19th Int. Conf. on IFSs, Burgas, 4–6 June 2015
Notes on Intuitionistic Fuzzy Sets
ISSN 1310–4926
Vol. 21, 2015, No. 2, 35–42

# Intuitionistic fuzzy implications and Kolmogorov's and Lukasiewisz–Tarski's axioms of logic

**Nora Angelova, Evgeniy Marinov and Krassimir Atanassov**

Dept. of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering
Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria
e-mails: nora.angelova@biomed.bas.bg,
evgeni.marinov@gmail.com, krat@bas.bg

**Abstract:** During years of research, there have been defined intuitionistic fuzzy implications. In the paper, it is checked which of the basic 149 implications satisfy which Kolmogorov's and which Lukasiewisz–Tarski's axioms of logic, whether as (classical) tautologies or as intuitionistic fuzzy tautologies.

**Keywords:** Implication, Intuitionistic fuzzy logic, Kolmogorov's axioms, Lukasiewisz–Tarski's axioms.
**AMS Classification:** 03E72.

## 1 Introduction

In a series of publications, (see, e.g. [1, 2]), a lot of different operations "implication" in intuitionistic fuzzy propositional calculus, have been intordouced. Here, we study two properties of them. They have been found to generate 45 operations "negation" over intuitionistic fuzzy sets.

In the present paper, we study which of these implications satisfy which axioms of Kolmogorov's and which – of Lukasiewisz–Tarski's axioms of logic, and whether, doing so, they behave tautologies or as intuitionistic fuzzy tautologies.

In intuitionistic fuzzy propositional calculus, if $x$ is a variable, then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b \rangle,$$

so that $a, b, a + b \in [0, 1]$, where $a$ and $b$ are degrees of validity and of non-validity of $x$.

For the needs of the discussion below, we shall define the notion of tautology and of Intuitionistic Fuzzy Tautology (IFT, see, e.g. [1, 2]) by:

*$x$ is an IFT if and only if for $V(x) = \langle a, b \rangle$ it holds that $a \geq b$,*

and

*$x$ will be a (classical) tautology if and only if $a = 1$ and $b = 0$.*

Obiously, each tautology is intuitionistic fuzzy tautology, but the opposite is not true.

# 2 On the operations "implication" and "negation" in intuitionistic fuzzy propositional calculus

Below, we shall assume that for the three variables $x$ and $y$ the equalities: $V(x) = \langle a, b \rangle, V(y) = \langle c, d \rangle$ $(a, b, c, d, a + b, c + d \in [0, 1])$ hold.

First, for the two variables $x$ and $y$, we define the operations "conjunction" (&) and "disjunction" ($\vee$) (see, e.g. [1, 2]) by:

$$V(x \& y) = \langle \min(a, c), \max(b, d) \rangle,$$

$$V(x \vee y) = \langle \max(a, c), \min(b, d) \rangle.$$

In some definitions, we need to use the auxiliary functions sg and $\overline{\text{sg}}$ defined by,

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}, \qquad \overline{\text{sg}}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

The list of all currently existing intuitionistic fuzzy implications and negations are given in [1, 6, 7].

# 3 Intutionistic fuzzy implications and Kolmogorov's and Lukasiewisz–Tarski's axioms of logic

Here, we check the validity of Kolmogorov's and Lukasiewisz–Tarski's axioms of logic (see, e.g., [5]). The first group of axioms is

(K1) $A \to (B \to A)$,

(K2) $(A \to (A \to B)) \to (A \to B))$,

(K3) $(A \to (B \to C)) \to (B \to (A \to C))$,

(K4) $(B \to C) \to ((A \to B) \to (A \to C))$,

(K5) $(A \to B) \to ((A \to \neg B) \to \neg A)$.

**Theorem 1.** The intuitionistic fuzzy implications that satisfy Kolmogorov's axioms as (classical) tautologies, are marked in Table 1 by " • " and these that satisfy the same axioms (only) as IFTs – by " ∘ ".

