**Notes on Intuitionistic Fuzzy Sets**
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# Elliptic intuitionistic fuzzy Bonferroni mean operator and its application in MCDM

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**Abstract:** The representation of elliptic intuitionistic fuzzy sets employs an elliptic framework to depict uncertainty across membership and non-membership functions, effectively capturing the inherent vagueness in these degrees. In this research article, an innovative approach is explored to tackle the critical challenge of personnel selection in today's fiercely competitive markets. Introducing a novel elliptic intuitionistic fuzzy multi criteria decision making (MCDM) framework, the paper incorporates the Bonferroni mean operator to overcome the limitations of traditional averaging methods. This elliptic intuitionistic fuzzy MCDM approach provides a comprehensive representation of collective opinions within an elliptic framework. Leveraging elliptic intuitionistic fuzzy sets proves effective in managing the uncertainties associated with personnel selection.

**Keywords:** Intuitionistic fuzzy set, Elliptic intuitionistic fuzzy set, Bonferroni mean operator, Elliptic intuitionistic fuzzy Bonferroni mean operator.
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# 1 Introduction

The foundation of fuzzy sets [33] was laid by Zadeh in 1965. Atanassov's seminal 1986 work in fuzzy sets and systems expanded this concept by introducing Intuitionistic Fuzzy Sets (IFSs) [2], where elements are characterized by degrees of membership and non-membership. In 1989, Atanassov further advanced the field with Interval-Valued Intuitionistic Fuzzy Sets (IVIFS) [5]. Later, in 2020, he proposed Circular Intuitionistic Fuzzy Sets (CIFS) [3] as a geometric representation of IFSs. These circles provide a visual interpretation, demonstrating that within a margin of error, a precise pair is enclosed by the "correct" pair defined by membership and non-membership.

Building on the hesitant intuitionistic fuzzy sets introduced by Beg and Rashid [6], as well as Chen, Li, Qian, and Hu [11], Atanassov later introduced Elliptic Intuitionistic Fuzzy Sets (EIFSs) [4], extending the geometric concept from circles to ellipses. This progression highlights the continuous refinement and expansion of fuzzy set theory.

The Bonferroni Mean Operator (BMO), introduced by Bonferroni [7], has been widely utilized in fuzzy logic and its extensions across various applications. In decision-making scenarios where uncertainty and vagueness play a significant role, IFSs can be used to model the imprecision in the information. The BMO can be applied to combine different pieces of information or criteria, taking into account their respective degrees of importance or reliability [12,18,19,27]. In medical diagnosis, where uncertainty and imprecision are common, IFSs can be employed to represent the uncertainty associated with symptoms and test results. The BMO can then be used to aggregate information from different medical tests or diagnostic criteria, considering the varying degrees of confidence associated with each [15,21,22,32]. In risk assessment applications, IFSs can be used to model uncertainties related to different risk factors, and the BMO can assist in combining risk assessments from various sources while considering the importance or credibility of each source [8, 14, 24]. In pattern recognition, IFSs can be utilized to handle uncertainty in feature values or classification results. The BMO can be employed to aggregate information from different features or classifiers, considering their individual reliability. In situations involving multiple criteria for decision-making, IFSs can be applied to represent the imprecision in evaluating each criterion. The BMO can then be used to combine the evaluations, accounting for the varying degrees of confidence or importance attached to each criterion [13, 26, 31].

The key idea is that IFSs allow for a more nuanced representation of uncertainty, hesitation, and imprecision, while the BMO provides a mechanism to combine such information in a systematic way, considering the associated degrees of confidence or reliability. The specific application will depend on the context and requirements of the problem at hand.

The following motivates the introduction of an Elliptic Intuitionistic Fuzzy Bonferroni Mean (E-IFBM) operator. In [36], the efficacy of search and rescue robots during emergencies is of paramount importance, particularly when employing MCDM methodologies with intuitionistic fuzzy logic and the Bonferroni operator. Four decision-makers assessed four alternatives against four criteria, yielding an overall performance score derived from the average of their evaluations. Similarly, [34] investigated the application of MCDM in system analysis engineering, where three decision-makers evaluated four alternatives across five criteria, culminating in an overall performance score computed from the averaged evaluations. Additionally, [35] addressed a supplier selection quandary, wherein four decision-makers scrutinized five alternatives based on five criteria, leading to an overall performance score calculated from the averaged assessments. In this paper, we introduce the concept of an E-IFBM operator for representing a decision-maker's choices geometrically, akin to an ellipse within the framework of IFSs. The EIFS approach offers a solution to the limitation of simply averaging the decision values of the decision-maker, as discussed in [1,9,10,12,16–19,21,24,25,28–30,34].

The following contributions have been made to achieve the objectives of the study:

- Introduction of E-IFBM operator: Within the domain of MCDM, this paper delves into the E-IFBMO operator, employing score function to illustrate evaluation values of alternatives across various criteria. Integration of this operator into decision analysis methodologies advances theoretical comprehension and practical applications in decision-making processes.

- Practical benefits of EIFS in decision effectiveness: Through a numerical case study, the practical utility and effectiveness of EIFS and the E-IFBM operator are demonstrated in decision-making contexts. The study showcases how EIFS surpasses traditional fuzzy sets by enabling intuitive data interpretation and facilitating visualization of intricate decision landscapes, thereby enhancing decision effectiveness.

# 2 Preliminary concepts

**Definition 2.1.** [7] Let $\rho, \sigma \geq 0$, and $\alpha_i$ $(i = 1, 2, 3, \ldots, \psi)$ be a collection of non-negative numbers, If

$$R^{\rho,\sigma}(\alpha_1, \alpha_2, \ldots, \alpha_\psi) = \left( \frac{1}{\psi(\psi-1)} \sum_{\substack{i,j=1 \\ i\neq j}}^{\psi} \alpha_i^{\rho} \alpha_j^{\sigma} \right)^{\frac{1}{\rho+\sigma}}$$

then $R^{\rho,\sigma}(\alpha_1, \alpha_2, \ldots, \alpha_\psi)$ is called Bonferroni mean.

**Definition 2.2.** [2] Let $E$ be a fixed universe and $A \subseteq E$ be a subset. An Elliptic Intuitionistic Fuzzy Set (EIFS) $A_{\lambda,\gamma}$ is defined as: $A_{\lambda,\gamma} = \{\langle x, \mu_A(x), \nu_A(x), \lambda, \gamma\rangle \mid x \in E\}$ , where:

- $\mu_A : E \to [0, 1]$ is the degree of membership of $x \in E$, $\nu_A : E \to [0, 1]$ is the degree of non-membership of $x \in E$, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in E$, we can define the degree of indeterminacy (uncertainty) $\pi_A(x)$ as: $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

- The center $\mu(x) = \frac{a(x)+c(x)}{2}$, $\nu(x) = \frac{b(x)+d(x)}{2}$, and $\lambda, \gamma \in [0, 1]$ are the semi-major and semi-minor axes

$$\lambda(x)=\sqrt{a(x)^2 + \left(\frac{c(x)-a(x)}{d(x)-b(x)}\right)^2 b(x)^2}, \quad \gamma(x)=\sqrt{b(x)^2 + \left(\frac{d(x)-b(x)}{c(x)-a(x)}\right)^2 a(x)^2}$$

of the ellipse associated with $x \in E$, where $a(x)=\min_{1\leq i\leq k_x} \mu(x_i)$, $b(x)=\min_{1\leq i\leq k_x} \nu(x_i)$, $c(x)=\max_{1\leq i\leq k_x} \mu(x_i)$, and $d(x)=\max_{1\leq i\leq k_x} \nu(x_i)$.

