

ON THE INTUITIONISTIC Q-FUZZY IDEALS OF NEAR-RINGS

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Abstract

In this paper, we introduce the notion of intuitionistic Q-fuzzification of ideals in a near-ring and investigate some related properties. Characterization of intuitionistic Q-fuzzy ideals is given.

Key words and phrases: Q-fuzzy set, Near-rings, Intuitionistic Q-fuzzy set, intuitionistic Q-fuzzy ideals.

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1 Introduction and preliminaries:

Zadeh [13] in 1965 introduced fuzzy sets after which several researchers explored on the generalizations of the notion of fuzzy sets and its application to many mathematical branches. Abou-Zaid[1], introduced the notion of a fuzzy subnear-ring and studied fuzzy ideals of a near-ring. This concept is further discussed by many researchers, among whom Cho, Davvaz, Biswas, Jun, Kim.[3,4,5,6,12]

The idea of intuitionistic fuzzy sets was introduced by Atanassov [2] as a generalization of the notion of fuzzy sets. In[3], Biswas applied the concept of intuitionistic fuzzy set to the theory of groups and studied intuitionistic fuzzy subgroups of a group. The notion of an intuitionistic fuzzy R-subgroups of a near-ring is given by Jun, Yon, Cho in [4,12]. Zhan Jianming and Ma Xueling[14], also discussed the various properties on intuitionistic fuzzy ideals of near-rings. Also Cho *et al* in [4] the notion of a normal intuitionistic fuzzy N-subgroup in a near-ring is introduced and related properties are investigated. Recently, Roh *et al* in [11] discussed the intuitionistic Q-fuzzification of the concept of subalgebras in BCK/BCI-algebra and investigated various properties. Osman Kazanci *et al* in [7] have introduced the notion of intuitionistic Q-fuzzification of N-subgroups(subnear-rings) in a near-ring

and investigated some related properties. In this paper, we introduce the concept of intuitionistic Q-fuzzification of ideals in a near-ring and investigate some related properties.

By a near-ring we mean a nonempty set N with two binary operations $+$ and \cdot satisfying the following axioms

- (i) $(N, +)$ is a group
- (ii) (N, \cdot) is a semigroup
- (iii) $(x + y) \cdot z = x \cdot z + y \cdot z$ for all $x, y, z \in N$

Precisely speaking it is a right near-ring because it satisfies the right distributive law. We use xy to denote $x \cdot y$. A near-ring N is called zero symmetric if $x \cdot 0 = 0$ for all $x \in N$. An ideal of a near-ring N is a subset I of N such that

- (i) $(I, +)$ is a normal subgroup of $(N, +)$
- (ii) $IN \subseteq I$
- (iii) $y(x + i) - y \cdot x \in I$ for all $x, y \in N$ and $i \in I$

By a fuzzy set μ in a nonempty set X , we mean a function $\mu: X \rightarrow [0, 1]$ and the complement of μ denoted by $\bar{\mu}$ is the fuzzy set in X given by $\bar{\mu}(x) = 1 - \mu(x)$, for $x \in X$. For any $t \in [0, 1]$, and a fuzzy set μ in a nonempty set X , the set $U(\mu; t) = \{x \in X | \mu(x) \geq t\}$ is called an upper t -level cut of μ and the set $L(\mu; t) = \{x \in X | \mu(x) \leq t\}$ is called a lower t -level cut of μ .

An intuitionistic fuzzy set (IFS) A in a nonempty set X is an object having the form $IFSA = \{(x, \alpha_A(x), \beta_A(x)) | x \in X\}$ where the functions $\alpha_A: X \rightarrow [0, 1]$ and $\beta_A: X \rightarrow [0, 1]$ denote the degree of membership and the degree of non membership respectively, and $0 \leq \alpha_A(x) + \beta_A(x) \leq 1, x \in X$.