Table 1

| | $K1$ | $K2$ | $K3$ | $K4$ | $K5$ | | $K1$ | $K2$ | $K3$ | $K4$ | $K5$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rightarrow_{1}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{37}$ | | • | | | |
| $\rightarrow_{2}$ | | • | | • | • | $\rightarrow_{38}$ | ∘ | ∘ | ∘ | | |
| $\rightarrow_{3}$ | • | • | • | • | • | $\rightarrow_{39}$ | | | | | |
| $\rightarrow_{4}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{40}$ | | • | | | |
| $\rightarrow_{5}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{41}$ | | | | | |
| $\rightarrow_{6}$ | ∘ | ∘ | ∘ | | | $\rightarrow_{42}$ | • | • | • | | |
| $\rightarrow_{7}$ | ∘ | ∘ | | | ∘ | $\rightarrow_{43}$ | | • | • | | |
| $\rightarrow_{8}$ | | • | | | • | $\rightarrow_{44}$ | ∘ | ∘ | ∘ | | |
| $\rightarrow_{9}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{45}$ | • | • | • | | |
| $\rightarrow_{10}$ | | | | | | $\rightarrow_{46}$ | | ∘ | ∘ | ∘ | ∘ |
| $\rightarrow_{11}$ | • | • | • | • | • | $\rightarrow_{47}$ | | • | | | • |
| $\rightarrow_{12}$ | | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{48}$ | | • | • | • | • |
| $\rightarrow_{13}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{49}$ | | ∘ | • | • | ∘ |
| $\rightarrow_{14}$ | • | • | • | • | • | $\rightarrow_{50}$ | | ∘ | ∘ | ∘ | ∘ |
| $\rightarrow_{15}$ | | • | | | • | $\rightarrow_{51}$ | | ∘ | ∘ | ∘ | ∘ |
| $\rightarrow_{16}$ | | | | | • | $\rightarrow_{52}$ | | • | | | • |
| $\rightarrow_{17}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{53}$ | | ∘ | ∘ | ∘ | ∘ |
| $\rightarrow_{18}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{54}$ | | | | | |
| $\rightarrow_{19}$ | | | | | • | $\rightarrow_{55}$ | | • | | • | • |
| $\rightarrow_{20}$ | • | • | • | • | • | $\rightarrow_{56}$ | | | | | • |
| $\rightarrow_{21}$ | ∘ | ∘ | | ∘ | ∘ | $\rightarrow_{57}$ | | • | • | • | • |
| $\rightarrow_{22}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{58}$ | | | | | |
| $\rightarrow_{23}$ | • | • | • | • | • | $\rightarrow_{59}$ | | | | | |
| $\rightarrow_{24}$ | | ∘ | | • | ∘ | $\rightarrow_{60}$ | | | | | |
| $\rightarrow_{25}$ | | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{61}$ | ∘ | ∘ | ∘ | ∘ | ∘ |
| $\rightarrow_{26}$ | | | | | ∘ | $\rightarrow_{62}$ | | | | ∘ | |
| $\rightarrow_{27}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{63}$ | | | | | |
| $\rightarrow_{28}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{64}$ | ∘ | ∘ | ∘ | | |
| $\rightarrow_{29}$ | ∘ | ∘ | ∘ | ∘ | ∘ | $\rightarrow_{65}$ | | • | | | |
| $\rightarrow_{30}$ | ∘ | ∘ | ∘ | | | $\rightarrow_{66}$ | ∘ | ∘ | ∘ | ∘ | ∘ |
| $\rightarrow_{31}$ | • | • | • | | | $\rightarrow_{67}$ | | | | | |
| $\rightarrow_{32}$ | • | • | • | | | $\rightarrow_{68}$ | | | | | |
| $\rightarrow_{33}$ | ∘ | ∘ | ∘ | | | $\rightarrow_{69}$ | | | | | |
| $\rightarrow_{34}$ | • | | • | | | $\rightarrow_{70}$ | | | | | |
| $\rightarrow_{35}$ | ∘ | ∘ | ∘ | | | $\rightarrow_{71}$ | ∘ | ∘ | ∘ | ∘ | ∘ |
| $\rightarrow_{36}$ | ∘ | ∘ | ∘ | | | $\rightarrow_{72}$ | ∘ | ∘ | ∘ | | |

