Notes on Intuitionistic Fuzzy Sets
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A short note on intuitionistic fuzzy operators $X_{a,b,c,d,e,f}$ and $x_{a,b,c,d,e,f}$

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Abstract: An ommision of a condition in the definitions of operators $X_{a,b,c,d,e,f}$ and $x_{a,b,c,d,e,f}$, described in [2, 3] is discussed.
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1 Introduction

In two books of mine, [2, 3], the basic elements of the theory of the Intuitionistic Fuzzy Sets (IFSs) are introduced. In these books, modal operators (for them see, e.g., [4]) and a series of their extensions are described. One of the most powerful extensions of the modal operators is operator $X_{a,b,c,d,e,f}$.

Unfortunately, in both cited books, one of the conditions, that is necessary for the validity of the definition of operator $X_{a,b,c,d,e,f}$ has been omitted. Now, it is explicitly included in the definition, and discussed.

Let us have a fixed universe E and its subset A. The set

$$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in E\},$$

where

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

is called IFS and functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ represent the *degree of membership (validity, etc.)* and *non-membership (non-validity, etc.)*. Now, we can define also function $\pi_A : E \to [0, 1]$ by means of

$$\pi(x) = 1 - \mu(x) - \nu(x)$$

and it corresponds to *degree of indeterminacy (uncertainty, etc.).*

Below, we write A instead of A^*.

Let $\alpha, \beta \in [0, 1]$ and let:

$$\square A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\},$$

$$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\},$$

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1 - \alpha).\pi_A(x)\rangle | x \in E\},$$

$$F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\}, \text{ where } \alpha + \beta \leq 1,$$

$$G_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \beta.\nu_A(x)\rangle | x \in E\},$$

$$H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\},$$

$$H^*_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.(1 - \alpha.\mu_A(x) - \nu_A(x))\rangle | x \in E\},$$

$$J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x)\rangle | x \in E\},$$

$$J^*_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.(1 - \mu_A(x) - \beta.\nu_A(x)), \beta.\nu_A(x)\rangle | x \in E\},$$

$$f_{\alpha,\beta}(A) = \{\langle x, \nu_A(x) + \alpha.\pi_A(x), \mu_A(x) + \beta.\pi_A(x)\rangle \mid x \in E\}, \text{ where } \alpha + \beta \leq 1;$$

$$g_{\alpha,\beta}(A) = \{\langle x, \alpha.\nu_A(x), \beta.\mu_A(x)\rangle \mid x \in E\};$$

$$h_{\alpha,\beta}(A) = \{\langle x, \alpha.\nu_A(x), \mu_A(x) + \beta.\pi_A(x)\rangle \mid x \in E\};$$

$$h^*_{\alpha,\beta}(A) = \{\langle x, \alpha.\nu_A(x), \mu_A(x) + \beta.(1 - \alpha.\nu_A(x) - \mu_A(x))\rangle \mid x \in E\};$$

$$j_{\alpha,\beta}(A) = \{\langle x, \nu_A(x) + \alpha.\pi_A(x), \beta.\mu_A(x)\rangle \mid x \in E\};$$

$$j^*_{\alpha,\beta}(A) = \{\langle x, \nu_A(x) + \alpha.(1 - \nu_A(x) - \beta.\mu_A(x)), \beta.\mu_A(x)\rangle \mid x \in E\};$$

The operators from modal type were extended in some directions, as described in [2, 3]. Here, we mention only one of the directions of their extensions.

The first of the next operators includes as a partial case the operators $D_\alpha, ..., J^*_{\alpha,\beta}$ and the second one includes as a partial case the operators $d_\alpha, ..., j^*_{\alpha,\beta}$:

$$X_{a,b,c,d,e,f}(A) = \{\langle x, a.\mu_A(x) + b.(1 - \mu_A(x) - c.\nu_A(x)),$$
$$d.\nu_A(x) + e.(1 - f.\mu_A(x) - \nu_A(x))\rangle | x \in E\}$$

and

$$x_{a,b,c,d,e,f}(A) = \{\langle x, a.\nu_A(x) + b.(1 - \nu_A(x) - c.\mu_A(x)),$$
$$d.\mu_A(x) + e.(1 - f.\nu_A(x) - \mu_A(x))\rangle | x \in E\},$$

respectively, where $a, b, c, d, e, f \in [0, 1]$. For the first operator, in [2, 3], and for the second operator, in [3], the following two conditions are given:

$$a + e - e.f \leq 1, \tag{1}$$

$$b + d - b.c \leq 1. \tag{2}$$

Now, we show that the following third condition is necesary to be added

$$b + e \leq 1, \tag{3}$$

because for the IFS

$$U^* = \{\langle x, 0, 0\rangle | x \in E\}$$

we obtain

$$X_{0,1,0,0,1,0}(U^*) = \{\langle x, 1, 1\rangle | x \in E\},$$

$$x_{0,1,0,0,1,0}(U^*) = \{\langle x, 1, 1\rangle | x \in E\},$$

that is impossible.

On the other hand, this condition is valid in all cases when operators $X_{a,b,c,d,e,f}$ and $x_{a,b,c,d,e,f}$ represent some of the above described modal type of operators.

References

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