**Definition 2.3.** [23] Let $\theta_i = \langle \mu_{\theta_i}, \nu_{\theta_i}, \lambda_{\theta_i}, \gamma_{\theta_i} \rangle$ $(i = 1, 2, 3, \ldots, \psi)$ be EIFSs over the universal set $E$, and let $\alpha > 0$. We define the following operations:

1. $\theta_1 \oplus \theta_2 = \langle \mu_{\theta_1} + \mu_{\theta_2} - \mu_{\theta_1}\mu_{\theta_2}, \nu_{\theta_1}\nu_{\theta_2}, \lambda_{\theta_1} + \lambda_{\theta_2} - \lambda_{\theta_1}\lambda_{\theta_2}, \gamma_{\theta_1} + \gamma_{\theta_2} - \gamma_{\theta_1}\gamma_{\theta_2} \rangle$.
2. $\bigoplus_{i=1}^{\psi} \theta_i = \left\langle 1 - \prod_{i=1}^{\psi}(1 - \mu_{\theta_i}), \prod_{i=1}^{\psi} \nu_{\theta_i}, 1 - \prod_{i=1}^{\psi}(1 - \lambda_{\theta_i}), 1 - \prod_{i=1}^{\psi}(1 - \gamma_{\theta_i}) \right\rangle$.
3. $\theta_1 \otimes \theta_2 = \langle \mu_{\theta_1}\mu_{\theta_2}, \nu_{\theta_1} + \nu_{\theta_2} - \nu_{\theta_1}\nu_{\theta_2}, \lambda_{\theta_1}\lambda_{\theta_2}, \gamma_{\theta_1}\gamma_{\theta_2} \rangle$.
4. $\bigotimes_{i=1}^{\psi} \theta_i = \left\langle \prod_{i=1}^{\psi} \mu_{\theta_i}, 1 - \prod_{i=1}^{\psi}(1 - \nu_{\theta_i}), \prod_{i=1}^{\psi} \lambda_{\theta_i}, \prod_{i=1}^{\psi} \gamma_{\theta_i} \right\rangle$.
5. $\alpha\theta_1 = \left\langle 1 - (1 - \mu_{\theta_1})^{\alpha}, \nu_{\theta_1}^{\alpha}, 1 - (1 - \lambda_{\theta_1})^{\alpha}, 1 - (1 - \gamma_{\theta_1})^{\alpha} \right\rangle$.
6. ${\theta_1}^{\alpha} = \left\langle \mu_{\theta_1}^{\alpha}, 1 - (1 - (\nu_{\theta_1}))^{\alpha}, \lambda_{\theta_1}^{\alpha}, \gamma_{\theta_1}^{\alpha} \right\rangle$.

**Example 2.4.** Let $A = \{\langle 0.4, 0.05\rangle, \langle 0.2, 0.1\rangle, \langle 0.1, 0.15\rangle\}$ be the collection of IFSs. To construct EIFS, we determine the following values:

$$a(x) = \min_{1 \le i \le k_x} \mu(x_i) = \min\{0.4, 0.2, 0.1\} = 0.1,$$
$$b(x) = \min_{1 \le i \le k_x} \nu(x_i) = \min\{0.05, 0.1, 0.15\} = 0.05,$$
$$c(x) = \max_{1 \le i \le k_x} \mu(x_i) = \max\{0.4, 0.2, 0.1\} = 0.4,$$
$$d(x) = \max_{1 \le i \le k_x} \nu(x_i) = \max\{0.05, 0.1, 0.15\} = 0.15.$$
$$\mu(x) = \frac{a(x) + c(x)}{2} = \frac{0.1 + 0.4}{2} = 0.25, \quad \nu(x) = \frac{b(x) + d(x)}{2} = \frac{0.05 + 0.15}{2} = 0.1.$$
$$e(x) = \frac{d(x) - b(x)}{c(x) - a(x)} = \frac{0.15 - 0.05}{0.4 - 0.1} = 0.33 < 1.$$
$$\lambda(x) = \sqrt{a(x)^2 + \left(\frac{c(x) - a(x)}{d(x) - b(x)}\right)^2 b(x)^2} = \sqrt{(0.1)^2 + \left(\frac{0.4 - 0.1}{0.15 - 0.05}\right)^2 (0.05)^2} = 0.18,$$
$$\gamma(x) = \sqrt{b(x)^2 + \left(\frac{d(x) - b(x)}{c(x) - a(x)}\right)^2 a(x)^2} = \sqrt{(0.05)^2 + \left(\frac{0.15 - 0.05}{0.4 - 0.1}\right)^2 (0.1)^2} = 0.06.$$

Then the EIFS is $A_{\lambda,\gamma} = \langle \mu(x), \nu(x), \lambda(x), \gamma(x) \rangle = \langle 0.25, 0.1; 0.18, 0.06 \rangle$

# 3 Elliptic intuitionistic fuzzy Bonferroni mean operator

In this section, we introduce elliptic intuitionistic fuzzy Bonferroni mean operator via elliptic intuitionistic fuzzy sets.

**Definition 3.1.** Let $r_\epsilon = \langle \mu_{r_\epsilon}, \nu_{r_\epsilon}, \lambda_{r_\epsilon}, \gamma_{r_\epsilon} \rangle$ here $\epsilon = 1, 2, \ldots, \psi$ is a collection of EIFSs. For any $\rho, \sigma > 0$, if

$$E - IFBM^{\rho,\sigma}(r_1, r_2, \ldots, r_\psi) = \left( \frac{1}{\psi(\psi - 1)} \bigoplus_{\substack{\epsilon,\delta=1 \\ \epsilon \ne \delta}}^{\psi} (r_\epsilon^{\rho} \otimes r_\delta^{\sigma}) \right)^{\frac{1}{\rho+\sigma}}, \tag{1}$$

then $E - IFBM^{\rho,\sigma}(r_1, r_2, \ldots, r_\psi)$ is called the elliptic intuitionistic fuzzy Bonferroni mean operator.

**Theorem 3.2.** *Consider* $\rho, \sigma > 0$ *and* $r_\epsilon = \langle \mu_{r_\epsilon}, \nu_{r_\epsilon}, \lambda_{r_\epsilon}, \gamma_{r_\epsilon} \rangle$ *here* $\epsilon = 1, 2, \ldots, \psi$ *is a collection of elliptic intuitionistic fuzzy numbers. Then the aggregation through elliptic intuitionistic fuzzy Bonferroni mean operator is again an elliptic intuitionistic fuzzy number and*

$$E\text{-}IFBM^{\rho,\sigma}(r_1, r_2, \ldots, r_\psi) = \Bigg\langle \left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - \mu_{r_\epsilon}^{\rho} \mu_{r_\delta}^{\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}}, 1 - \left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - (1-\nu_{r_\epsilon})^{\rho}(1-\nu_{r_\delta})^{\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}},$$
$$\left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - \lambda_{r_\epsilon}^{\rho} \lambda_{r_\delta}^{\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}}, \left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - \gamma_{r_\epsilon}^{\rho} \gamma_{r_\delta}^{\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}} \Bigg\rangle \tag{2}$$