| | $K1$ | $K2$ | $K3$ | $K4$ | $K5$ |
|---|---|---|---|---|---|
| $\rightarrow_{73}$ | | | | | |
| $\rightarrow_{74}$ | ● | ● | ● | ● | ● |
| $\rightarrow_{75}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{76}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{77}$ | ● | ● | ● | ● | ● |
| $\rightarrow_{78}$ | | | | | |
| $\rightarrow_{79}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{80}$ | ○ | ○ | ○ | | |
| $\rightarrow_{81}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{82}$ | ○ | ○ | ○ | | |
| $\rightarrow_{83}$ | | | | | |
| $\rightarrow_{84}$ | | | | | |
| $\rightarrow_{85}$ | ○ | ○ | ○ | | |
| $\rightarrow_{86}$ | | | | | |
| $\rightarrow_{87}$ | | | | | |
| $\rightarrow_{88}$ | ● | ● | ● | | |
| $\rightarrow_{89}$ | ○ | ○ | ○ | | |
| $\rightarrow_{90}$ | ○ | ● | ● | | |
| $\rightarrow_{91}$ | | ○ | ○ | ○ | ○ |
| $\rightarrow_{92}$ | | | | | |
| $\rightarrow_{93}$ | | | | | |
| $\rightarrow_{94}$ | | ○ | ○ | ○ | ○ |
| $\rightarrow_{95}$ | | | | | |
| $\rightarrow_{96}$ | | | | | |
| $\rightarrow_{97}$ | | ● | ● | ● | ● |
| $\rightarrow_{98}$ | | ○ | | | |
| $\rightarrow_{99}$ | | | | | |
| $\rightarrow_{100}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{101}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{102}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{103}$ | ○ | ○ | ○ | | |
| $\rightarrow_{104}$ | ○ | ○ | ○ | | ○ |
| $\rightarrow_{105}$ | ○ | ○ | ○ | | ○ |
| $\rightarrow_{106}$ | | ○ | ○ | | ○ |
| $\rightarrow_{107}$ | | ○ | ○ | | |
| $\rightarrow_{108}$ | | ○ | | | |
| $\rightarrow_{109}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{110}$ | ○ | ○ | ○ | ○ | ○ |

| | $K1$ | $K2$ | $K3$ | $K4$ | $K5$ |
|---|---|---|---|---|---|
| $\rightarrow_{111}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{112}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{113}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{114}$ | ○ | ○ | ○ | | |
| $\rightarrow_{115}$ | ○ | ○ | ○ | | |
| $\rightarrow_{116}$ | ○ | ○ | ○ | | |
| $\rightarrow_{117}$ | ○ | ○ | ○ | | |
| $\rightarrow_{118}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{119}$ | | ○ | | | ○ |
| $\rightarrow_{120}$ | | ○ | ○ | | |
| $\rightarrow_{121}$ | | ○ | ○ | ○ | ○ |
| $\rightarrow_{122}$ | | ○ | ○ | | |
| $\rightarrow_{123}$ | | | | | |
| $\rightarrow_{124}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{125}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{126}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{127}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{128}$ | ○ | ○ | ○ | ○ | ○ |
| $\rightarrow_{129}$ | ○ | ○ | ○ | | |
| $\rightarrow_{130}$ | ○ | ○ | ○ | | |
| $\rightarrow_{131}$ | ○ | ○ | ○ | | |
| $\rightarrow_{132}$ | ○ | ○ | ○ | | |
| $\rightarrow_{133}$ | ○ | ○ | ○ | ○ | |
| $\rightarrow_{134}$ | | ○ | ○ | ○ | ○ |
| $\rightarrow_{135}$ | | ○ | ○ | ○ | ○ |
| $\rightarrow_{136}$ | | ○ | ○ | ○ | ○ |
| $\rightarrow_{137}$ | | ○ | ○ | ○ | ○ |
| $\rightarrow_{138}$ | | | | | |
| $\rightarrow_{139}$ | | | | | |
| $\rightarrow_{140}$ | | | | | |
| $\rightarrow_{141}$ | | | | | |
| $\rightarrow_{142}$ | | | | | |
| $\rightarrow_{143}$ | | | | | |
| $\rightarrow_{145}$ | | | | | |
| $\rightarrow_{146}$ | | | | | |
| $\rightarrow_{147}$ | | | | | |
| $\rightarrow_{148}$ | | | | | |
| $\rightarrow_{149}$ | | | | | |