In what follows, let Q and N denote a set and a near-ring, respectively, unless otherwise stated. A mapping $\mu: N \times Q \rightarrow [0, 1]$ is called a Q -fuzzy set in N . For any Q -fuzzy set μ in N and any $t \in [0, 1]$ we define two sets $U(\mu; t) = \{x \in X | \mu(x, q) \geq t, q \in Q\}$ and $L(\mu; t) = \{x \in X | \mu(x, q) \leq t, q \in Q\}$ which are called an upper and lower t -level cut of μ and can be used to give the characterization of μ . An intuitionistic Q -fuzzy set (IQFS for short) A is an object having the form $A = \{(x, q), \alpha_A(x, q), \beta_A(x, q)) | x \in X, q \in Q\}$, where the functions $\alpha_A: X \times Q \rightarrow [0, 1]$ and $\beta_A: X \times Q \rightarrow [0, 1]$ denote the degree of membership and the degree of non membership for each element (x, q) to the set A respectively, and $0 \leq \alpha_A(x, q) + \beta_A(x, q) \leq 1$ for all $x \in X$ and $q \in Q$. For the sake of simplicity we shall use the symbol $A = (\alpha_A, \beta_A)$ for the $IQFSA = \{((x, q), \alpha_A(x, q), \beta_A(x, q)) | x \in X, q \in Q\}$.

Definition 1.1 (O. Kazanci *et al* [7]) *A Q -fuzzy set μ in a near-ring N is called Q -fuzzy subnear-ring of N if*

$$(QF1) \quad \mu(x - y, q) \geq \min\{\mu(x, q), \mu(y, q)\}$$

$$(QF2) \quad \mu(xy, q) \geq \min\{\mu(x, q), \mu(y, q)\} \text{ for all } x, y \in N \text{ and } q \in Q.$$

Definition 1.2 (O. Kazanci et al [7]) A Q -fuzzy set μ in a near-ring N is called Q -fuzzy N -subgroup of N if μ satisfies (QF1) and

$$(QF3) \mu(nx, q) \geq \mu(x, q)$$

$$(QF4) \mu(xn, q) \geq \mu(x, q) \text{ for all } x, n \in N \text{ and } q \in Q.$$

Definition 1.3 A Q -fuzzy set μ in a near-ring N is called Q -fuzzy ideal of N if μ satisfies (QF1), (QF4) and

$$(QF5) \mu\{n(y+x) - ny, q\} \geq \mu(x, q) \text{ for all } x, y, n \in N \text{ and } q \in Q.$$

Definition 1.4 Let θ be a mapping from X to Y . For any $IFSB = (\alpha_B, \beta_B)$ in Y , we define a new IFS denoted as $\theta^{-1}(B) = (\alpha_{\theta^{-1}(B)}, \beta_{\theta^{-1}(B)})$ in X where $\alpha_{\theta^{-1}(B)}(x, q) = \alpha_B(\theta(x), q)$, $\beta_{\theta^{-1}(B)}(x, q) = \beta_B(\theta(x), q)$ for all $x \in X, q \in Q$. For any $IFSA = (\alpha_A, \beta_A)$ in X we define $\theta(A)$ denoted as $\theta(A) = (\alpha_{\theta(A)}, \beta_{\theta(A)})$ in Y by

$$\alpha_{\theta(A)}(y, q) = \begin{cases} \sup_{x \in \theta^{-1}(y)} \alpha_A(x, q), & \text{if } \theta^{-1}(y) \neq \phi; \\ 0, & \text{otherwise.} \end{cases}$$

$$\beta_{\theta(A)}(y, q) = \begin{cases} \inf_{x \in \theta^{-1}(y)} \beta_A(x, q), & \text{if } \theta^{-1}(y) \neq \phi; \\ 1, & \text{otherwise.} \end{cases}$$

for all $y \in Y, q \in Q$

2 Intuitionistic Q -fuzzy ideals of near-rings:

In what follows, let Q and N denote a set and a near-ring, respectively, unless otherwise specified.

Definition 2.1 (O. Kazanci et al [7]) An $IQFSA = (\alpha_A, \beta_A)$ in N is called an intuitionistic Q -fuzzy subnear-ring of N if

$$(IQF1) \alpha_A(x-y, q) \geq \min\{\alpha_A(x, q), \alpha_A(y, q)\} \text{ and } \beta_A(x-y, q) \leq \max\{\beta_A(x, q), \beta_A(y, q)\}$$

$$(IQF2) \alpha_A(xy, q) \geq \min\{\alpha_A(x, q), \alpha_A(y, q)\} \text{ and } \beta_A(xy, q) \leq \max\{\beta_A(x, q), \beta_A(y, q)\} \\ \text{for all } x, y \in N \text{ and } q \in Q.$$

Definition 2.2 (O. Kazanci et al [7]) An $IQFSA = (\alpha_A, \beta_A)$ in N is called an intuitionistic Q -fuzzy N -subgroup of N if A satisfies (IQF1) and

$$(IQF3) \alpha_A(nx, q) \geq \alpha_A(x, q) \text{ and } \beta_A(nx, q) \leq \beta_A(x, q)$$

$$(IQF4) \alpha_A(xn, q) \geq \alpha_A(x, q) \text{ and } \beta_A(xn, q) \leq \beta_A(x, q) \text{ for all } x, n \in N \text{ and } q \in Q.$$

If A satisfies (IQF1) and (IQF3), then A is called an intuitionistic Q -fuzzy left N -subgroup of N , and if A satisfies (IQF1) and (IQF4), then A is called an intuitionistic Q -fuzzy right N -subgroup of N .