Proof: From the basic operations, we get $r_\epsilon^\rho = \langle \mu_{r_\epsilon}^\rho, 1 - (1-\nu_{r_\epsilon})^\rho, \lambda_{r_\epsilon}^\rho, \gamma_{r_\epsilon}^\rho \rangle$, and $r_\epsilon^\sigma = \langle \mu_{r_\delta}^\sigma, 1-(1-\nu_{r_\delta})^\sigma, \lambda_{r_\delta}^\sigma, \gamma_{r_\delta}^\sigma \rangle$. Then $r_\epsilon^\rho \otimes r_\delta^\sigma = \langle \mu_{r_\epsilon}^\rho \mu_{r_\delta}^\sigma, 1-(1-\nu_{r_\epsilon})^\rho(1-\nu_{r_\delta})^\sigma, \lambda_{r_\epsilon}^\rho \lambda_{r_\delta}^\sigma, \gamma_{r_\epsilon}^\rho \gamma_{r_\delta}^\sigma \rangle$. We need to prove that

$$\bigoplus_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (r_\epsilon^\rho \otimes r_\delta^\sigma) = \Bigg\langle 1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - \mu_{r_\epsilon}^\rho \mu_{r_\delta}^\sigma), \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - (1-\nu_{r_\epsilon})^\rho (1-\nu_{r_\delta})^\sigma),$$
$$1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - \lambda_{r_\epsilon}^\rho \lambda_{r_\delta}^\sigma), 1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - \gamma_{r_\epsilon}^\rho \gamma_{r_\delta}^\sigma) \Bigg\rangle \tag{3}$$

By mathematical induction principle on $\psi$, for $\psi = 2$, we get

$$\bigoplus_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (r_\epsilon^\rho \otimes r_\delta^\sigma) = (r_1^\rho \otimes r_2^\sigma) \oplus (r_2^\rho \otimes r_1^\sigma)$$
$$(r_1^\rho \otimes r_2^\sigma) \oplus (r_2^\rho \otimes r_1^\sigma) = \big\langle 1 - (1 - \mu_{r_1}^\rho \mu_{r_2}^\sigma)(1 - \mu_{r_2}^\rho \mu_{r_1}^\sigma), (1 - (1-\nu_{r_1})^\rho (1-\nu_{r_2})^\sigma) \times$$
$$(1 - (1-\nu_{r_2})^\rho (1-\nu_{r_1})^\sigma), 1 - (1 - \lambda_{r_1}^\rho \lambda_{r_2}^\sigma)(1 - \lambda_{r_2}^\rho \lambda_{r_1}^\sigma),$$
$$1 - (1 - \gamma_{r_1}^\rho \gamma_{r_2}^\sigma)(1 - \gamma_{r_2}^\rho \gamma_{r_1}^\sigma) \big\rangle \tag{4}$$

Assume that equation (2) holds for $\psi = \omega$, that is,

$$\bigoplus_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\omega} (r_\epsilon^\rho \otimes r_\delta^\sigma) = \Bigg\langle 1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\omega} (1 - \mu_{r_\epsilon}^\rho \mu_{r_\delta}^\sigma), \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\omega} (1 - (1-\nu_{r_\epsilon})^\rho (1-\nu_{r_\delta})^\sigma),$$
$$1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\omega} (1 - \lambda_{r_\epsilon}^\rho \lambda_{r_\delta}^\sigma), 1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\omega} (1 - \gamma_{r_\epsilon}^\rho \gamma_{r_\delta}^\sigma) \Bigg\rangle \tag{5}$$

Now let $\psi = \omega + 1$, we have

$$\bigoplus_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega+1}(r_\epsilon^\rho \otimes r_\delta^\sigma) = \left(\bigoplus_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega}(r_\epsilon^\rho \otimes r_\delta^\sigma)\right) \oplus \left(\bigoplus_{\epsilon=1}^{\omega}(r_\epsilon^\rho \otimes r_{\omega+1}^\sigma)\right) \oplus \left(\bigoplus_{\delta=1}^{\omega}(r_{\omega+1}^\rho \otimes r_\delta^\sigma)\right) \tag{6}$$

We claim that

$$\begin{aligned}\bigoplus_{\epsilon=1}^{\omega}(r_\epsilon^\rho \otimes r_{\omega+1}^\sigma) = \Bigg\langle 1 - \prod_{\epsilon=1}^{\omega}(1-\mu_{r_\epsilon}^\rho \mu_{r_{\omega+1}}^\sigma), \prod_{\epsilon=1}^{\omega}(1-(1-\nu_{r_\epsilon})^\rho(1-\nu_{r_{\omega+1}})^\sigma),\\ 1-\prod_{\epsilon=1}^{\omega}(1-\lambda_{r_\epsilon}^\rho \lambda_{r_{\omega+1}}^\sigma), 1-\prod_{\epsilon=1}^{\omega}(1-\gamma_{r_\epsilon}^\rho \gamma_{r_{\omega+1}}^\sigma)\Bigg\rangle\end{aligned} \tag{7}$$

Let us prove the result (7) by mathematical induction principle, for $\omega = 2$,
we get

$$r_\epsilon^\rho \otimes r_{2+1}^\sigma = \langle \mu_{r_\epsilon}^\rho \mu_{r_{2+1}}^\sigma, 1-(1-\nu_{r_\epsilon})^\rho(1-\nu_{r_{2+1}})^\sigma, \lambda_{r_\epsilon}^\rho \lambda_{r_{2+1}}^\sigma, \gamma_{r_\epsilon}^\rho \gamma_{r_{2+1}}^\sigma\rangle \tag{8}$$

and

$$\begin{aligned}\bigoplus_{\epsilon=1}^{2}(r_\epsilon^\rho \otimes r_{2+1}^\sigma) = \Bigg\langle 1 - \prod_{\epsilon=1}^{2}(1-\mu_{r_\epsilon}^\rho \mu_{r_3}^\sigma), \prod_{\epsilon=1}^{2}(1-(1-\nu_{r_\epsilon})^\rho(1-\nu_{r_3})^\sigma),\\ 1-\prod_{\epsilon=1}^{2}(1-\lambda_{r_\epsilon}^\rho \lambda_{r_3}^\sigma), 1-\prod_{\epsilon=1}^{2}(1-\gamma_{r_\epsilon}^\rho \gamma_{r_3}^\sigma)\Bigg\rangle\end{aligned} \tag{9}$$

Assume that the result (7) holds for $\omega = \omega_0$, that is,

$$\begin{aligned}\bigoplus_{\epsilon=1}^{\omega_0}(r_\epsilon^\rho \otimes r_{\omega_0+1}^\sigma) = \Bigg\langle 1 - \prod_{\epsilon=1}^{\omega_0}(1-\mu_{r_\epsilon}^\rho \mu_{r_{\omega_0+1}}^\sigma), \prod_{\epsilon=1}^{\omega_0}(1-(1-\nu_{r_\epsilon})^\rho(1-\nu_{r_{\omega_0+1}})^\sigma),\\ 1-\prod_{\epsilon=1}^{\omega_0}(1-\lambda_{r_\epsilon}^\rho \lambda_{r_{\omega_0+1}}^\sigma), 1-\prod_{\epsilon=1}^{\omega_0}(1-\gamma_{r_\epsilon}^\rho \gamma_{r_{\omega_0+1}}^\sigma)\Bigg\rangle\end{aligned} \tag{10}$$

When $\omega = \omega_0 + 1$, we get

$$\bigoplus_{\epsilon=1}^{\omega_0+1}(r_\epsilon^\rho \otimes r_{\omega_0+2}^\sigma) = \left(\bigoplus_{\epsilon=1}^{\omega_0}(r_\epsilon^\rho \otimes r_{\omega_0+2}^\sigma)\right) \oplus \left(r_{\omega_0+1}^\rho \otimes r_{\omega_0+2}^\sigma\right) \tag{11}$$

$$\begin{aligned}\bigoplus_{\epsilon=1}^{\omega_0}(r_\epsilon^\rho \otimes r_{\omega_0+2}^\sigma) \oplus (r_{\omega_0+1}^\rho \otimes r_{\omega_0+2}^\sigma) = \Bigg\langle 1 - \prod_{\epsilon=1}^{\omega_0+1}(1-\mu_{r_\epsilon}^\rho \mu_{r_{\omega_0+2}}^\sigma),\\ \prod_{\epsilon=1}^{\omega_0+1}(1-(1-\nu_{r_\epsilon})^\rho(1-\nu_{r_{\omega_0+2}})^\sigma),\\ 1-\prod_{\epsilon=1}^{\omega_0+1}(1-\lambda_{r_\epsilon}^\rho \lambda_{r_{\omega_0+2}}^\sigma), 1-\prod_{\epsilon=1}^{\omega_0+1}(1-\gamma_{r_\epsilon}^\rho \gamma_{r_{\omega_0+2}}^\sigma)\Bigg\rangle\end{aligned} \tag{12}$$