The second group of axioms is:

(LT1) $A \supset (B \supset A)$,

(LT2) $(A \supset B) \supset ((B \supset C) \supset (A \supset C))$,

(LT3) $\neg A \supset (\neg B \supset (B \supset A))$,

(LT4) $((A \supset \neg A) \supset A) \supset A$.

**Theorem 2.** The intuitionistic fuzzy implications that satisfy Lukasiewicz–Tarski's axioms as (classical) tautologies, are marked in Table 2 by " $\bullet$ " and these that satisfy the same axioms (only) as IFTs – by " $\circ$ ".

Table 2

| | $LT1$ | $LT2$ | $LT3$ | $LT4$ | | $LT1$ | $LT2$ | $LT3$ | $LT4$ |
|---|---|---|---|---|---|---|---|---|---|
| $\rightarrow_{1}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{29}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ |
| $\rightarrow_{2}$ | | $\bullet$ | $\bullet$ | | $\rightarrow_{30}$ | $\circ$ | | $\circ$ | $\circ$ |
| $\rightarrow_{3}$ | $\bullet$ | $\bullet$ | $\bullet$ | | $\rightarrow_{31}$ | $\bullet$ | | $\bullet$ | |
| $\rightarrow_{4}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{32}$ | $\bullet$ | | $\bullet$ | |
| $\rightarrow_{5}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{33}$ | $\circ$ | | $\circ$ | $\circ$ |
| $\rightarrow_{6}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{34}$ | $\bullet$ | | $\bullet$ | $\circ$ |
| $\rightarrow_{7}$ | $\circ$ | | | $\circ$ | $\rightarrow_{35}$ | $\circ$ | | $\circ$ | $\circ$ |
| $\rightarrow_{8}$ | | | $\bullet$ | | $\rightarrow_{36}$ | $\circ$ | | $\circ$ | $\circ$ |
| $\rightarrow_{9}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{37}$ | | | $\bullet$ | |
| $\rightarrow_{10}$ | | | | | $\rightarrow_{38}$ | $\circ$ | | $\circ$ | $\circ$ |
| $\rightarrow_{11}$ | $\bullet$ | $\bullet$ | $\bullet$ | | $\rightarrow_{39}$ | | | | |
| $\rightarrow_{12}$ | | $\circ$ | $\circ$ | | $\rightarrow_{40}$ | | | $\bullet$ | |
| $\rightarrow_{13}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{41}$ | | | $\bullet$ | |
| $\rightarrow_{14}$ | $\bullet$ | $\bullet$ | $\bullet$ | | $\rightarrow_{42}$ | $\bullet$ | | $\bullet$ | $\bullet$ |
| $\rightarrow_{15}$ | | | $\bullet$ | | $\rightarrow_{43}$ | | | $\bullet$ | |
| $\rightarrow_{16}$ | | | $\bullet$ | | $\rightarrow_{44}$ | $\circ$ | | $\bullet$ | $\circ$ |
| $\rightarrow_{17}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{45}$ | $\bullet$ | | $\bullet$ | $\circ$ |
| $\rightarrow_{18}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{46}$ | | $\circ$ | $\circ$ | |
| $\rightarrow_{19}$ | | | $\bullet$ | | $\rightarrow_{47}$ | | | $\bullet$ | |
| $\rightarrow_{20}$ | $\bullet$ | $\bullet$ | $\bullet$ | $\bullet$ | $\rightarrow_{48}$ | | $\bullet$ | $\bullet$ | |
| $\rightarrow_{21}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{49}$ | | $\bullet$ | $\bullet$ | |
| $\rightarrow_{22}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{50}$ | | $\circ$ | $\circ$ | |
| $\rightarrow_{23}$ | $\bullet$ | $\bullet$ | $\bullet$ | $\bullet$ | $\rightarrow_{51}$ | | $\circ$ | $\circ$ | |
| $\rightarrow_{24}$ | | $\bullet$ | $\bullet$ | | $\rightarrow_{52}$ | | | $\bullet$ | |
| $\rightarrow_{25}$ | | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{53}$ | | $\circ$ | $\circ$ | |
| $\rightarrow_{26}$ | | | $\circ$ | | $\rightarrow_{54}$ | | | | |
| $\rightarrow_{27}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{55}$ | | $\bullet$ | $\bullet$ | |
| $\rightarrow_{28}$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\rightarrow_{56}$ | | | $\bullet$ | |

