKNOWLEDGEMENTS

The author is grateful for the support provided by the project of the Bulgarian Ministry of Education and Science (Grant BIn-2/09).

## References


[1] Atanassov K. Intuitionistic fuzzy sets and expert estimations. II, BUSEFAL, Vol. 59, 1994, 64-69.

[2] Atanassov K. Intuitionistic Fuzzy Sets, Springer Heidelberg, 1999.

[3] Atanassov, K. Remark on a property of the intuitionistic fuzzy interpretation triangle. Notes on Intuitionistic Fuzzy Sets, Vol. 8 (2002), No. 1, 34-36. http.//ifigenia.org/wiki/issue:nifs/8/1/34-36

[4] Gargov G., Knowledge, uncertainty and ignorance in logic: bilattices and beyond, Journal of Applied Non-Classical Logics, Vol. 9, 1999, No. 2-3, 195-283.