Similarly,

$$\bigoplus_{\delta=1}^{\omega}(r_{\omega+1}^{\rho}\otimes r_{\delta}^{\sigma})=\Bigg\langle 1-\prod_{\epsilon=1}^{\omega}(1-\mu_{r_{\omega+1}}^{\rho}\mu_{r_{\delta}}^{\sigma}),\prod_{\epsilon=1}^{\omega}(1-(1-\nu_{r_{\omega+1}})^{\rho}(1-\nu_{r_{\delta}})^{\sigma}),\\ 1-\prod_{\epsilon=1}^{\omega}(1-\lambda_{r_{\omega+1}}^{\rho}\lambda_{r_{\delta}}^{\sigma}),1-\prod_{\epsilon=1}^{\omega}(1-\gamma_{r_{\omega+1}}^{\rho}\gamma_{r_{\delta}}^{\sigma})\Bigg\rangle \tag{13}$$

Now, we get

$$\begin{aligned}\bigoplus_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega+1}(r_{\epsilon}^{\rho}\otimes r_{\delta}^{\sigma})=&\Bigg\langle 1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega}(1-\mu_{r_{\epsilon}}^{\rho}\mu_{r_{\delta}}^{\sigma}),\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega}(1-(1-\nu_{r_{\epsilon}})^{\rho}(1-\nu_{r_{\delta}})^{\sigma}),\\ &1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega}(1-\lambda_{r_{\epsilon}}^{\rho}\lambda_{r_{\delta}}^{\sigma}),1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega}(1-\gamma_{r_{\epsilon}}^{\rho}\gamma_{r_{\delta}}^{\sigma})\Bigg\rangle\oplus\\ &\Bigg\langle 1-\prod_{\epsilon=1}^{\omega}(1-\mu_{r_{\epsilon}}^{\rho}\mu_{r_{\omega+1}}^{\sigma}),\prod_{\epsilon=1}^{\omega}(1-(1-\nu_{r_{\epsilon}})^{\rho}(1-\nu_{r_{\omega+1}})^{\sigma}),\\ &1-\prod_{\epsilon=1}^{\omega}(1-\lambda_{r_{\epsilon}}^{\rho}\lambda_{r_{\omega+1}}^{\sigma}),1-\prod_{\epsilon=1}^{\omega}(1-\gamma_{r_{\epsilon}}^{\rho}\gamma_{r_{\omega+1}}^{\sigma})\Bigg\rangle\oplus\\ &\Bigg\langle 1-\prod_{\delta=1}^{\omega}(1-\mu_{r_{\omega+1}}^{\rho}\mu_{r_{\delta}}^{\sigma}),\prod_{\delta=1}^{\omega}(1-(1-\nu_{r_{\omega+1}})^{\rho}(1-\nu_{r_{\delta}})^{\sigma}),\\ &1-\prod_{\delta=1}^{\omega}(1-\lambda_{r_{\omega+1}}^{\rho}\lambda_{r_{\delta}}^{\sigma}),1-\prod_{\delta=1}^{\omega}(1-\gamma_{r_{\omega+1}}^{\rho}\gamma_{r_{\delta}}^{\sigma})\Bigg\rangle\\ =&\Bigg\langle 1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega+1}(1-\mu_{r_{\epsilon}}^{\rho}\mu_{r_{\delta}}^{\sigma}),\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega+1}(1-(1-\nu_{r_{\epsilon}})^{\rho}(1-\nu_{r_{\delta}})^{\sigma}),\\ &1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega+1}(1-\lambda_{r_{\epsilon}}^{\rho}\lambda_{r_{\delta}}^{\sigma}),1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\omega+1}(1-\gamma_{r_{\epsilon}}^{\rho}\gamma_{r_{\delta}}^{\sigma})\Bigg\rangle\end{aligned} \tag{14}$$

Therefore the result hold for $\psi=\omega+1$, by mathematical induction result (7) holds for all $\psi$. Now,

$$\begin{aligned}&\frac{1}{\psi(\psi-1)}\bigoplus_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(r_{\epsilon}^{\rho}\otimes r_{\delta}^{\sigma})\\ &=\Bigg\langle 1-\left(\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-\mu_{r_{\epsilon}}^{\rho}\mu_{r_{\delta}}^{\sigma})\right)^{\frac{1}{\psi(\psi-1)}},\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-(1-\nu_{r_{\epsilon}})^{\rho}(1-\nu_{r_{\delta}})^{\sigma})^{\frac{1}{\psi(\psi-1)}},\\ &\quad 1-\left(\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-\lambda_{r_{\epsilon}}^{\rho}\lambda_{r_{\delta}}^{\sigma})\right)^{\frac{1}{\psi(\psi-1)}},1-\left(\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-\gamma_{r_{\epsilon}}^{\rho}\gamma_{r_{\delta}}^{\sigma})\right)^{\frac{1}{\psi(\psi-1)}}\Bigg\rangle\end{aligned} \tag{15}$$

$$E-IFBM^{\rho,\sigma}(r_1,r_2,\ldots,r_\psi)$$
$$= \left\langle \left(1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-\mu_{r_\epsilon}^{\rho}\mu_{r_\delta}^{\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}}, 1-\left(1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-(1-\nu_{r_\epsilon})^{\rho}(1-\nu_{r_\delta})^{\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}}, \right.$$
$$\left. \left(1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-\lambda_{r_\epsilon}^{\rho}\lambda_{r_\delta}^{\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}}, \left(1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-\gamma_{r_\epsilon}^{\rho}\gamma_{r_\delta}^{\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}} \right\rangle \tag{16}$$

**Definition 3.3.** Let $r_\epsilon = \langle \mu_{r_\epsilon}, \nu_{r_\epsilon}, \lambda_{r_\epsilon}, \gamma_{r_\epsilon}\rangle$ here $\epsilon = 1,2,\ldots,\psi$ is a collection of EIFSs. For any $\rho,\sigma,w > 0$ and $\sum_{\epsilon=1}^{\psi} w_\epsilon = 1$, if

$$E-IFWBM^{\rho,\sigma}(r_1,r_2,\ldots,r_\psi) = \left(\frac{1}{\psi(\psi-1)}\bigoplus_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}((w_\epsilon r_\epsilon)^{\rho}\otimes(w_\delta r_\delta)^{\sigma})\right)^{\frac{1}{\rho+\sigma}} \tag{17}$$

then $E-IFWBM^{\rho,\sigma}(r_1,r_2,\ldots,r_\psi)$ is called the elliptic intuitionistic fuzzy weighted Bonferroni mean operator.

Similar to Theorem 3.3, we have Theorem 3.4.

