

Incorporating hesitation in intuitionistic fuzzy slack based measure efficiency with an application to MGNREGA

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Abstract: The intuitionistic fuzzy set theory was introduced by Attanassov to lay focus on the hesitation present in a variable while deciding the degree of membership in fuzzy sets. The studies done on intuitionistic fuzzy sets deal extensively with the membership and non-membership function of the variables and pays little attention to the hesitation. The present study tries to dig deeper into the hesitation of triangular intuitionistic fuzzy numbers. We also try to establish the effect of hesitation on efficiency of decision making units. The hesitation and relative efficiency are inversely related. The presence of higher hesitation in a firm would lead to lower efficiency. In this study we have developed a mathematical model using SBM model to calculate the relative efficiency incorporated with hesitation in the decision making units and proposed a ranking algorithm to compare them. The proposed method has been applied to the scheme of MGNREGA to calculate the efficiencies of Indian States. The best performers have come out to be the States of Tamil Nadu, Andhra Pradesh and Tripura, whereas, the worst performers are the States of Manipur, Gujarat and Nagaland.

Keywords: Fuzzy sets, Intuitionistic fuzzy sets, Data envelopment analysis, Intuitionistic fuzzy



1 Introduction

The intuitionistic fuzzy set (IFS) theory was introduced by Atanassov [7]. The notion behind introduction to IFS theory was to mathematically present the indeterminacy present in data. Crisp data for any variable is easily available through the firms, or various internet websites and government agencies. But this data hides the confusion present in the mind of data collector. For more realistic mathematical models and results, the decision maker needs to consider the extent of vagueness present in data. IFS theory presents the level of impreciseness present in data with the help of membership function and non-membership function. It can also express the hesitation in defining the values to membership function and non-membership function of a variable. For comparison of efficiencies of different decision-making units (DMUs) using homogeneous (crisp) inputs to produce homogeneous (crisp) outputs, the non-radial non-parametric method of (crisp) Slack based measure (SBM) [33] is widely used. The IFSBM method was developed by [3] for the DMUS using intuitionistic fuzzy inputs and outputs. While calculating the efficiency of firms with IF variables, the researchers have given huge weightage to the non-membership function, whereas, the hesitation present in the variable has been ignored. The effect of hesitation on efficiency has not been considered by the researchers yet. This paper explores deeper in the hesitation present in the input and output variables and the effect of this hesitation on the efficiency of firm.

The IFS theory given by Atanassov [7] is an extension or generalization of the fuzzy set theory given by Zadeh [45]. Dubois et al. [14] and Atanassov [8] discussed in detail about the motivation and underlying mathematical structures behind the terminologies of intuitionistic fuzzy sets and fuzzy logic. To find the efficiencies of firms, Charnes, Cooper and Rhodes [12] introduced the DEA model, which is widely used and has numerous applications. The DEA model is a radial, non-parametric in nature and measures the efficiencies of homogeneous firms. Another non-radial, non-parametric model used widely for finding the efficiencies of firms is SBM Model given by Tone [33]. The SBM model given by Tone is for crisp data, which was extended by Sasti [26] to fuzzy data and was further extended to IF data by Alka [3]. Another model widely used for finding efficiencies of various firms is additive model [12]. Meena and Shiv Prasad [42] calculated the relative efficiency of DMUs with IF data in the form of an intervals. Various applications of IFS include the banking sector [24], in health institutions [4], in finance and credit firms [15], township development [25] and many others (see [17, 19, 28, 44]). Ever since the beginning of IFS theory, not much have been studied about hesitation other than its significance and formula.

The paper has been organized as follows: Section 2 contains basic definitions and formulas related to the IFS that will be used in rest of the paper. Section 3 contains the proposed methodology. Section 4 explains the methodology with an example, followed by an application of the proposed method in Section 5 and a conclusion in Section 6.

2 Preliminaries

This section defines some basic terminology and models that will be used later to establish a background for the present research.

Definition 2.1 (Efficiency, [11]). *The efficiency of a firm is defined as the ratio of the weighted sum of inputs to the weighted sum of outputs. Relative efficiency of a firm can be calculated by dividing the efficiency of the firm by the maximum efficiency of the firm in the set of the firms. Relative efficiencies always lies in (0,1]. The DEA Model and SBM Model are used to find the relative efficiency of DMUs and compare their performances.*

Definition 2.2 (Intuitionistic Fuzzy Set (IFS), [6]). *Let X be the universe of discourse. Then, an IFS $\tilde{A}^I$ is defined as follows*

$$\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)) | x \in X\}$$

where $\mu_{\tilde{A}^I}, \nu_{\tilde{A}^I} : X \rightarrow [0, 1]$ are called the membership and non-membership functions respectively and satisfy $0 < \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1$. The lack of information or degree of indeterminacy is defined by $\pi_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x) - \nu_{\tilde{A}^I}(x)$. It is also called hesitation of element x in $\tilde{A}^I$. Note that $0 \leq \pi_{\tilde{A}^I}(x) < 1 \forall x \in X$. If $\pi_{\tilde{A}^I}(x) = 0 \forall x$ in X , the IFS is reduced to a fuzzy set.

