10th Int. Workshop on IFSs, Banská Bystrica, 29 Sept. 2014
Notes on Intuitionistic Fuzzy Sets
ISSN 1310–4926
Vol. 20, 2014, No. 4, 41–46

Sum- and average-based approach to criteria shortlisting in the InterCriteria Analysis

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Abstract: Here we discuss an alternative approach to shortlisting the *k* out of *n* best correlating criteria at the end of the InterCriteria Analysis. We propose to use the operation 'average-row-aggregation', instead of 'max-row-aggregation' as done in previous research, and compare and discuss the results produced with both approaches. For illustration of the proposed approach, data are used from the World Economic Forum's annual Global Competitiveness Reports, for the 28 EU member states from year 2008–2009 to year 2014–2015.
Keywords: Index matrix, Intercriteria decision making, Intuitionistic fuzzy sets.
AMS Classification: 03E72.

1 Introduction

In a recent leg of research, published in a series of papers, Atanassova and coauthors have been investigating the InterCriteria Analysis approach to decision making, based on the idea that from data sets of evaluations of multiple objects against multiple criteria, information is derived about exhibited dependencies / correlations among the criteria themselves.

The approach employs the concept of index matrices, making particular use of some of the operations introduced over them, and the concept of intuitionistic fuzziness, giving us the tools, to construct the index matrices of intuitionistic fuzzy pairs, defining the presence or absence of dependency / correlation between any pair of criteria within the set. The role of both concepts for the ICA approach has been presented in details in [2].

From the various aspects of this new approach, which need investigation, one particular question is related to the determination of the two threshold values, named α and β, which respectively help discriminating among the membership and non-membership values of the intercriteria correlations in the resultant index matrix. With their help, we can finally determine which pairs of criteria are in the so-called *positive consonance*, in *negative consonance* or in *dissonance* – the three possible states of correlation we have introduced within the ICA approach.

In previous studies, there have been proposed several approaches to defining the two threshold values. In our line of reasoning, the immediate first idea was letting the decision maker define the thresholds based on their expertise, or by setting a series of predefined pairs of values, which are gradually changing: α changing in a top-down, and β changing in a bottom-up manner, [3, 5].

In a next step, [6] it was noticed that this approach may yield some rather different results for α and β. Some finer approach was needed for the case when one of the values, interchangeably, was fixed, and the other one has to be accordingly determined, in order to yield commensurate (if not identical) results. This also lead to defining the two thresholds based on a possible decision maker's need to identify, to *shortlist* the set of k out of n criteria, which exhibit the most expressed positive consonance. For this sake, a simple algorithm has been developed, which for every separate criteria calculates its correlation with each of the rest $(n - 1)$ criteria, and takes the maximal value; and then sorts these n values in a descending way, selecting the top k of them.

This approach required the decision maker to set in advance the number $0 < k < n$ and work with the values of α and β that correspond to k. However, we can notice that in this way it is quite possible to "draw the line" at a wrong place: the differences between the α-s and β-s of the first $(k + 1)$ and between the α-s and β-s of the first k criteria might be negligible, while the differences between the α-s and β-s of the first $(k - 1)$ and between the α-s and β-s of the first k criteria might be quite well expressed. Thus, in [4] we started discussing the *homogeneity* of the criteria, in order to abstract from a particular number k, but attempt to identify the (unknown in advance) "most strongly correlating criteria". This question has not been exhaustively explored yet, thus giving the possibility for further discussions.

2 Main results

In this paper, we return back to the idea of shortlisting of k best correlating out of n criteria, and make a further discussion on an alternative way for their determining. It is noteworthy that at this point we cannot claim that this alternative yields better results than the approach proposed so far, but it is worth considering, exploring and comparing it.

The algorithm proposed in [6] calculates for every separate criterion its positive and negative consonance with each of the rest $n - 1$ criteria, and sorts the positive consonances with respect to the exhibited maximal value before selecting the top k of the n criteria. Taking the maximum value is equivalent to applying the 'max-row-aggregation' operation from the augmented calculus of index matrices, as presented in [1], page 8.