**Theorem 3.4.** *Consider $\rho,\sigma > 0$ and $r_\epsilon = \langle \mu_{r_\epsilon}, \nu_{r_\epsilon}, \lambda_{r_\epsilon}, \gamma_{r_\epsilon}\rangle$ here $\epsilon = 1,2,\ldots,\psi$ is a collection of elliptic intuitionistic fuzzy numbers, whose weight vector is $w = (w_1,w_2,\ldots,w_n)^T$, which satisfies $w_i > 0 (i = 1,2,\ldots,n)$ and $\sum_{\epsilon=1}^{\psi} w_\epsilon = 1$, Then the aggregation through elliptic intuitionistic fuzzy weighted Bonferroni mean operator is again an elliptic intuitionistic fuzzy number and*

$$E\text{-}IFWBM^{\rho,\sigma}(r_1,r_2,\ldots,r_\psi)$$
$$= \left\langle \left(1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-\mu_{r_\epsilon}^{w_\epsilon\rho}\mu_{r_\delta}^{w_\delta\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}}, 1-\left(1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-(1-\nu_{r_\epsilon}^{w_\epsilon})^{\rho}(1-\nu_{r_\delta}^{w_\delta})^{\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}}, \right.$$
$$\left. \left(1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-\lambda_{r_\epsilon}^{w_\epsilon\rho}\lambda_{r_\delta}^{w_\delta\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}}, \left(1-\prod_{\substack{\epsilon,\delta=1\\ \epsilon\neq\delta}}^{\psi}(1-\gamma_{r_\epsilon}^{w_\epsilon\rho}\gamma_{r_\delta}^{w_\delta\sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}} \right\rangle \tag{18}$$

# 4 Numerical example

In the realm of software development, a company's triumph hinges on its technical team's prowess. The system analysis engineer, a pivotal role, spearheads intricate software system design and implementation. As this role is pivotal, the company's quest to fill it is paramount. Employing MCDM methods, the company meticulously evaluates candidates, considering technical skills and interpersonal qualities. A seasoned committee utilizes MCDM to ensure an objective assessment, leading to the selection of the most adept candidate. This paper delves into MCDM's significance in hiring system analysis engineers, exploring its rationale, criteria and role in enhancing transparency and efficacy in decision-making for contemporary software companies.

We now employ the proposed approach to determine the optimal system analysis engineer using the following MCDM approach.

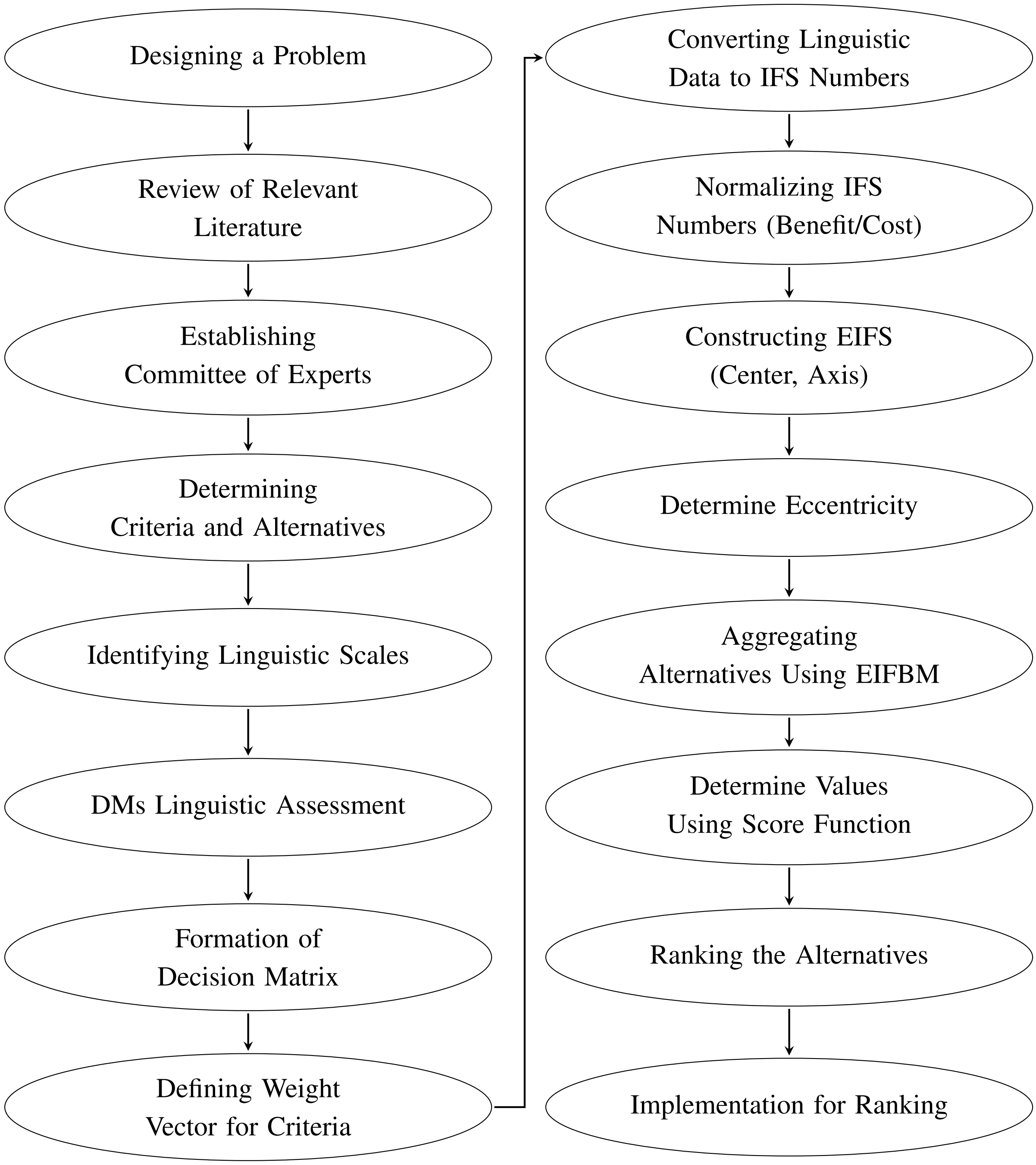


Figure 1. MCDM approach to determining the optimal system analysis engineer

**Algorithm 1 Algorithm for integrating MCDM Techniques using EIFSs**

1: Start

2: Input: To select the best alternative.

3: Identify and define key evaluation criteria $\{C_i\}$ $(i = 1, 2, \ldots, n)$ relevant to the decision-making problem.

4: Choose decision makers $\{R^i\}$ $(i = 1, 2, \ldots, n)$ based on expertise, impartiality, diversity and knowledge of the decision context. Ensure decision makers are familiar with the evaluation criteria and procedures.

5: Each decision maker $\{R^i\}$ $(i = 1, 2, \ldots, n)$ independently assesses alternatives $\{x_i\}$ $(i = 1, 2, \ldots, n)$ based on predefined criteria $\{C_i\}$ $(i = 1, 2, \ldots, n)$.

6: Use linguistic terms or IFS scales to express preferences or performance for each criterion $\{C_i\}$ $(i = 1, 2, \ldots, n)$ and alternative $\{x_i\}$ $(i = 1, 2, \ldots, n)$. Convert linguistic terms to IFS value representations if used.

7: Convert decision makers' IFSs into EIFSs by calculating the center $(\mu(x), \nu(x))$, eccentricity $e(x)$ minor axis and major axis $(\lambda(x), \gamma(x))$ to represent each evaluation as an EIFS $\mathbb{A}^*_{\lambda,\gamma}$, where

$$a(x) = \min_{1 \le i \le k_x} \mu(x_i), \quad b(x) = \min_{1 \le i \le k_x} \nu(x_i), \quad c(x) = \max_{1 \le i \le k_x} \mu(x_i), \quad d(x) = \max_{1 \le i \le k_x} \nu(x_i),$$

$$\mu(x) = \frac{a(x) + c(x)}{2}, \quad \nu(x) = \frac{b(x) + d(x)}{2}, \quad e(x) = \frac{d(x) - b(x)}{c(x) - a(x)},$$

$$\lambda(x) = \sqrt{a(x)^2 + \left(\frac{c(x) - a(x)}{d(x) - b(x)}\right)^2 b(x)^2}, \quad \gamma(x) = \sqrt{b(x)^2 + \left(\frac{d(x) - b(x)}{c(x) - a(x)}\right)^2 a(x)^2}.$$

8: Aggregate EIFS scores for each alternative using the E-IFWBM Operator.

$$E - IFWBM^{\rho,\sigma}(r_1, r_2, \ldots, r_\psi) = \left( \frac{1}{\psi(\psi - 1)} \bigoplus_{\substack{\epsilon,\delta=1 \\ \epsilon \neq \delta}}^{\psi} (w_\epsilon r_\epsilon^\rho \otimes w_\delta r_\delta^\sigma) \right)^{\frac{1}{\rho+\sigma}}$$

9: Calculate score function

$$S = \frac{\mu - \nu + \lambda + \gamma}{3}, S \in [-1, 1]$$

10: Analyze the scores to rank alternatives based on performance or suitability.