Definition 2.3 (Triangular intuitionistic fuzzy number (TIFN), [45]). *A TIFN $\tilde{A}^I = (a, b, c; a', b, c')$, where $a' \leq a \leq b \leq c \leq c'$, is defined by the membership function, $\mu_{\tilde{A}^I}$ and the non-membership function, $\nu_{\tilde{A}^I}$ as given below:*

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-a}{b-a}, & a < x \leq b, \\ \frac{c-x}{c-b}, & b \leq x < c, \\ 0, & \text{elsewhere,} \end{cases} \quad \nu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-b}{a'-b}, & a' < x \leq b, \\ \frac{x-b}{c-b}, & b \leq x < c', \\ 1, & \text{elsewhere,} \end{cases}$$

for all $x \in \mathbb{R}$,

Definition 2.4 (α -cut, [7]). *The α -cut of an IFS $\tilde{M}^I$, denoted by A_α^I , is defined by*

$$A_\alpha^I = \{x \in X : \mu_{\tilde{A}^I}(x) \geq \alpha\} \quad \forall \alpha \in [0, 1].$$

Definition 2.5 (β -cut, [7]). *The β -cut of an IFS $\tilde{A}^I$, denoted by A_β^I , is defined by*

$$A_\beta^I = \{x \in X : \mu_{\tilde{A}^I}(x) \leq \beta\} \quad \forall \beta \in [0, 1].$$

Definition 2.6 (Positive TIFN, [45]). *A TIFN $\tilde{A}^I = (a, b, c; a', b, c')$ is said to be positive TIFN if $a' > 0$.*

3 Methodology

The main idea of this research is divided into two parts: first, understanding the hesitation present in each DMU and second, understanding the effect of this hesitation on efficiency.

3.1 Hesitation present in a DMU

To understand the hesitation present in a DMU first we will explore the hesitation present in a TIFN. Suppose $\tilde{A}^I = (a, b, c; a', b, c')$ is a TIFN. Then, as given in the definition 2.2, its hesitation function is defined as $\pi_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x) - \nu_{\tilde{A}^I}(x)$. Using the membership function and non-membership function of TIFN, the hesitation function can be written as below.

$$\pi_{\tilde{A}^I}(x) = \begin{cases} \frac{x - a'}{b - a'}, & a' \leq x \leq a, \\ \frac{(a - a')(b - x)}{(b - a')(b - a)}, & a \leq x \leq b, \\ \frac{(c' - c)(x - b)}{(c' - b)(c - b)}, & b \leq x \leq c, \\ \frac{c' - x}{c' - b}, & c \leq x \leq c', \\ 0, & \text{otherwise.} \end{cases}$$

As seen in Figure 1, the hesitation function is union of two triangular functions: one with base $a'b$ and other with base bc' . The function has two local maxima or two peaks P_1 and P_2 , obtained at points a and c respectively. One can observe that $P_1 = \frac{a - a'}{b - a'}$ and $P_2 = \frac{c' - c}{c' - b}$. Note that there are possibilities of $P_1 < P_2$ or $P_1 > P_2$ or $P_1 = P_2$.

Also, the hesitation function can never take value 1 as for that one must have both membership function and non-membership function equal to zero simultaneously, which is not possible. Hence, $\pi_{\tilde{A}^I}(x) \in [0, \max(P_1, P_2)] \subset [0, 1)$.

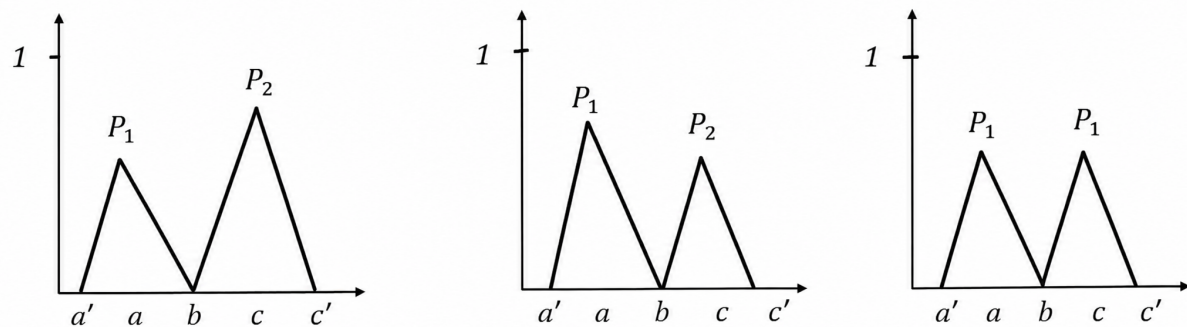


Figure 1. The hesitation in a TIFN

Define $P = \max\{P_1, P_2\}$ obtained above as the hesitation present in the TIFN $\tilde{A}^I$.

