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# The most general form of one type of intuitionistic fuzzy modal operators

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## 1 Introduction

Over Intuitionistic Fuzzy Sets (IFSs, see [2]) there have been defined not only operations and relations similar to the ordinary fuzzy set ones, but also operators that cannot be defined in case of ordinary fuzzy sets.

In the present paper we shall discuss a new modal-like type of operator.

## 2 Basic concepts

Let a set $E$ be fixed. An IFS $A$ in $E$ is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function $\pi$ determines the degree of uncertainty.

Obviously, for every ordinary fuzzy set $\pi_A(x) = 0$ for each $x \in E$ and these sets have the form:

$$\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\}.$$

For every two IFSs $A$ and $B$ a lot of relations and operations are defined (see, e.g. [2]), the most important of which are:

$$\begin{array}{lcl}
A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \\
A \supset B & \text{iff} & B \subset A; \\
A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)); \\
\overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}; \\
A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\};
\end{array}$$

$$A \cup B \quad = \quad \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\};$$

We shall define the following operators (see, e.g., [2]):

$$P_{\alpha,\beta}(A) = \{\langle x, \max(\alpha, \mu_A(x)), \min(\beta, \nu_A(x))\rangle | x \in E\}, \text{ where } \alpha + \beta \leq 1;$$
$$Q_{\alpha,\beta}(A) = \{\langle x, \min(\alpha, \mu_A(x)), \max(\beta, \nu_A(x))\rangle | x \in E\}, \text{ where } \alpha + \beta \leq 1;$$

We will describe couples of four types of other modal-like operators, following [1, 2, 3, 4]. We shall start with the first two simplest operators:

$$\boxplus A = \{\langle x, \frac{\mu_A(x)}{2}, \frac{\nu_A(x)+1}{2}\rangle | x \in E\},$$
$$\boxtimes A = \{\langle x, \frac{\mu_A(x)+1}{2}, \frac{\nu_A(x)}{2}\rangle | x \in E\}.$$

Let $\alpha \in [0, 1]$ and let $A$ be an IFS. Then we can define the first extension:

$$\boxplus_\alpha A = \{\langle x, \alpha.\mu_A(x), \alpha.\nu_A(x) + 1 - \alpha\rangle | x \in E\},$$

$$\boxtimes_\alpha A = \{\langle x, \alpha.\mu_A(x) + 1 - \alpha, \alpha.\nu_A(x)\rangle | x \in E\}.$$

The second extension of operators $\boxplus$ and $\boxtimes$ is introduced in [4] by Katerina Dencheva. She extended the last two operators to the forms:

$$\boxplus_{\alpha,\beta} A = \{\langle x, \alpha.\mu_A(x), \alpha.\nu_A(x) + \beta\rangle | x \in E\},$$

$$\boxtimes_{\alpha,\beta} A = \{\langle x, \alpha.\mu_A(x) + \beta, \alpha.\nu_A(x)\rangle | x \in E\},$$

where $\alpha, \beta, \alpha + \beta \in [0, 1]$.

The third extensions of the above operators is described in [3]. They will have the forms:

$$\boxplus_{\alpha,\beta,\gamma} A = \{\langle x, \alpha.\mu_A(x), \beta.\nu_A(x) + \gamma\rangle | x \in E\},$$

$$\boxtimes_{\alpha,\beta,\gamma} A = \{\langle x, \alpha.\mu_A(x) + \gamma, \beta.\nu_A(x)\rangle | x \in E\},$$

where $\alpha, \beta, \gamma \in [0, 1]$ and $\max(\alpha, \beta) + \gamma \leq 1$.

# 3 Main results

A natural extension of both later operators is the operator

$$\boxdot_{\alpha,\beta,\gamma,\delta} A = \{\langle x, \alpha.\mu_A(x) + |ga, \beta.\nu_A(x) + \delta\rangle | x \in E\},$$

where $\alpha, \beta, \gamma, \delta \in [0, 1]$ and $\max(\alpha + \beta) + \gamma + \delta \leq 1$.