11: Output: The best alternative.

12: End.

This algorithm provides a systematic approach to integrating MCDM techniques using EIFS, promoting transparency, objectivity and informed decision-making.

In MCDM, the decision maker's involvement plays a crucial role in determining the weights and preferences associated with different criteria. It has been suggested that the decision maker's influence should be greater than 1 to enable the creation of an elliptic representation in EIFS. This requirement reflects the need for a significant level of involvement to ensure the meaningful representation of preferences and uncertainties. Understanding the limitations of EIFS is essential for their effective utilization in MCDM. To ensure a valid elliptic representation in EIFS, it is essential for the decision maker to maintain $e(x) < 1$.

**Step 1.** The numerical example is adapted from [34]. The software company is in the process of hiring a system analysis engineer and after initial screening, four candidates (alternatives) have been shortlisted: $x_1$, $x_2$, $x_3$, and $x_4$. To aid in the selection process, a committee consisting of three decision makers, namely $R^1$, $R^2$ and $R^3$, has been assembled. The committee will conduct interviews and evaluate the candidates based on five criteria obtained from [9]: emotional steadiness (C1), oral communication skills (C2), personality (C3), past experience (C4) and self-confidence (C5). These criteria will serve as the basis for determining the most suitable candidate for the role. Three Decision markers namely $R^1$, $R^2$ and $R^3$, evaluated four candidates (alternatives) $x_1$, $x_2$, $x_3$, and $x_4$ with five criteria $C1, C2, C3, C4, \&C5$ using the following Table 1 linguistic variables.

| Linguistic variables | Abbreviation | IFNs $(\mu, \nu)$ |
|---|---|---|
| Extremely Poor | EP | (0.05, 0.95) |
| Very Poor | VP | (0.15, 0.80) |
| Poor | P | (0.25, 0.65) |
| Medium Poor | MP | (0.35, 0.55) |
| Medium | M | (0.50, 0.40) |
| Medium Good | MG | (0.65, 0.25) |
| Good | G | (0.75, 0.15) |
| Very Good | VG | (0.85, 0.10) |
| Extremely Good | EG | (0.95, 0.05) |

Table 1. Comparative analysis of ranking

The results of decision makers decision in linguistic variables are given in the following matrix.

$$\begin{bmatrix} \textit{DM} & \textit{Candidate} & C1 & C2 & C3 & C4 & C5 \\ & x_1 & VG & MG & G & EG & EG \\ & x_2 & MG & G & VG & G & G \\ R^1 & x_3 & M & VG & VG & VG & VG \\ & x_4 & M & MG & M & G & G \\ \\ & x_1 & G & MG & G & G & VG \\ R^2 & x_2 & M & G & G & VG & G \\ & x_3 & M & MG & MP & M & M \\ & x_4 & G & MG & MG & M & G \\ \\ & x_1 & G & G & VG & EG & VG \\ R^3 & x_2 & MG & VG & VG & MG & G \\ & x_3 & G & G & MP & VG & MG \\ & x_4 & VG & G & M & VG & MG \end{bmatrix}$$

**Step 2.** Construct the intuitionistic fuzzy decision matrices of each decision maker. Converting the linguistic evaluation shown in the above matrix into IFNs by using Table 1. Then, the intuitionistic fuzzy decision matrices $R^i (i = 1, 2, 3)$ of each decision maker are formed.

| *DM* | *Candidate* | $C1$ | $C2$ | $C3$ | $C4$ | $C5$ |
|---|---|---|---|---|---|---|
| | $x_1$ | $(0.85, 0.10)$ | $(0.65, 0.25)$ | $(0.75, 0.15)$ | $(0.95, 0.05)$ | $(0.95, 0.05)$ |
| | $x_2$ | $(0.65, 0.25)$ | $(0.75, 0.15)$ | $(0.85, 0.10)$ | $(0.75, 0.15)$ | $(0.75, 0.15)$ |
| $R^1$ | $x_3$ | $(0.50, 0.40)$ | $(0.85, 0.10)$ | $(0.85, 0.10)$ | $(0.85, 0.10)$ | $(0.85, 0.10)$ |
| | $x_4$ | $(0.50, 0.40)$ | $(0.65, 0.25)$ | $(0.50, 0.40)$ | $(0.75, 0.15)$ | $(0.75, 0.15)$ |
| | $x_1$ | $(0.75, 0.15)$ | $(0.65, 0.25)$ | $(0.75, 0.15)$ | $(0.75, 0.15)$ | $(0.85, 0.10)$ |
| | $x_2$ | $(0.50, 0.40)$ | $(0.75, 0.15)$ | $(0.75, 0.15)$ | $(0.85, 0.10)$ | $(0.75, 0.15)$ |
| $R^2$ | $x_3$ | $(0.50, 0.40)$ | $(0.65, 0.25)$ | $(0.35, 0.55)$ | $(0.50, 0.40)$ | $(0.50, 0.40)$ |
| | $x_4$ | $(0.75, 0.15)$ | $(0.65, 0.25)$ | $(0.65, 0.25)$ | $(0.50, 0.40)$ | $(0.75, 0.15)$ |
| | $x_1$ | $(0.75, 0.15)$ | $(0.75, 0.15)$ | $(0.85, 0.10)$ | $(0.95, 0.05)$ | $(0.85, 0.10)$ |
| $R^3$ | $x_2$ | $(0.65, 0.25)$ | $(0.85, 0.10)$ | $(0.85, 0.10)$ | $(0.65, 0.25)$ | $(0.75, 0.15)$ |
| | $x_3$ | $(0.75, 0.15)$ | $(0.75, 0.15)$ | $(0.35, 0.55)$ | $(0.85, 0.10)$ | $(0.65, 0.25)$ |
| | $x_4$ | $(0.85, 0.10)$ | $(0.75, 0.15)$ | $(0.50, 0.40)$ | $(0.85, 0.10)$ | $(0.65, 0.25)$ |

**Step 3.** Convert decision makers' IFSs into EIFSs by calculating the center $(\mu(x), \nu(x))$, eccentricity $e(x)$ minor axis and major axis $(\lambda(x), \gamma(x))$ to represent each evaluation as an EIFS $\mathbb{A}^*_{\lambda,\gamma}$. Where

$$a(x) = \min_{1\leq i\leq k_x} \mu(x_i) = \min\{0.85, 0.75, 0.75\} = 0.75$$

$$b(x) = \min_{1\leq i\leq k_x} \nu(x_i) = \min\{0.10, 0.15, 0.15\} = 0.10$$

$$c(x) = \max_{1\leq i\leq k_x} \mu(x_i) = \max\{0.85, 0.75, 0.75\} = 0.85$$

$$d(x) = \max_{1\leq i\leq k_x} \nu(x_i) = \max\{0.10, 0.15, 0.15\} = 0.15$$

$$\mu(x) = \frac{a(x) + c(x)}{2} = \frac{0.75 + 0.85}{2} = 0.80$$

$$\nu(x) = \frac{b(x) + d(x)}{2} = \frac{0.10 + 0.15}{2} = 0.13$$

$$e(x) = \frac{d(x) - b(x)}{c(x) - a(x)} = \frac{0.15 - 0.10}{0.85 - 0.75} = 0.50$$

$$\lambda(x) = \sqrt{a(x)^2 + \left(\frac{c(x) - a(x)}{d(x) - b(x)}\right)^2 b(x)^2} = \sqrt{(0.75)^2 + \left(\frac{0.85 - 0.75}{0.15 - 0.10}\right)^2 (0.10)^2} = 0.78.$$

$$\gamma(x) = \sqrt{b(x)^2 + \left(\frac{d(x) - b(x)}{c(x) - a(x)}\right)^2 a(x)^2} = \sqrt{(0.10)^2 + \left(\frac{0.15 - 0.10}{0.85 - 0.75}\right)^2 (0.75)^2} = 0.39.$$