Here, we note that there are two other operations in the index matrix calculus, namely 'sum-row-aggregation' and 'average-row-aggregation', which may be used instead of the 'max-row-aggregation' operation in the above discussed algorithm. Practically, due to the constant number, $n - 1$, in the denominator, when constructing the average, there will be no difference whether we apply the 'sum' or the 'average' operation: both will result in the same ordering of the n criteria, and the same sets of shortlisted k out of n criteria. The only difference is that the average values will be in the [0; 1] interval, i.e. will directly produce the respective values for the threshold values α and β. For this reason, we choose to (interchangeably) take for our further discussions the 'average-row-aggregation' operation, or briefly, the 'ave'. The modified, 'ave'-based algorithm is given below.

'Ave'-based Algorithm

1. For each criterion C_i, $i = 1, \ldots, n$, we find $\text{ave}_\mu(C_i) = \frac{1}{n-1}\sum_{j=1, j\neq i}^{n} \mu(C_i, C_j)$, i.e. the sum of the discovered correlations of C_i with the rest criteria C_j, divided by $n - 1$.
2. We create a table like the one shown on Table 1.

Column (1)	Column (2)
C_1	$\text{ave}_\mu(C_1)$
C_2	$\text{ave}_\mu(C_2)$
…	…
C_n	$\text{ave}_\mu(C_n)$

Table 1.

3. We sort the whole Table 1 by Column (2) in descending order.
4. We shortlist the first k criteria from Column (1) in the resultant sorted table.

By analogy, the algorithm can be formulated for the case of the negative consonance, but there the sorting in Step 3. will be in ascending order. In comparison with the max-based algorithm from [6], here we cannot claim that "[the] sought value of the threshold α is then the respective value in Column (2) on k-th place top down, in the resultant sorted table", because here we deal with mean values.

Example and Discussion

It is interesting to compare the orderings of the n criteria, as produced by the application of the 'ave' and the 'max' operations. Our preliminary expectation is that the results should be different (due to the fact that they are completely data-driven), but will nevertheless exhibit high similarity.

For the purpose of illustration of the herewith theoretically proposed approach, we will again rely on previously used data from the Global Competitiveness Reports of the World Economic Forum, [7]. We have chosen to use the data from years 2008–2009 to 2013–2014 for the 28 Member-states of the European Union, as evaluated against the 12 criteria in the WEF methodology, known as 'pillars of competitiveness'.

Since in [6] the demonstrated results for the 'max' operation are for the two extreme years of the period (2008–2009 and 2013–2014), we will repeat them here and compare with those obtained for the 'ave' operation. As in [6], using the 'max'-based algorithm there, we determine columns (3) "Criteria ordered by max positive (resp. min negative) consonance" and (4) "True when $\alpha \geq$ (resp. $\beta \leq$)", and on this basis generate the data in columns (1) "Number of correlating criteria" and (2) "Number of pairs of correlating criteria". When using the 'average' operation, however, the meaning of column (4) “True when $\alpha \geq$ (resp. $\beta \leq$)” is not the same, because it already does not reflect the maximal threshold α for positive consonance, respectively the minimal threshold β for negative consonance, but some mean values, calculated on the basis of all the n criteria. For this reason, the columns (1) “Number of correlating criteria” and (2) “Number of pairs of correlating criteria” for the ‘average’-based ordering are already irrelevant, and will not be given. We will only concentrate on the resultant ordering of the criteria, by average positive consonance and by average negative consonance, as shown in Table 4 for the year 2008–2009, and in Table 5 for the year 2013–2014.

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by max positive consonance	True when $\alpha \geq$
2	1	11	0.860
		12	
4	2	1	0.844
		2	
5	3	6	0.833
6	5	8	0.828
7	6	9	0.823
8	14	5	0.796
9	18	4	0.780
10	37	3	0.693
11	41	7	0.664
12	45	10	0.648

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by min negative consonance	True when $\beta \leq$
2	1	1	0.077
		6	
4	2	11	0.079
		12	
5	3	8	0.09
6	4	9	0.095
8	5	4	0.108
		5	
9	8	2	0.114
11	35	3	0.204
		7	
12	54	10	0.307

Table 2. Results for year 2008–2009, as produced by the ‘max’ operation, [6].