Now, we can define the γ - cut of a hesitation function as

$$(\pi_{\tilde{A}^I}(x))_\gamma = \{x \in X | \pi_{\tilde{A}^I}(x) \geq \gamma, \gamma \in [0, P]\}$$

- For $\gamma \leq \min\{P_1, P_2\}$, the γ - cut can be calculated by simplifying the expressions

$$\frac{x - a'}{b - a'} \geq \gamma, \frac{(a - a')(b - x)}{(b - a')(b - a)} \geq \gamma$$

and

$$\frac{(c' - c)(x - b)}{(c' - b)(c - b)} \geq \gamma, \frac{c' - x}{c' - b} \geq \gamma$$

After simplification, the $(\pi_{\tilde{A}^I}(x))_\gamma$ comes out to be the union of two intervals I_1 and I_2 , where

$$I_1 = [\gamma(b - a') + a', b - \frac{\gamma(b - a')(b - a)}{(a - a')}] \text{ and } I_2 = [b + \frac{\gamma(c' - b)(c - b)}{(c' - c)}, c' - \gamma(c' - b)]$$

- For $P_1 < \gamma < P_2$, i.e., if $\min\{P_1, P_2\} = P_1$, the γ - cut is given by the interval

$$I_2 = [b + \frac{\gamma(c' - b)(c - b)}{(c' - c)}, c' - \gamma(c' - b)]$$

- For $P_1 > \gamma > P_2$, i.e., if $\min\{P_1, P_2\} = P_2$, the γ - cut is given by the interval

$$I_1 = [\gamma(b - a') + a', b - \frac{\gamma(b - a')(b - a)}{(a - a')}]$$

Hence, for $\min\{P_1, P_2\} < \gamma < \max(P_1, P_2)$, $(\pi_{\tilde{A}^I}(x))_\gamma$ is a single interval I_1 or I_2 .

Result. I_1 and I_2 as defined above are intervals.

Proof. To prove that I_1 is an interval, we must show that $\gamma(b - a') + a' \leq b - \frac{\gamma(b - a')(b - a)}{(a - a')}$. Further simplification shows that this inequality holds if $(a - a') \geq \gamma(b - a')$, which is true as $\gamma \leq \max\{P_1, P_2\}$. On the similar lines, it can be easily proved that I_2 is also an interval. $\square$

Now, suppose a DMU, DMU_k , is using m number of TIFN inputs $\tilde{x}_{ik}^I, i = 1, 2, \dots, m$ to produce s number of TIFN outputs $\tilde{y}_{rk}^I, r = 1, 2, \dots, s$. We wish to determine the hesitation present in DMU_k . Let P_{ik} be the hesitation present in each input variable $\tilde{x}_{ik}^I, i = 1, 2, \dots, m$ and P'_{rk} be the hesitation present in each output variable $\tilde{y}_{rk}^I, r = 1, 2, \dots, s$. Define

$$H_k = \min_{i,r} \{P_{ik}, P'_{rk}\}$$

as the minimum hesitation in DMU_k . In H_k , we minimize the hesitation over all inputs and outputs. Define $H = \min_k H_k$, as the minimum hesitation present in DMU_k .

3.2 Effect of hesitation on efficiency

In this section, we will understand the relation between the efficiency and the hesitation present in a DMU. The presence of high levels of hesitation in a variable, points to the high level of indeterminacy present in the variable. Hence, high hesitation value shows high amount of

unreliable information. Similarly, the small levels of hesitation in a DMU will signify high accuracy in efficiency calculations, whereas, high extent of hesitation in a DMU signifies that major information available about the DMU is unreliable and hence the efficiency results are unreliable. So, we define a penalty on each DMU based on the hesitation levels of the DMU. Consider a set of n DMUs using m inputs (TIFNs) to produce s number of outputs (TIFNs). Let the inputs be $\tilde{x}_{ik}^I, i = 1, 2, \dots, m$ where $\tilde{x}_{ik}^I = (x_{ik}^l, x_{ik}^m, x_{ik}^u; x_{ik}^l, x_{ik}^m, x_{ik}^u)$ and outputs be $\tilde{y}_{rk}^I, r = 1, 2, \dots, s$ where $\tilde{y}_{rk}^I = (y_{rk}^l, y_{rk}^m, y_{rk}^u; y_{rk}^l, y_{rk}^m, y_{rk}^u)$ for $DMU_k, k = 1, 2, \dots, n$. The non-radial IFSBM Model for IF inputs and outputs as given by Arya [3] is given as follows:

Model 1.

$$\tilde{E}_k^I = \min \rho - \frac{1}{m} \sum_{i=1}^m \frac{S_{ik}^-}{\tilde{x}_{ik}^I}$$

subject to

$$\rho + \frac{1}{s} \sum_{r=1}^s \frac{S_{rk}^+}{\tilde{y}_{rk}^I} = 1$$

$$\rho \tilde{x}_{ik}^I = \sum_{j=1}^n \tilde{x}_{ij}^I \lambda_j' + S_{ik}^-, \quad \rho \tilde{y}_{rk}^I = \sum_{j=1}^n \tilde{y}_{rj}^I \lambda_j' + S_{rk}^+$$

$$\sum_{j=1}^n \lambda_j' = \rho$$

$\rho > 0, S_{ik}^- \geq 0, S_{rk}^+ \geq 0, \lambda_j' \geq 0$ for all $i = 1, 2, \dots, m$ and $r = 1, 2, \dots, s$ and $j = 1, 2, \dots, n$. Now, the α -cuts of inputs and outputs are given by $(\tilde{x}_{ij}^I)_\alpha = [\alpha x_{ij}^m + (1 - \alpha)x_{ij}^l, \alpha x_{ij}^m + (1 - \alpha)x_{ij}^u]$ and $(\tilde{y}_{ij}^I)_\alpha = [\alpha y_{ij}^m + (1 - \alpha)y_{ij}^l, \alpha y_{ij}^m + (1 - \alpha)y_{ij}^u]$. Based on the favorable and unfavorable approach, we have upper and lower efficiency models (Model 2, 3) given below.