Definition 2.3 An $IQFSA = (\alpha_A, \beta_A)$ in N is called an intuitionistic Q -fuzzy ideal of N if A satisfies (IQF1), (IQF4) and

$$(IQF5) \quad \alpha_A(y + x - y, q) \geq \alpha_A(x, q) \text{ and } \beta_A(y + x - y, q) \leq \beta_A(x, q)$$

$$(IQF6) \quad \alpha_A\{n(x + y) - nx, q\} \geq \alpha_A(x, q) \text{ and } \beta_A\{n(x + y) - nx, q\} \leq \beta_A(x, q) \text{ for all } x, y, n \in N \text{ and } q \in Q.$$

Example 1 Let $N = \{a, b, c, d\}$ be set with two binary operations as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	b
d	a	a	a	b

Then $(N, +, \cdot)$ is a near-ring. Let $Q = \{p, q\}$. We define an $IQFSA = (\alpha_A, \beta_A)$ in N as follows:

$$\alpha_A(a, p) = \frac{3}{4}, \alpha_A(a, q) = 1, \alpha_A(b, p) = \frac{1}{3}, \alpha_A(b, q) = \frac{1}{4},$$

$$\alpha_A(c, p) = \frac{1}{5} = \alpha_A(d, p), \alpha_A(c, q) = 0 = \alpha_A(d, q).$$

$$\beta_A(a, p) = \frac{1}{4}, \beta_A(a, q) = 0, \beta_A(b, p) = \frac{1}{4}, \beta_A(b, q) = \frac{1}{3},$$

$$\beta_A(c, p) = \frac{4}{5} = \beta_A(d, p), \beta_A(c, q) = 1 = \beta_A(d, q).$$

It is easy to verify that $IQFSA = (\alpha_A, \beta_A)$ is an intuitionistic Q -fuzzy ideal of N .

Lemma 2.4 If an $IQFSA = (\alpha_A, \beta_A)$ satisfies the condition (IQF1), then

$$(i) \quad \alpha_A(0, q) \geq \alpha_A(x, q) \text{ and } \beta_A(0, q) \leq \beta_A(x, q)$$

$$(ii) \quad \alpha_A(-x, q) = \alpha_A(x, q) \text{ and } \beta_A(-x, q) = \beta_A(x, q) \text{ for all } x \in N \text{ and } q \in Q.$$

Lemma 2.5 If an $IQFSA = (\alpha_A, \beta_A)$ satisfies the condition IQF5, then $\alpha_A(x + y, q) = \alpha_A(y + x, q)$ and $\beta_A(x + y, q) = \beta_A(y + x, q)$ for all $x, y \in N$ and $q \in Q$.

Proof: It follows by using the condition IQF5 twice.

Proposition 2.6 If an $IQFSA = (\alpha_A, \beta_A)$ satisfies the condition (IQF1), then

$$(i) \quad \alpha_A(x - y, q) \geq \alpha_A(0, q) \text{ implies } \alpha_A(x, q) = \alpha_A(y, q)$$

$$(ii) \quad \beta_A(x - y, q) \leq \beta_A(0, q) \text{ implies } \beta_A(x, q) = \beta_A(y, q) \text{ for all } x, y \in N \text{ and } q \in Q.$$

Proof: It is clear by using lemma 2.4.

Theorem 2.7 Let $\{A_i\} = \{(\alpha_i, \beta_i) \mid i \in I\}$ be a family of intuitionistic Q -fuzzy ideals in a near-ring N . Then $(\bigcap_{i \in I} A_i)$ is an intuitionistic Q -fuzzy ideal of N where $(\bigcap_{i \in I} A_i) = \{(\bigcap_{i \in I} \alpha_{A_i}, \bigcup_{i \in I} \beta_{A_i})\}$.