| | $LT1$ | $LT2$ | $LT3$ | $LT4$ |
|---|---|---|---|---|
| $\rightarrow_{57}$ | | ● | ● | |
| $\rightarrow_{58}$ | | | | |
| $\rightarrow_{59}$ | | | | |
| $\rightarrow_{60}$ | | | | |
| $\rightarrow_{61}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{62}$ | | ○ | ● | |
| $\rightarrow_{63}$ | | | ● | |
| $\rightarrow_{64}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{65}$ | | | ● | |
| $\rightarrow_{66}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{67}$ | | | | |
| $\rightarrow_{68}$ | | | ● | |
| $\rightarrow_{69}$ | | | ● | |
| $\rightarrow_{70}$ | | | | |
| $\rightarrow_{71}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{72}$ | ○ | ○ | | ○ |
| $\rightarrow_{73}$ | | | | |
| $\rightarrow_{74}$ | ● | ● | ● | ● |
| $\rightarrow_{75}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{76}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{77}$ | ● | ● | ● | ● |
| $\rightarrow_{78}$ | | | | |
| $\rightarrow_{79}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{80}$ | ○ | ○ | | ○ |
| $\rightarrow_{81}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{82}$ | ○ | | ○ | ○ |
| $\rightarrow_{83}$ | | | ● | |
| $\rightarrow_{84}$ | | | ● | |
| $\rightarrow_{85}$ | ○ | | ○ | ○ |
| $\rightarrow_{86}$ | | | | |
| $\rightarrow_{87}$ | | | | |
| $\rightarrow_{88}$ | ● | | ● | ● |
| $\rightarrow_{89}$ | ○ | | | ○ |
| $\rightarrow_{90}$ | ○ | | ● | ● |
| $\rightarrow_{91}$ | | ○ | ○ | |
| $\rightarrow_{92}$ | | | | |
| $\rightarrow_{93}$ | | | ● | |
| $\rightarrow_{94}$ | | ○ | ○ | |
| $\rightarrow_{95}$ | | | | |
| $\rightarrow_{96}$ | | | | |

| | $LT1$ | $LT2$ | $LT3$ | $LT4$ |
|---|---|---|---|---|
| $\rightarrow_{97}$ | | ● | ● | |
| $\rightarrow_{98}$ | | | | |
| $\rightarrow_{99}$ | | | | |
| $\rightarrow_{100}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{101}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{102}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{103}$ | ○ | | ○ | ○ |
| $\rightarrow_{104}$ | ○ | | ○ | ○ |
| $\rightarrow_{105}$ | ○ | | ○ | ○ |
| $\rightarrow_{106}$ | | | | |
| $\rightarrow_{107}$ | | | ○ | |
| $\rightarrow_{108}$ | | | | |
| $\rightarrow_{109}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{110}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{111}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{112}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{113}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{114}$ | ○ | | ○ | ○ |
| $\rightarrow_{115}$ | ○ | | ○ | ○ |
| $\rightarrow_{116}$ | ○ | | ○ | ○ |
| $\rightarrow_{117}$ | ○ | | ○ | ○ |
| $\rightarrow_{118}$ | ○ | ○ | ○ | |
| $\rightarrow_{119}$ | | | | |
| $\rightarrow_{120}$ | | ○ | | |
| $\rightarrow_{121}$ | | ○ | ○ | |
| $\rightarrow_{122}$ | | ○ | | |
| $\rightarrow_{123}$ | | | | |
| $\rightarrow_{124}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{125}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{126}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{127}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{128}$ | ○ | ○ | ○ | ○ |
| $\rightarrow_{129}$ | ○ | | ○ | ○ |
| $\rightarrow_{130}$ | ○ | | ○ | ○ |
| $\rightarrow_{131}$ | ○ | | ○ | ○ |
| $\rightarrow_{132}$ | ○ | | ○ | ○ |
| $\rightarrow_{133}$ | ○ | | ○ | ○ |
| $\rightarrow_{134}$ | | ○ | ○ | |
| $\rightarrow_{135}$ | | ○ | ○ | |
| $\rightarrow_{136}$ | | ○ | ○ | |