Model 2.

$$E_{k,\alpha}^l = \min \rho - \frac{1}{m} \sum_{i=1}^m \frac{S_{ik}^-}{\alpha x_{ik}^m + (1 - \alpha)x_{ik}^u}$$

subject to

$$\rho + \frac{1}{m} \sum_{r=1}^s \frac{S_{rk}^+}{\alpha y_{rk}^m + (1 - \alpha)y_{rk}^l} = 1$$

$$\rho(\alpha x_{ik}^m + (1 - \alpha)x_{ik}^u) = \sum_{j=1, j \neq k}^n (\alpha x_{ij}^m + (1 - \alpha)x_{ij}^l) \lambda_j' + (\alpha x_{ik}^m + (1 - \alpha)x_{ik}^u) \lambda_k' + S_{ik}^-,$$

where $i = 1, 2, \dots, m$,

$$\rho(\alpha y_{rk}^m + (1 - \alpha)y_{rk}^l) = \sum_{j=1, j \neq k}^n (\alpha y_{rj}^m + (1 - \alpha)y_{rj}^u) \lambda_j' + (\alpha y_{rk}^m + (1 - \alpha)y_{rk}^l) \lambda_k' + S_{rk}^+,$$

where $r = 1, 2, \dots, s$,

$$\sum_{j=1}^n \lambda_j' = \rho$$

$$\rho > 0, S_{ik}^- \geq 0 \text{ for } i = 1, 2, \dots, m$$

$$S_{rk}^+ \geq 0 \quad \text{for } r = 1, 2, \dots, s$$

$$\lambda_j' \geq 0 \quad \text{for } j = 1, 2, \dots, n$$

Model 3.

$$E_{k,\alpha}^u = \min \rho - \frac{1}{m} \sum_{i=1}^m \frac{S_{ik}^-}{\alpha x_{ik}^m + (1-\alpha)x_{ik}^l}$$

subject to

$$\rho + \frac{1}{m} \sum_{r=1}^s \frac{S_{rk}^+}{\alpha y_{rk}^m + (1-\alpha)y_{rk}^u} = 1$$

$$\rho(\alpha x_{ik}^m + (1-\alpha)x_{ik}^l) = \sum_{j=1, j \neq k}^n (\alpha x_{ij}^m + (1-\alpha)x_{ij}^u) \lambda_j' + (\alpha x_{ik}^m + (1-\alpha)x_{ik}^l) \lambda_k' + S_{ik}^-,$$

where $i = 1, 2, \dots, m$,

$$\rho(\alpha y_{rk}^m + (1-\alpha)y_{rk}^u) = \sum_{j=1, j \neq k}^n (\alpha y_{rj}^m + (1-\alpha)y_{rj}^l) \lambda_j' + (\alpha y_{rk}^m + (1-\alpha)y_{rk}^u) \lambda_k' + S_{rk}^+,$$

where $r = 1, 2, \dots, s$,

$$\sum_{j=1}^n \lambda_j' = \rho$$

$$\rho > 0, S_{ik}^- \geq 0 \quad \text{for } i = 1, 2, \dots, m$$

$$S_{rk}^+ \geq 0 \quad \text{for } r = 1, 2, \dots, s$$

$$\lambda_j' \geq 0 \quad \text{for } j = 1, 2, \dots, n$$

$[E_{k,\alpha}^l, E_{k,\alpha}^u]$ forms the efficiency interval corresponding to $\alpha \in (0, 1]$. Note that $[E_{k,\alpha}^l, E_{k,\alpha}^u] \subset (0, 1]$.

A DMU is said to be SBM-efficient if $E_{k,\alpha}^l = E_{k,\alpha}^u = 1$ ([3]). For an inefficient DMU, the input excess and output shortfalls are given by $S_{ik}^- = \rho^* \tilde{x}_{ik}^I - \sum_{j=1}^n \tilde{x}_{ij}^I \lambda_j'^*$ and $S_{rk}^+ = \rho^* \tilde{y}_{rk}^I - \sum_{j=1}^n \tilde{y}_{ij}^I \lambda_j'^*$, where ρ^* and $\lambda_j'^*$ are optimal values of ρ and λ_j' . However, this definition does not consider the hesitation present in the DMU.

Using the α -cuts, we can easily find the triangular fuzzy membership function of efficiency for each DMU. Let this triangular fuzzy membership function be (E_k^l, E_k^m, E_k^u) . The accuracy function of DMU_k is defined as $A_k = \frac{E_k^l + 2E_k^m + E_k^u}{4}$ [40].

Now, we will associate a penalty on the DMU's accuracy function based on the hesitation present in it. The penalty function should be such that it can impose higher penalty for high hesitation and lower penalty for low hesitation present in the DMU. Define the penalty function as $\frac{H_k}{H}$.