Proof: Let $(\bigcap_{i \in I} \alpha_{A_i}, \bigcup_{i \in I} \beta_{A_i}) = (\alpha, \beta)$ and for all $x, y, n \in N$, and $q \in Q$

$$\begin{aligned} \alpha(x - y, q) &= \inf_{i \in I} \alpha_{A_i}(x - y, q) \\ &\geq \inf_{i \in I} \{\min\{\alpha_{A_i}(x, q), \alpha_{A_i}(y, q)\}\} \\ &= \min\{\inf_{i \in I} \alpha_{A_i}(x, q), \inf_{A_i \in I} \alpha_{A_i}(y, q)\} \\ &= \min\{\alpha(x, q), \alpha(y, q)\} \end{aligned}$$

and

$$\begin{aligned} \beta(x - y, q) &= \sup_{i \in I} \beta_{A_i}(x - y, q) \\ &\leq \sup_{i \in I} \{\max\{\beta_{A_i}(x, q), \beta_{A_i}(y, q)\}\} \\ &= \max\{\sup_{i \in I} \beta_{A_i}(x, q), \sup_{i \in I} \beta_{A_i}(y, q)\} \\ &= \max\{\beta(x, q), \beta(y, q)\} \end{aligned}$$

Also,

$$\begin{aligned} \alpha(xn, q) &= \inf_{i \in I} \alpha_{A_i}(xn, q) \\ &\geq \inf_{i \in I} \{\alpha_{A_i}(x, q)\} = \alpha(x, q) \end{aligned}$$

and,

$$\begin{aligned} \beta(xn, q) &= \sup_{i \in I} \beta_i(xn, q) \\ &\leq \sup_{i \in I} \beta_i(x, q) \\ &= \beta(x, q) \end{aligned}$$

Again,

$$\begin{aligned} \alpha\{n(x + y) - nx, q\} &= \inf_{i \in I} \alpha_{A_i}\{n(x + y) - nx, q\} \\ &\geq \inf_{i \in I} \{\alpha_{A_i}(y, q)\} \\ &= \alpha(y, q) \end{aligned}$$

and

$$\begin{aligned} \beta\{n(x + y) - nx, q\} &= \sup_{i \in I} \beta_i\{n(x + y) - nx, q\} \\ &\leq \sup_{i \in I} \{\beta_i(y, q)\} \\ &= \beta(y, q) \end{aligned}$$

Thus, $(\bigcap_{i \in I} A_i)$ is an intuitionistic Q -fuzzy ideal of N .

Lemma 2.8 *An IQFSA (α_A, β_A) is an intuitionistic Q -fuzzy ideal of a near-ring N if and only if α_A and β_A are Q -fuzzy ideals of N .*

Proof: It is trivial.

Theorem 2.9 Let $IQFSA = (\alpha_A, \beta_A)$ be in N . Then $A = (\alpha_A, \beta_A)$ is an intuitionistic Q -fuzzy ideal of N if and only if $\diamond A = (\alpha_A, \overline{\alpha_A})$ and $\square A = (\overline{\beta_A}, \beta_A)$ are intuitionistic Q -fuzzy ideals of N .

Proof: It follows from lemma 2.8.

Theorem 2.10 Let $IQFSA = (\alpha_A, \beta_A)$ be an intuitionistic Q -fuzzy ideal of N . Then the sets

$$N_{\alpha_A} = \{x \in N | \alpha_A(x, q) = \alpha_A(0, q)\} \text{ and } N_{\beta_A} = \{x \in N | \beta_A(x, q) = \beta_A(0, q)\}$$

are ideals of N for all $q \in Q$.

Proof: Let $x, y \in N_{\alpha_A}$ and $q \in Q$. Then $\alpha_A(x, q) = \alpha_A(0, q)$, $\alpha_A(y, q) = \alpha_A(0, q)$. Since $A = (\alpha_A, \beta_A)$ is an intuitionistic Q -fuzzy ideal of N , we get $\alpha_A(x - y, q) \geq \min\{\alpha_A(x, q), \alpha_A(y, q)\} = \alpha_A(0, q)$. By using lemma 2.4 we get $\alpha_A(x - y, q) = \alpha_A(0, q)$. Hence $x - y \in N_{\alpha_A}$. For $n \in N, x \in N_{\alpha_A}$ we have $\alpha_A(xn, q) \geq \alpha_A(x, q) = \alpha_A(0, q)$. By using lemma 2.4 we get $\alpha_A(xn, q) = \alpha_A(0, q)$. Hence $xn \in N_{\alpha_A}$. Similarly, we can show that for $y \in N_{\alpha_A}, n, x \in N$ and $n(x + y) - nx \in N_{\alpha_A}$. Therefore, N_{α_A} is an ideal of N . Similarly, N_{β_A} is an ideal of N .