| | $LT1$ | $LT2$ | $LT3$ | $LT4$ |
|---|---|---|---|---|
| $\rightarrow_{137}$ | | $\circ$ | $\circ$ | |
| $\rightarrow_{138}$ | | | | |
| $\rightarrow_{139}$ | | | $\circ$ | |
| $\rightarrow_{140}$ | | | | |
| $\rightarrow_{141}$ | | | | |
| $\rightarrow_{142}$ | | | | |
| $\rightarrow_{143}$ | | | | |

| | $LT1$ | $LT2$ | $LT3$ | $LT4$ |
|---|---|---|---|---|
| $\rightarrow_{144}$ | | | | |
| $\rightarrow_{145}$ | | | | |
| $\rightarrow_{146}$ | | | | |
| $\rightarrow_{147}$ | | | | |
| $\rightarrow_{148}$ | | | | |
| $\rightarrow_{149}$ | | | | |

The check of the validity of the assertions in Theorems 1 and 2 was made by the software application IFSTool [4, 8], developed as a tool for automatic check of the properties of intuitionistic fuzzy implications and negations. The software has an option – to either check for intuitionistic fuzzy tautologies or only for fuzzy tautologies. For the needs of the present paper, each axiom was tested with all implications and their corresponding negations, and manual backup checks of some of the properties were made, as well.

# 4 Conclusion: Open problems

We finish with the following interesting open problem.

**Open problem:** To determine criteria that show the most suitable axioms that can have real applications.

In a next research, other properties of the implications will be studied.

# Acknowledgements

The authors are thankful for the support provided by the Bulgarian National Science Fund under Grant Ref. No. DFNI-I-02-5 “InterCriteria Analysis: A New Approach to Decision Making”.

# References

[1] Atanassov, K. (2012) *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin.

[2] Atanassov, K. (2014) On Intuitionistic Fuzzy Logics: Results and Problems. In: *Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics, Volume 1: Foundations.* (Atanassov, K., M. Baczynski, J. Drewniak, J. Kacprzyk, M. Krawczak, E. Szmidt, M. Wygralak, S. Zadrozny, eds.), SRI-PAS, Warsaw, pages 23–49.

[3] Atanassov, K. (2015) On a new intuitionistic fuzzy implication. *Proc. of IFSA-EUSFLAT 2015* (accepted).

[4] Dimitrov, D. (2011) IFSTool – Software for intuitionistic fuzzy sets. *Issues in Intuitionistic Fuzzy Sets and Generalized Nets*, 9, 61–69.

[5] Tabakov, M. (1986) *Logics and axiomatics*, Nauka i Izkustvo, Sofia.

[6] Implications over intuitionistic fuzzy sets. *Ifigenia.net*. Available at: http://www.ifigenia.org/wiki/IF_implications

[7] Negations over intuitionistic fuzzy sets. *Ifigenia.net*. Available at: http://www.ifigenia.org/wiki/IF_negations

[8] IFSTool. *Ifigenia.net*. Available at: http://www.ifigenia.org/wiki/IFSTool