Then the $E\text{-}IFS$ is $\mathbb{A}^*_{\lambda,\gamma} = \langle \mu(x), \nu(x), \lambda(x), \gamma(x) \rangle = \langle 0.80, 0.13, 0.78, 0.39 \rangle$

The EIFS for the given matrix is

$$\begin{bmatrix} \textit{Candidate} & C1 & C2 & C3 \\ x_1 & (0.80, 0.13, 0.78, 0.39) & (0.70, 0.20, 0.67, 0.67) & (0.80, 0.13, 0.78, 0.39) \\ x_2 & (0.58, 0.33, 0.56, 0.56) & (0.80, 0.13, 0.78, 0.39) & (0.80, 0.13, 0.78, 0.39) \\ x_3 & (0.63, 0.28, 0.52, 0.52) & (0.75, 0.18, 0.66, 0.50) & (0.60, 0.33, 0.37, 0.33) \\ x_4 & (0.68, 0.25, 0.51, 0.44) & (0.70, 0.20, 0.67, 0.67) & (0.58, 0.33, 0.56, 0.56) \\ \\ & C4 & C5 & \\ x_1 & (0.85, 0.10, 0.76, 0.38) & (0.90, 0.08, 0.86, 0.43) & \\ x_2 & (0.75, 0.18, 0.66, 0.50) & (0.77, 0.15, 0.76, 0.76) & \\ x_3 & (0.68, 0.25, 0.51, 0.44) & (0.68, 0.25, 0.51, 0.44) & \\ x_4 & (0.68, 0.25, 0.51, 0.44) & (0.70, 0.20, 0.67, 0.67) & \end{bmatrix}$$

**Step 4.** We use Theorem 3.4, to aggregate the elliptic intuitionistic fuzzy decision matrix.

$$E\text{-}IFWBM^{\rho,\sigma}(x_1, x_2, \ldots, x_\psi) =$$

$$\Bigg\langle \left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - \mu_{x_\epsilon}^{w_\epsilon \rho} \mu_{x_\delta}^{w_\epsilon \sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}}, 1 - \left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - (1 - \nu_{x_\epsilon}^{w_\epsilon})^\rho (1 - \nu_{x_\delta}^{w_\delta})^\sigma)^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}},$$

$$\left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - \lambda_{x_\epsilon}^{w_\epsilon \rho} \lambda_{x_\delta}^{w\ \sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}}, \left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi} (1 - \gamma_{x_\epsilon}^{w_\epsilon \rho} \gamma_{x_\delta}^{w\ \sigma})^{\frac{1}{\psi(\psi-1)}}\right)^{\frac{1}{\rho+\sigma}} \Bigg\rangle$$

when $\rho = \sigma = 1,$ with weights $w_1 = 0.0342; w_2 = 0.1687; w_3 = 0.1088; w_4 = 0.3561; w_5 = 0.3322$

$$E\text{-}IFWBM^{\rho=1,\sigma=1}(x_1, x_2, x_3, x_4) =$$

$$\Bigg\langle \left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi=4} (1 - \mu_{x_\epsilon}^{w_\epsilon} \mu_{x_\delta}^{w_\delta})^{\frac{1}{12}}\right)^{\frac{1}{2}}, 1 - \left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi=4} (1 - (1 - \nu_{x_\epsilon}^{w_\epsilon})(1 - \nu_{x_\delta}^{w_\delta}))^{\frac{1}{12}}\right)^{\frac{1}{2}},$$

$$\left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi=4} (1 - \lambda_{x_\epsilon}^{w_\epsilon} \lambda_{x_\delta}^{w_\delta})^{\frac{1}{12}}\right)^{\frac{1}{2}}, \left(1 - \prod_{\substack{\epsilon,\delta=1 \\ \epsilon\neq\delta}}^{\psi=4} (1 - \gamma_{x_\epsilon}^{w_\epsilon} \gamma_{x_\delta}^{w_\delta})^{\frac{1}{12}}\right)^{\frac{1}{2}} \Bigg\rangle$$

$$\begin{aligned} \mu_{x_1} = \Big(1 - \big((1 - (0.80^{0.0342} \times 0.70^{0.1687})) \times (1 - (0.80^{0.0342} \times 0.80^{0.1088})) \times \\ (1 - (0.80^{0.0342} \times 0.85^{0.3561})) \times (1 - (0.80^{0.0342} \times 0.90^{0.3322})) \times \\ (1 - (0.70^{0.1687} \times 0.80^{0.1088})) \times (1 - (0.70^{0.1687} \times 0.85^{0.3561})) \times \\ (1 - (0.70^{0.1687} \times 0.90^{0.3322})) \times (1 - (0.80^{0.1088} \times 0.85^{0.3561})) \times \\ (1 - (0.80^{0.1088} \times 0.90^{0.3322})) \times (1 - (0.85^{0.3561} \times 0.90^{0.3322}))\big)^{\frac{1}{12}}\Big)^{\frac{1}{2}} \\ = 0.946 \end{aligned}$$

$$\nu_{x_1} = 1-\Big(1-\Big(1-\big((1-0.13^{0.0342})\times(1-0.20^{0.1687})\big)\times(1-(1-0.13^{0.0342})\times(1-0.13^{0.1088}))\times$$
$$(1-(1-0.13^{0.0342})\times(1-0.10^{0.3561}))\times(1-(1-0.13^{0.0342})\times(1-0.08^{0.3322}))\times$$
$$(1-(1-0.20^{0.1687})\times(1-0.13^{0.1088}))\times(1-(1-0.20^{0.1687})\times(1-0.10^{0.3561}))\times$$
$$(1-(1-0.20^{0.1687})\times(1-0.08^{0.3322}))\times(1-(1-0.13^{0.1088})\times(1-0.10^{0.3561}))\times$$
$$(1-(1-0.13^{0.1088})\times(1-0.08^{0.3322}))\times(1-(1-0.10^{0.3561})\times(1-0.08^{0.3322}))\Big)^{\frac{1}{12}}\Big)^{\frac{1}{2}}$$
$$= 0.706$$
$$\lambda_{x_1} = \Big(1-\big((1-(0.78^{0.0342}\times 0.67^{0.1687}))\times(1-(0.78^{0.0342}\times 0.78^{0.1088}))\times$$
$$(1-(0.78^{0.0342}\times 0.76^{0.3561}))\times(1-(0.78^{0.0342}\times 0.86^{0.3322}))\times$$
$$(1-(0.67^{0.1687}\times 0.78^{0.1088}))\times(1-(0.67^{0.1687}\times 0.76^{0.3561}))\times$$
$$(1-(0.67^{0.1687}\times 0.86^{0.3322}))\times(1-(0.78^{0.1088}\times 0.76^{0.3561}))\times$$
$$(1-(0.78^{0.1088}\times 0.86^{0.3322}))\times(1-(0.86^{0.3561}\times 0.86^{0.3322}))\big)^{\frac{1}{12}}\Big)^{\frac{1}{2}}$$
$$= 0.931$$
$$\gamma_{x_1} = \Big(1-\big((1-(0.39^{0.0342}\times 0.70^{0.1687}))\times(1-(0.39^{0.0342}\times 0.80^{0.1088}))\times$$
$$(1-(0.39^{0.0342}\times 0.67^{0.3561}))\times(1-(0.39^{0.0342}\times 0.39^{0.3322}))\times$$
$$(1-(0.67^{0.1687}\times 0.38^{0.1088}))\times(1-(0.67^{0.1687}\times 0.43^{0.3561}))\times$$
$$(1-(0.67^{0.1687}\times 0.90^{0.3322}))\times(1-(0.39^{0.1088}\times 0.38^{0.3561}))\times$$
$$(1-(0.39^{0.1088}\times 0.43^{0.3322}))\times(1-(0.38^{0.3561}\times 0.43^{0.3322}))\big)^{\frac{1}{12}}\Big)^{\frac{1}{2}}$$
$$= 0.829$$