The efficiency after considering the effect of hesitation or the penalty is given by

$$\bar{E}_k = A_k \cdot \frac{1}{H_k/H} = \frac{(E_k^l + 2E_k^m + E_k^u)H}{4H_k}.$$

We define $\bar{E}_k$ as the overall efficiency of DMU_k .

Definition 3.1. DMU_k , is said to be efficient if its overall efficiency is one and it has lowest hesitation among all of the DMUs, i.e., $\bar{E}_k = 1$ (or its accuracy value, A_k , is 1) and $H = H_k$. Otherwise, the DMU is said to be inefficient.

One can easily determine the source of inefficiency—either because of low SBM efficiency (i.e., $A_k < 1$) or because of the amount of hesitation present or both. If the inefficiency is due to low accuracy score, then the DMU can become efficient by the increasing the output shortfall and decreasing the inputs excess. If the inefficiency is due to presence of high hesitation, than the decision maker needs to collect more precise data of the DMU. We can now rank the DMUs in decreasing order of their efficiency incorporated with hesitation, i.e., in decreasing order of $\bar{E}_k$.

3.3 Numerical example

Consider the example of 15 DMUs using two IF inputs and two IF outputs given in Table 1. Apply the IFSBM model for different values of $\alpha \in (0, 1]$ After compiling different cuts to find the membership function of DMUs in the form of triangular fuzzy number, their accuracy function has been calculated and given in Table 2. Now, calculate the maximum of P_1 and P_2 for each input and output for a particular DMU as given by the formula in Section 1 and then take minimum over all these four maximas obtained to get the value of H_k . For example, for DMU_6 , the maximum peaks are 0.714, 0.714, 0.56 and 0.583 respectively. The minimum hesitation present in DMU_6 is $\min(0.714, 0.714, 0.56, 0.583)$ i.e., $H_6 = 0.56$. We can also find minimum hesitation in the set of 15 DMUs given by $H = 0.4$ corresponding to DMU_3 . Now calculate the efficiency incorporated with hesitation, i.e., $\bar{E}_k$.

Table 1. Hypothetical data of DMUs in Example 1

DMU	Input 1	Input 2	Output 1	Output 2
1	(140,144,150;135,144,155)	(72,75,80;70,75,85)	(83085,83089,83092;83082,83089,83096)	(48,52,56;45,52,60)
2	(18,20,25;15,20,28)	(20,22,25;15,22,30)	(14105,14107,14110;14103,14107,14115)	(892,897,901;890,897,905)
3	(442,446,449;440,446,451)	(120,126,130;115,126,135)	(115855,115858,115862;115852,115858,115865)	(2,3,5;1,3,9)
4	(145,149,152;140,149,155)	(16,19,23;12,19,27)	(165120,165121,165125;165118,165121,165129)	(476,480,485;472,480,489)
5	(58,60,63;55,60,65)	(4,8,12;2,8,15)	(34095,34099,34105;34092,34099,34110)	(4275,4278,4282;4271,4278,4286)
6	(65,68,70;62,68,75)	(8,10,12;5,10,17)	(12185,12189,12195;12180,12189,12199)	(392,397,402;385,397,408)
7	(122,124,126;120,124,130)	(70,74,80;65,74,85)	(65290,65293,65298;65286,65293,65302)	(1150,1156,1160;1145,1156,1165)
8	(162,166,168;160,166,172)	(20,25,30;16,25,35)	(258750,258754,258758;258745,258754,258765)	(950,955,959;945,955,965)
9	(65,68,75;62,68,80)	(10,12,15;7,12,20)	(77852,77856,77859;77850,77856,77865)	(4430,4432,4437;4426,4432,4442)
10	(98,100,105;95,100,115)	(25,28,32;20,28,35)	(79720,79725,79729;79718,79725,79735)	(1235,1240,1245;1230,1240,1249)
11	(130,132,135;125,132,145)	(76,80,85;72,80,90)	(25385,25387,25392;25380,25387,25397)	(1090,1092,1097;1085,1092,1102)
12	(97,100,105;95,100,110)	(29,32,37;25,32,42)	(297445,297449,297453;297440,297449,297458)	(4545,4548,4553;4540,4548,4558)
13	(248,250,255;245,250,260)	(150,155,159;145,155,165)	(61190,61192,61196;61185,61192,61202)	(2692,2696,2699;2687,2696,2705)
14	(248,250,255;245,250,258)	(35,39,45;30,39,49)	(129432,129435,129438;129430,129435,129445)	(500,502,507;495,502,511)
15	(90,93,95;85,93,100)	(15,18,22;12,18,27)	(78275,78278,78282;78272,78278,78288)	(1662,1666,1670;1658,1666,1675)

By Table 2, we can see that $\bar{E}_k < 1$ for all DMUs. Though DMUs 2, 8 and 11 have accuracy value equal to 1, but they have hesitation higher than $H = 0.4$. Hence the result suggests that these DMUs are working efficiently if the hesitation is ignored but as they have to pay the penalty

of lack of accurate information about the data. After the penalty, the efficiency of DMUs reduces to 0.8 each. DMU_{10} and DMU_{13} have almost same accuracy, however, DMU_{13} has high level of hesitation (0.6) than DMU_{10} (0.6) and hence DMU_{13} faces higher penalty than DMU_{12} . As a result, the overall efficiency of DMU_{13} (0.05) is less than DMU_{10} (0.059). We can say that the relative best performers are DMUs 2, 8 and 11 as they have highest overall efficiency scores (0.8). The worst performer is DMU_1 (0.006).