Obviously, for every IFS $A$:

$$\boxplus A = \boxdot_{0.5,0.5,0,0.5} A,$$

$$\boxtimes A = \boxdot_{0.5,0.5,0.5,0} A,$$

$$\boxplus_\alpha A = \boxdot_{\alpha,\alpha,0,1-\alpha} A,$$

$$\boxtimes_{\alpha} A = \boxdot_{\alpha,\alpha,1-\alpha,0} A,$$
$$\boxplus_{\alpha,\beta} A = \boxdot_{\alpha,\alpha,0,\beta} A,$$
$$\boxtimes_{\alpha,\beta} A = \boxdot_{\alpha,\alpha,\beta,0} A.$$
$$\boxplus_{\alpha,\beta,\gamma} A = \boxdot_{\alpha,\beta,0,\gamma} A,$$
$$\boxtimes_{\alpha,\beta,\gamma} A = \boxdot_{\alpha,\beta,\gamma,0} A.$$

The following assertions hold for the new operator.

**Theorem 1**: For every IFS $A$ and for every $\alpha, \beta, \gamma, \delta \in [0,1]$ for which $\max(\alpha+\beta)+\gamma+\delta \leq 1$:

**(a)** $\neg \boxdot_{\alpha,\beta,\gamma,\delta} \neg A = \boxdot_{\beta,\alpha,\delta,\gamma} A$,

**(b)** $\boxdot_{\alpha,\beta,\gamma,\delta}(\boxdot_{\varepsilon,\zeta,\eta,\theta} A) = \boxdot_{\alpha\varepsilon,\beta\zeta,\alpha\eta+\gamma,\beta\theta+\delta} A$,

**(c)** $\boxdot_{\alpha,\beta,\gamma,\delta} \Box A \supset \Box \boxdot_{\alpha,\beta,\gamma,\delta} A$,

**(d)** $\boxdot_{\alpha,\beta,\gamma,\delta} \Diamond A \subset \Diamond \boxdot_{\alpha,\beta,\gamma,\delta} A$.

**Theorem 2**: For every two IFSs $A$ and $B$ and for every $\alpha, \beta, \gamma, \delta \in [0,1]$ for which $\max(\alpha+\beta)+\gamma+\delta \leq 1$:

**(a)** $\boxdot_{\alpha,\beta,\gamma,\delta}(A \cap B) = \boxdot_{\alpha,\beta,\gamma,\delta} A \cap \boxdot_{\alpha,\beta,\gamma,\delta} B$,

**(b)** $\boxdot_{\alpha,\beta,\gamma,\delta}(A \cup B) = \boxdot_{\alpha,\beta,\gamma,\delta} A \cup \boxdot_{\alpha,\beta,\gamma,\delta} B$,

**(c)** $\boxdot_{\alpha,\beta,\gamma,\delta}(A@B) = \boxdot_{\alpha,\beta,\gamma,\delta} A @ \boxdot_{\alpha,\beta,\gamma,\delta} B$.

**Theorem 3**: For every IFS $A$ and for every $\alpha, \beta, \gamma, \delta \in [0,1]$ for which $\max(\alpha+\beta)+\gamma+\delta \leq 1$:

**(a)** $\boxdot_{\alpha,\beta,\gamma,\delta} C(A) = C(\boxdot_{\alpha,\beta,\gamma,\delta} A)$,

**(b)** $\boxdot_{\alpha,\beta,\gamma,\delta} I(A) = I(\boxdot_{\alpha,\beta,\gamma,\delta} A)$,

**(c)** $\boxdot_{\alpha,\beta,\gamma,\delta} P_{\varepsilon,\zeta}((A)) = P_{\alpha\varepsilon+\gamma,\beta\zeta+\delta}(\boxdot_{\alpha,\beta,\gamma,\delta} A)$,

**(d)** $\boxdot_{\alpha,\beta,\gamma,\delta} P_{\varepsilon,\zeta}((A)) = Q_{\alpha\varepsilon+\gamma,\beta\zeta+\delta}(\boxdot_{\alpha,\beta,\gamma,\delta} A)$.

# References


[1] Atanassov K., Some operators on intuitionistic fuzzy sets, Proceedings of the First International Conference on Intuitionistic Fuzzy Sets (J. Kacprzyk and K. Atanassov Eds.), Sofia, Oct 18-19, 1997; Notes on Intuitionistic Fuzzy Sets, Vol. 3 (1997), No. 4, 28-33.

[2] Atanassov K., Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, 1999.

[3] Atanassov K. On one type of intuitionistic fuzzy modal operators. Notes on Intuitionistic Fuzzy Sets, Vol. 11, 2005, No. 5, 24-28.

[4] Dencheva K. Extension of intuitionistic fuzzy modal operators $\boxplus$ and $\boxtimes$. Proceedings of the Second Int. IEEE Symposium: Intelligent Systems, Varna, June 22-24, 2004, Vol. 3, 21-22.