Theorem 2.11 Let $IQFSA = (\alpha_A, \beta_A)$ be an intuitionistic Q -fuzzy ideal of N . Then the sets $U(\alpha_A; t)$ and $L(\beta_A; t)$ are ideals of N for all for all $q \in Q, t \in Im(\alpha_A) \cap Im(\beta_A)$.

Proof: Let $t \in Im(\alpha_A) \cap Im(\beta_A) \subseteq [0, 1]$ and let $x, y \in U(\alpha_A; t)$ then $\alpha_A(x, q) \geq t, \alpha_A(y, q) \geq t$ so, $\alpha_A(x - y, q) \geq \min\{\alpha_A(x, q), \alpha_A(y, q)\} \geq t$ which implies that $x - y \in U(\alpha_A; t)$. For any $n \in N, x \in U(\alpha_A; t)$ then $\alpha_A(xn, q) \geq \alpha_A(x, q) \geq t$ which implies that $xn \in U(\alpha_A; t)$. Let $n, y \in N$ and $x \in U(\alpha_A; t)$ then, $\alpha_A(y + x - y, q) \geq \alpha_A(x, q) \geq t$ and $\alpha_A\{n(y + x) - ny, q\} \geq \alpha_A(x, q) \geq t$ which implies that $y + x - y, n(y + x) - ny \in U(\alpha_A; t)$. Thus, $U(\alpha_A; t)$ is an ideal of N . Similarly we can prove that $L(\beta_A; t)$ is an ideal of N .

Theorem 2.12 If $IQFSA = (\alpha_A, \beta_A)$ is an intuitionistic Q -fuzzy set in N such that all the non-empty level sets $U(\alpha_A; t)$ and $L(\beta_A; t)$ are ideals of N then $A = (\alpha_A, \beta_A)$ is an intuitionistic Q -fuzzy ideal of N .

Prof: Assume that all the non-empty level sets $U(\alpha_A; t)$ and $L(\beta_A; t)$ are ideals of N . If $t_1 = \min\{\alpha_A(x, q), \alpha_A(y, q)\}$ and $t_2 = \max\{\beta_A(x, q), \beta_A(y, q)\}$ for $x, y \in N, q \in Q$ then $x - y \in U(\alpha_A; t_1)$ and $x - y \in L(\beta_A; t_2)$. Hence, $\alpha_A(x - y, q) \geq \min\{\alpha_A(x, q), \alpha_A(y, q)\}$ and $\beta_A(x - y, q) \leq \max\{\beta_A(x, q), \beta_A(y, q)\}$. Let $t_3 = \alpha_A(x, q), t_4 = \beta_A(x, q)$ for some $x \in N, q \in Q$ then $x \in U(\alpha_A; t_3), x \in L(\beta_A; t_4)$. Since $U(\alpha_A; t_3)$ and $L(\beta_A; t_4)$ are ideals of N , $xn \in U(\alpha_A; t_3)$ and $xn \in L(\beta_A; t_4)$ which implies that $\alpha_A(xn, q) \geq \alpha_A(x, q)$ and $\beta_A(xn, q) \leq \beta_A(x, q)$. Also for any $y \in N$ $y + x - y \in U(\alpha_A; t_3)$ and $y + x - y \in L(\beta_A; t_4)$ which implies that $\alpha_A(y + x - y, q) \geq \alpha_A(x, q)$ and $\beta_A(y + x - y, q) \leq \beta_A(x, q)$. Lastly if $n \in N$, then $n(y + x) - ny \in U(\alpha_A; t_3)$ and $n(y + x) - ny \in L(\beta_A; t_4)$ so that $\alpha_A\{n(y + x) - ny, q\} \geq \alpha_A(x, q)$ and $\beta_A\{n(y + x) - ny, q\} \leq \beta_A(x, q)$. Thus,

$A = (\alpha_A, \beta_A)$ is an intuitionistic Q -fuzzy ideal of N .