Table 2. Efficiency of DMUs in Example 1

DMU	Membership function of efficiency	Accuracy function	H_k	$\bar{E}_k$
1	[0.0059, 0.0075, 0.0094]	0.008	0.500	0.006
2	[1, 1, 1]	1	0.500	0.800
3	[0.119, 0.201, 1]	0.381	0.400	0.381
4	[0.582, 1, 1]	0.895	0.500	0.716
5	[0.085, 0.124, 1]	0.333	0.429	0.311
6	[0.123, 0.141, 0.163]	0.142	0.556	0.102
7	[0.241, 1, 1]	0.810	0.556	0.583
8	[1, 1, 1]	1	0.500	0.800
9	[0.195, 0.244, 0.302]	0.246	0.500	0.197
10	[0.0667, 0.0729, 0.082]	0.074	0.500	0.059
11	[1, 1, 1]	1	0.500	0.800
12	[0.086, 0.0926, 0.102]	0.093	0.500	0.075
13	[0.0587, 0.074, 0.0926]	0.075	0.600	0.050
14	[0.292, 0.383, 1]	0.514	0.556	0.370
15	[0.117, 0.136, 0.155]	0.136	0.556	0.098

4 Application of MGNREGA 2021–2022

The Mahatma Gandhi National Rural Employment Guarantee Act 2005 (MGNREGA) was started by Government of India in September 2005. The aim of the scheme is to provide at least guaranteed 100 days of employment to unemployed youths who are willing to work but are unable to find it. Initially, the scheme was started for rural areas but it was later extended to many urban areas as well. The applicants are given unskilled manual work to complete. The main objective behind the launch of the scheme of MGNREGA is to provide livelihood security and a sense of independence especially in the people living below poverty line. The scheme provides employment to everyone, including women and physically challenged people. The

desiring applicants are given job cards by the Gram Panchayat, who provides unskilled manual work within fifteen days of the application and preferably within 5km radius of the village they reside in. The identification and allotment of work and estimated cost are pre-planned by different Government levels, i.e., by Gram Panchayat, Gram Sabha, Panchayat Samiti, Block Programme Officer and District Programme Coordinator. The major works included under the scheme are construction and renovation of government buildings, roads, food grain storage structures, water harvesting structures, dams, anganwadis, playgrounds, afforestation and tree plantation, irrigation works, land development, agriculture related work, livestock related work, fisheries related work, rural drinking water related work, sanitation work etc.. If the government fails to provide employment within fifteen days of receipt of application. In our study, we will take Indian states and Union Territories as DMUs. We will consider two inputs and two IF outputs for the study and check the efficiency of states performing under MGNREGA in the financial year 2021–22. The inputs and outputs used are given as follows:

Inputs:

1. **Employment demanded:** It is the total number of people who demanded employment.
2. **Work taken up:** It is the total amount of work that was taken up to be completed.

Outputs:

1. **Employment offered:** It is total number of people that were given employment.
2. **Work completed:** It refers to the work completed.

The data has been collected from official website of MGNREGA and is given in Table 3. Out of 29 DMUs, 10 DMUs have efficiency membership function as $[1, 1, 1]$ i.e., the states of Andhra Pradesh, Chhattisgarh, Goa, Jharkhand, Karnataka, Madhya Pradesh, Mizoram, Tamil Nadu, Tripura and West Bengal. All these DMUs have accuracy function equal to 1. For the state of Andhra Pradesh, the minimum hesitation is $\min(0.487, 0.244, 0.713, 0.429) = 0.244$. The minimum hesitation in all the DMUs is 0.219 (for the state of Uttarakhand). So, the penalty faced by state of Andhra Pradesh is $0.244/0.219 = 1.115$.

Similarly, for other states, the accuracy function, the minimum hesitation, penalty and the overall efficiency are given in Table 4.

Both Chhattisgarh and Goa have efficiency membership value as 1, however, the hesitation in Chhattisgarh is higher than that of Goa, hence it faces higher penalty, resulting in lesser overall efficiency than Goa. No state has overall efficiency equal to one as no state is able to excel in both fields, i.e., have highest efficiency function and lowest hesitation simultaneously. By the ranking based on overall efficiency, the best performer is the state of Tamil Nadu, followed by Andhra Pradesh and Tripura. The worst performing states are Manipur, Gujarat, Nagaland and Arunachal Pradesh.

Table 3. Data for inputs and outputs for MGNREGA.

DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
ANDHRA PRADESH	(7279000,7279422,7279600,7278600,7279422,7279750)	(402860,403009,403040,402819,403009,403050)	(7252150,7252512,7252650,7251250,7252512,7252980)	(168126,168158,168176,168102,168158,168180)
ARUNACHAL PRADESH	(98000,98716,99000,97965,98716,99150)	(4920,4932,4950,4890,4932,5000)	(92680,93894,94250,92150,93894,94560)	(2461,2468,2472,2452,2468,2479)
ASSAM	(2146120,2146283,2146350,2145880,2146283,2146550)	(282480,282517,282540,282420,282517,282550)	(2144650,2144838,2144960,2144050,2144838,2145160)	(106430,106459,106475,106390,106459,106480)
BIHAR	(2986400,2986574,2986750,2986310,2986574,2986850)	(459850,459895,459920,459810,459895,459940)	(2975602,2976042,2976052,2974205,2976042,2977960)	(229043,229099,229144,229019,229099,229168)
CHHATTISGARH	(4144450,4144538,4144650,4144390,4144538,4144700)	(190750,190780,190798,190720,190780,190823)	(4140950,4141579,4141900,4140260,4141579,4142260)	(91445,91498,91510,91410,91498,91520)
GOA	(170,224,270,153,224,300)	(215,238,245,200,238,260)	(205,214,260,190,214,290)	(125,130,132,121,130,138)
GUJARAT	(1113360,1113486,1113550,1113310,1113486,1113600)	(269157,269185,269202,269100,269185,269215)	(1110150,1110257,1110400,1110000,1110257,1110950)	(121320,121365,121390,121290,121365,121420)
HARYANA	(294600,295030,295200,294510,295030,295350)	(21756,21799,21812,21733,21799,21834)	(294050,294129,294560,293850,294129,294960)	(12940,12961,12980,12910,12961,12994)
HIMACHAL PRADESH	(541600,541819,541980,541520,541819,542150)	(128230,128267,128298,128198,128267,128323)	(541287,541489,542500,541020,541489,542840)	(66439,66457,66480,66410,66457,66510)
JAMMU AND KASHMIR	(262070,262120,262350,261952,262120,262500)	(147650,147685,147690,147610,147685,147729)	(260520,260921,261250,260000,260921,261560)	(54410,54472,54490,54390,54472,54510)
JHARKHAND	(1544640,1544891,1544940,1544480,1544891,1544990)	(695302,695312,695330,695298,695312,695340)	(1542960,1543438,1543584,1542890,1543438,1543680)	(321250,321279,321290,321230,321279,321312)
KARNATAKA	(3782810,3783013,3783180,3782740,3783013,3783250)	(853158,853179,853209,853127,853179,853224)	(3636870,3637160,3637850,3636520,3637160,3637960)	(574220,574271,574300,574170,574271,574350)
KERALA	(1478250,1478446,1478650,1478125,1478446,1478720)	(280833,280848,280894,280818,280848,280916)	(1477940,1478006,1478250,1477670,1478006,1478541)	(150430,150472,150499,150390,150472,150520)
MADHYA PRADESH	(5073958,5074048,5074245,5073852,5074048,5074290)	(1203012,1203042,1203062,1202992,1203042,1203079)	(5055420,5055794,5055980,5054450,5055794,5056120)	(788120,788124,788150,788100,788124,788190)
MAHARASHTRA	(2249550,2249761,2249860,2249480,2249761,2249920)	(399116,399173,399200,399093,399173,399219)	(2242890,2243136,2243250,2242450,2243136,2243670)	(115724,115760,115786,115680,115760,115798)
MANIPUR	(472690,472817,472950,472580,472817,472990)	(16500,16522,16543,16470,16522,16576)	(440560,440713,440950,440120,440713,441250)	(672,675,680,670,675,685)
MEGHALAYA	(404250,404531,404560,404160,404531,404582)	(30540,30593,30610,30500,30593,30634)	(404190,404238,404280,404102,404238,404500)	(4095,4108,4123,4070,4108,4130)
MIZORAM	(199650,199886,199960,199530,199886,199990)	(23460,23469,23480,23420,23469,23496)	(198650,199092,199650,198450,199092,199840)	(22947,22993,23120,22910,22993,23142)
NAGALAND	(242750,242930,243060,242680,242930,243150)	(3640,3658,3673,3620,3658,3684)	(183200,183371,183450,183100,183371,183680)	(1056,1069,1073,1034,1069,1085)
ODISHA	(2195000,2195178,2195520,2194920,2195178,2195600)	(316214,316243,316267,316180,316243,316284)	(2186950,2187549,2187940,2186450,2187549,2188450)	(133526,133567,133583,133489,133567,133600)
PUNJAB	(867490,867717,867960,867350,867717,868150)	(55934,55965,55987,55910,55965,55999)	(865657,865856,866245,865120,865856,866769)	(21420,21437,21460,21390,21437,21476)
RAJASTHAN	(7545480,7545564,7545820,7545360,7545564,7545910)	(520900,520934,520956,520896,520934,520974)	(7539565,7540032,7541234,7537842,7540032,7547654)	(193500,193542,193570,193460,193542,193593)
TAMIL NADU	(6079000,6079159,6079300,6078910,6079159,6079360)	(447858,447895,447912,447810,447895,447930)	(6076876,6077471,6077987,6076745,6077471,6078145)	(313829,313887,313927,313807,313887,313957)
TELANGANA	(4930470,4930591,4930750,4930360,4930591,4930790)	(353700,353744,353760,353680,353744,353782)	(4905245,4905634,4905970,4905000,4905634,4906345)	(158836,158870,158926,158810,158870,158960)
TRIPURA	(706520,706624,706850,706480,706624,706890)	(182920,182973,182999,182890,182973,183004)	(706125,706275,706439,705986,706275,706510)	(135827,135880,135928,135810,135880,135960)
UTTAR PRADESH	(5305120,5305290,5305460,5305060,5305290,5305499)	(1042985,1043010,1043120,1042960,1043010,1043180)	(5293145,5293913,5294024,5292860,5293913,5295342)	(626054,626107,626164,626018,626107,626242)
UTTARAKHAND	(397000,397250,397510,396930,397250,397560)	(98999,99026,99056,98986,99026,99071)	(396534,396733,396987,396100,396733,397324)	(41930,41956,41988,41916,41956,42017)
WEST BENGAL	(3214562,3214765,3214970,3214480,3214765,3215210)	(1232327,1232387,1232417,1232267,1232387,1232473)	(3210131,3210240,3210350,3210000,3210240,3210540)	(447075,447142,447182,447059,447142,447238)
ANDAMAN AND NICOBAR	(4613,4814,4900,4410,4814,4950)	(408,414,420,404,414,422)	(4640,4788,4850,4590,4788,4940)	(169,173,175,165,173,180)