The corresponding aggregated values is given by

$$\begin{bmatrix} x & \mu & \nu & \lambda & \gamma \\ x_1 & 0.946 & 0.706 & 0.931 & 0.829 \\ x_2 & 0.927 & 0.737 & 0.917 & 0.857 \\ x_3 & 0.902 & 0.793 & 0.852 & 0.823 \\ x_4 & 0.901 & 0.788 & 0.878 & 0.870 \end{bmatrix}$$

**Step 5.** Calculate score function

$$S = \frac{\mu - \nu + \lambda + \gamma}{3}; S \in [-1, 1]$$

now the scores of aggregated values is given by

$$\begin{bmatrix} x_1 = 0.667 & x_2 = 0.655 & x_3 = 0.595 & x_4 = 0.620 \end{bmatrix}$$

Here $x_1 > x_2 > x_4 > x_3$. Hence $x_1$ is the most suitable candidate.

# 5 Comparative analysis and limitations

We contrasted our approach with that of Zhang et. al. to further verify the method's efficacy. Table 1 shows that the ranking results achieved by Zhang *et al.* [34], Hwang and Yoon [17], Zhang and Xu [35] technique, Roopadevi *et al.* [23] approaches and proposed method are identical. $x_1$ is still the appropriate candidate.

Table 2. Comparative analysis of ranking

| **Method** | **Ranking** | **Appropriate candidate** |
|---|---|---|
| Shi-fang Zhang *et. al.* [34] | $x_1 > x_2 > x_3 > x_4$ | $x_1$ |
| Hwang and Yoon [17] | $x_1 > x_2 > x_4 > x_3$ | $x_1$ |
| Zhang and Xu [35] | $x_1 > x_2 > x_3 > x_4$ | $x_1$ |
| Roopadevi *et. al.* [23] | $x_1 > x_2 > x_4 > x_3$ | $x_1$ |
| Roopadevi *et. al.* [23] | $x_1 > x_2 > x_4 > x_3$ | $x_1$ |
| **Proposed Method** | $x_1 > x_2 > x_4 > x_3$ | $x_1$ |

To verify accuracy, we assess and present the combined values through a chart (Figure 2).

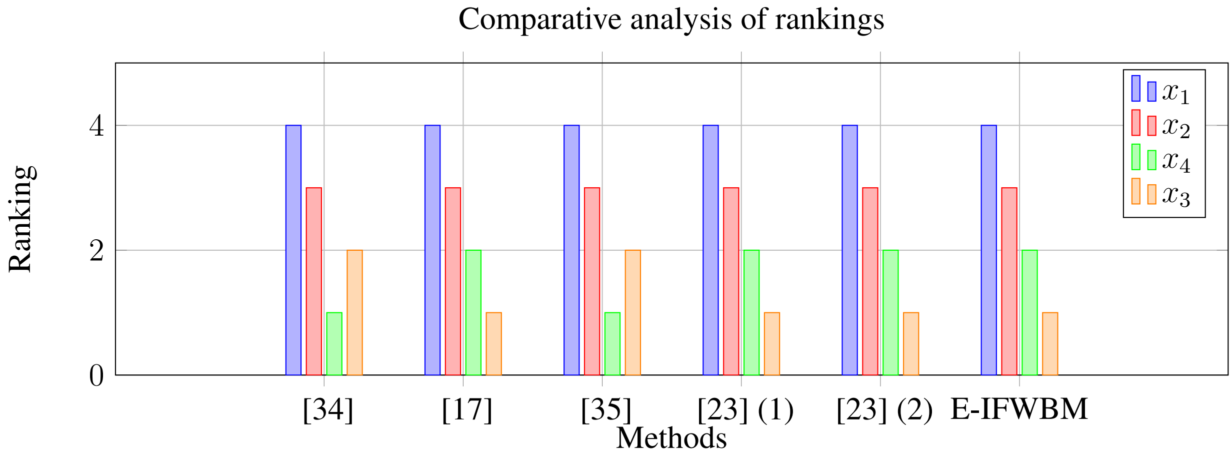


Figure 2. Comparative analysis of rankings for different methods

We utilize several values for the parameters of the $E - IFBM^{\rho,\sigma}$ operator to show how the parameters $\psi$ and $\eta$ influence the case. Table 3 displays the outcomes of the ranking. Table 3 illustrates how the ranking of the hyper parameter optimization and system analysis engineer selection using various values of parameters $\rho$ and $\sigma$ in the aggregation process differs slightly, but $x_1$ is the best system analysis engineer for all combinations of parameters.

Table 3. Ranking results of different parameters

| **Methods** | **Ranking** | **Appropriate candidate** |
|---|---|---|
| $E - IFBM^{\rho=1,\sigma=1}$ | $x_1 > x_2 > x_3 > x_4$ | $x_1$ |
| $E - IFBM^{\rho=1,\sigma=2}$ | $x_1 > x_2 > x_4 > x_3$ | $x_1$ |
| $E - IFBM^{\rho=2,\sigma=2}$ | $x_1 > x_2 > x_4 > x_3$ | $x_1$ |
| $E - IFBM^{\rho=2,\sigma=1}$ | $x_1 > x_2 > x_3 > x_4$ | $x_1$ |
| $E - IFBM^{\rho=2,\sigma=3}$ | $x_1 > x_2 > x_4 > x_3$ | $x_1$ |
| $E - IFBM^{\rho=3,\sigma=2}$ | $x_1 > x_2 > x_4 > x_3$ | $x_1$ |

## Limitations

One significant limitation of EIFS is related to the requirement for the eccentricity ($e(x)$) of the elliptic representation. According to the principles of geometry, the eccentricity should be less than 1 to form a valid ellipse. If $e(x)$ exceeds 1, the geometric representation will resemble a hyperbola and if it equals 1, it will resemble a parabola. In the context of MCDM, this limitation imposes constraints on the decision maker's involvement.

# 6 Conclusion

This research article introduces an innovative approach to address the critical challenge of personnel selection in today's competitive markets. Leveraging the concept of EIFS and the BMO within a MCDM framework, the study provides a comprehensive solution for decision-makers. The introduction of the E-IFBM operator offers a geometric representation that enhances decision-making processes by capturing uncertainties inherent in decision values. Through practical demonstrations and a numerical case study, the effectiveness of EIFS and the E-IFBM operator is showcased in enhancing decision effectiveness and transparency. The study emphasizes the significance of MCDM methodologies in personnel selection, particularly in roles critical to organizational success like system analysis engineering. By addressing the limitations of traditional methods and offering a robust framework for handling uncertainty, this research opens avenues for future applications across various decision-making domains, promising improved outcomes in project evaluation, supplier selection and other management contexts.

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