Example 2: Let I be an ideal of N and α_A and β_A be the Q -fuzzy sets in N given by

$$\alpha_A(x, q) = \begin{cases} t & \text{if } x \in I, \\ s & \text{otherwise.} \end{cases} \quad \beta_A(x, q) = \begin{cases} u & \text{if } x \in I, \\ v & \text{otherwise.} \end{cases}$$

for all $x \in N, q \in Q$ where $0 \leq s < t, 0 \leq u < v$ and $t + u \leq 1, s + v \leq 1$. Then $A = (\alpha_A, \beta_A)$ is an intuitionistic Q -fuzzy ideal of N . Since $U(\alpha_A; t) = I$ or N and $L(\beta_A; v) = I$ or N which are ideals of N , A is an intuitionistic Q -fuzzy ideal of N by theorem 2.12.

Theorem 2.13 *Let N and N' be two near-rings and $\theta : N \rightarrow N'$ a homomorphism. If $B = (\alpha_B, \beta_B)$ is an intuitionistic Q -fuzzy ideal in N' then the preimage $\theta^{-1}(B) = (\alpha_{\theta^{-1}(B)}, \beta_{\theta^{-1}(B)})$ of B under θ is an intuitionistic Q -fuzzy ideal of N .*

Proof: We assume that $B = (\alpha_B, \beta_B)$ is an intuitionistic Q -fuzzy ideal in N' and let $x, y, n \in N, q \in Q$. Then

$$\begin{aligned} \alpha_{\theta^{-1}(B)}\{n(x+y) - nx, q\} &= \alpha_B[\theta\{n(x+y) - nx, q\}] \\ &= \alpha_B[\theta(n)\{\theta(x) + \theta(y)\} - \theta(n)\theta(x), q] \\ &\geq \alpha_B\{\theta(y), q\} \\ &= \alpha_{\theta^{-1}(B)}(y, q) \end{aligned}$$

and,

$$\begin{aligned} \beta_{\theta^{-1}(B)}\{n(x+y) - nx, q\} &= \beta_B[\theta\{n(x+y) - nx, q\}] \\ &= \beta_B[\theta(n)\{\theta(x) + \theta(y)\} - \theta(n)\theta(x), q] \\ &\leq \alpha_B\{\theta(y), q\} \\ &= \beta_{\theta^{-1}(B)}(y, q) \end{aligned}$$

Similarly, the remaining properties can be easily verified.

We can also state the converse of the above theorem if we strengthen the condition of θ as follows.

Theorem 2.14 *Let N and N' be two near-rings and $\theta : N \rightarrow N'$ an epimorphism. If $B = (\alpha_B, \beta_B)$ is an intuitionistic Q -fuzzy set in N' such that the preimage $\theta^{-1}(B) = (\alpha_{\theta^{-1}(B)}, \beta_{\theta^{-1}(B)})$ of B under θ is an intuitionistic Q -fuzzy ideal of N , then $B = (\alpha_B, \beta_B)$ is an intuitionistic Q -fuzzy ideal in N' .*

Proof: Let $x, y, r \in N', q \in Q$. Then there exist $a, b, n \in N$ such that $\theta(a) = x, \theta(b) = y, \theta(n) = r$. We only check the condition $IQF6$ as the other conditions follow trivially.

$$\begin{aligned} \alpha_B\{r(x+y) - rx, q\} &= \alpha_B[\theta(n)\{\theta(a) + \theta(b)\} - \theta(n)\theta(a), q] \\ &= \alpha_B[\theta\{n(a+b) - na\}, q] \\ &= \alpha_{\theta^{-1}(B)}\{n(a+b) - na, q\} \\ &\geq \alpha_{\theta^{-1}(B)}(b, q) \end{aligned}$$

$$\begin{aligned}
&= \alpha_B\{\theta(b), q\} \\
&= \alpha_B(y, q)
\end{aligned}$$

and

$$\begin{aligned}
\beta_B\{r(x+y) - rx, q\} &= \beta_B[\theta(n)\{\theta(a) + \theta(b)\} - \theta(n)\theta(a), q] \\
&= \beta_B[\theta\{n(a+b) - na\}, q] \\
&= \beta_{\theta^{-1}(B)}\{n(a+b) - na, q\} \\
&\leq \beta_{\theta^{-1}(B)}(b, q) \\
&= \beta_B\{\theta(b), q\} \\
&= \beta_B(y, q)
\end{aligned}$$

Therefore $B = (\alpha_B, \beta_B)$ is an intuitionistic Q -fuzzy ideal in N' .

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