Table 4. Accuracy, hesitation, penalty, overall efficiency and ranks for states under MGNREGA.

States	Membership function of efficiency	Accuracy function	Hesitation	Penalty function	Overall efficiency	Rank
ANDHRA PRADESH	[1,1,1]	1	0.244	1.115	0.897	2
ARUNACHAL PRADESH	[0.102,0.112,0.117]	0.111	0.346	1.580	0.070	26
ASSAM	[0.409,0.410,0.410]	0.410	0.580	2.650	0.155	22
BIHAR	[0.419,0.420,0.420]	0.420	0.348	1.590	0.264	18
CHHATTISGARH	[1,1,1]	1	0.405	1.853	0.540	10
GOA	[1,1,1]	1	0.395	1.805	0.554	9
GUJARAT	[0.0965,0.097,0.097]	0.097	0.439	2.005	0.048	28
HARYANA	[0.490,0.493,0.497]	0.494	0.469	2.143	0.230	19
HIMACHAL PRADESH	[0.715,0.718,0.720]	0.718	0.464	2.120	0.339	14
JAMMU AND KASHMIR	[0.469,0.478,0.483]	0.477	0.526	2.406	0.198	21
JHARKHAND	[1,1,1]	1	0.357	1.633	0.613	8
KARNATAKA	[1,1,1]	1	0.295	1.350	0.741	4
KERALA	[0.522,0.523,0.524]	0.523	0.389	1.780	0.294	16
MADHYA PRADESH	[1,1,1]	1	0.459	2.100	0.476	12
MAHARASHTRA	[0.489,0.489,0.489]	0.489	0.377	1.725	0.284	17
MANIPUR	[0.049,0.049,0.050]	0.050	0.464	2.122	0.023	29
MEGHALAYA	[0.191,0.193,0.195]	0.193	0.431	1.972	0.098	25
MIZORAM	[1,1,1]	1	0.312	1.424	0.702	7
NAGALAND	[0.123,0.128,0.137]	0.129	0.409	1.870	0.069	27
ODISHA	[0.697,0.698,0.698]	0.698	0.310	1.417	0.492	11
PUNJAB	[0.278,0.281,0.284]	0.281	0.436	1.995	0.141	23
RAJASTHAN	[0.4267,0.4269,0.427]	0.427	0.450	2.057	0.208	20
TAMIL NADU	[1,1,1]	1	0.234	1.072	0.933	1
TELANGANA	[0.592,0.592,0.593]	0.592	0.433	1.981	0.299	15
TRIPURA	[1,1,1]	1	0.278	1.27	0.788	3
UTTAR PRADESH	[0.868,0.868,0.868]	0.868	0.261	1.193	0.728	6
UTTARAKHAND	[0.730,0.732,0.733]	0.732	0.219	1	0.732	5
WEST BENGAL	[1,1,1]	1	0.539	2.465	0.406	13
ANDAMAN AND NICOBAR	[0.197,0.237,0.259]	0.233	0.400	1.829	0.127	24

5 Conclusion

The present study focuses on the concept of hesitation in IFSs and its impact on decision-making and efficiency calculations. Specifically, the study delves deeper into the hesitation present in a triangular intuitionistic fuzzy variable and in a DMU with multiple TIFN inputs and outputs.

To evaluate the efficiency of a set of homogeneous DMUs using IF inputs and outputs, the study employs the IFSBM model and defines the overall efficiency of DMUs based on the degree of hesitation present in the DMU. This is done by defining a penalty function that takes into account the hesitation present in the decision-making process.

The proposed study aims to explore the fundamental ideas behind the foundation of IFSs and utilize them actively in decision-making and efficiency calculations. By understanding the role of hesitation in IFSs, the study hopes to improve the accuracy and effectiveness of decision-making processes.

In addition, the study suggests that there is ample opportunity to expand on the concept of hesitation in IFS and its impact on efficiency. For example, researchers could explore new applications of the concept in different fields or investigate the effect in different forms of hesitation on decision-making processes. Further research in this area could lead to new insights and applications in a variety of fields.

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