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by max positive consonance	True when $\alpha \geq$
2	1	11	0.873
		12	
4	2	1	0.854
		9	
5	3	5	0.847
6	11	2	0.804
7	13	6	0.788
9	20	7	0.749
		8	
10	25	4	0.730
11	37	3	0.675
12	39	10	0.661

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by min negative consonance	True when $\beta \leq$
2	1	11	0.071
		12	
4	2	1	0.077
		6	
5	3	5	0.079
6	4	9	0.090
8	13	2	0.135
		7	
9	17	4	0.143
10	19	8	0.146
11	38	3	0.251
12	45	10	0.286

Table 3. Results for year 2013–2014, as produced by the ‘max’ operation, [6].

Criteria ordered by ave positive consonance	ave_μ
1	0.771
12	0.769
11	0.768
9	0.765
6	0.751
2	0.748
5	0.744
8	0.743
4	0.722
3	0.665
7	0.617
10	0.563

Criteria ordered by ave negative consonance	ave_ν
6	0.147
12	0.155
1	0.163
9	0.165
11	0.166
4	0.175
5	0.175
8	0.194
2	0.195
3	0.249
7	0.261
10	0.376

Table 4. Results for year 2008–2009, as produced by the 'ave' operation.

Criteria ordered by ave positive consonance	ave_μ
1	0.763
12	0.758
11	0.754
9	0.75
5	0.734
6	0.726
2	0.707
8	0.687
4	0.662
7	0.661
3	0.577
10	0.563

Criteria ordered by ave negative consonance	ave_ν
11	0.174
9	0.175
12	0.175
6	0.176
1	0.179
5	0.183
8	0.224
2	0.225
4	0.230
7	0.244
3	0.361
10	0.371

Table 5. Results for year 2013–2014, as produced by the 'ave' operation.

As a general difference between the two algorithms, here we do not expect that the two top correlating criteria will always be 'twins', as seen for criteria 11/12 for positive consonance and criteria 1/6 for negative consonance for the year 2008–2009 in Table 2, and for criteria 11/12 for positive consonance and criteria 11/12 for negative consonance for the year 2013–2014 in Table 3. The reason is that the 'max-row-aggregation' operation simultaneously gives the information for two compared criteria at a time, while the 'average-row-aggregation' is obtained on the basis of the values of all $n - 1$ cells in each row of the index matrix, excluding the cell from the main diagonal, where, by definition, $\mu(C_i, C_j) = 1,\ \nu(C_i, C_j) = 0$.

Despite the obvious differences, we are interested to discuss the similarities in the results between the two algorithms. Since both approaches are completely data-driven and so far tested only over the data excerpted from the WEF Global Competitiveness Reports, we obviously cannot aim at strong generalizations.

We however note that there are strong similarities in the results yielded with both approaches, outlining the criteria of highest positive consonance '1. Infrastructure', '11. Business sophistication' and '12. Innovation', closely followed by criteria '6. Goods market efficiency' and '9. Technological readiness'. These findings are in support of Observations 3, 6, 7, 8 in [5], achieved using another approach with predefined threshold values of α and β. The criterion '5. Higher education and training' also appears to highly correlate in the end of the examined period.

In the other extreme, the criteria with highest negative consonance are criteria '10. Market size', '3. Macroeconomic stability', and '7. Labor market efficiency', as supported also by Observation 1 in [5], followed by another low-correlating criteria '4. Health and primary education'.

The middle zone of criteria with medium positive and negative consonance are ranked differently for every year and for each of the algorithms.

3 Conclusions

Testing the 'max'- and 'ave'-based approaches with different datasets will further help clarifying whether one of them is more adequate, in general or in some specific applications, or both approaches are to be applied simultaneously for better informed decision making. Under the stipulation that both approaches are data-driven by nature, we observe that the results produced with both of the algorithms exhibit significant similarities in the highest and lowest zones of the produced rankings, which also is confirmed with the findings of other approaches. This means that the new approach for ranking positive and negative consonances in the InterCriteria Analysis is a beneficial area of further research and has the capacity of discovering new patterns, undetected with other approaches, or reconfirm the findings of the others.

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